AMERICAN SOCIETY OF CIVIL ENGINEERS FOUNDED 1852

PROCEEDINGS
of the
American Society
of
Civil Engineers

2 PARTS PART 2

Vol. LIV DECEMBER, 1928 No. 10

Surveying in Texas Years Ago

Paul McCombs

UPON request, Mr. Paul McCombs, now seventy-one years old, and resident in Texas, prepared for the Texas Local Section of the Society a memorandum of some of his early railroad experiences.

Several incidents taken from the extended narrative are not at all technical but were none the less a part of the day's work.

"On my lower division", Mr. McCombs writes, "the contractors made their camp below Beaumont on what was called 'round mound', and which afterward, during the oil boom, acquired the name of 'Spindle Top'. At the base of this mound was a coffee-berry swamp or pond, at which the teams were watered. Along the edge of this pond there constantly arose bubbles of what we thought to be marsh gas; and it amused some of us to touch a match to these bubbles and see them burst into flames.

"One of the mule skinners, who had worked in the oil fields of Ohio, said he believed those bubbles came from crude oil down in the ground, and we all laughed him to scorn. It turned out afterward that this same fellow, many years later, came back with a partner and an old well drilling machine, and drilled a hole in Spindle Top, producing a gas well that was the inducement to the oil development at Beaumont."

"The mosquitoes were so bad that summer that it was necessary to wear gloves and netting over our hats; and to work toward the south, against the prevailing wind. Old McGee used to say 'Begorry, them mosquitoes is as big as a sand hill crane, and he has a brick under his wing to sharpen his bill on.' Mike Kelly, another section man, declared that after a very successful day's hunting he returned with a fine bunch of 'jack snipe' only to be told they were 'gallinippers'."

(*Continued on page 4*)

A Thoughtful Act

ABOUT 3 years ago the Board of Direction adopted a plan whereby members desiring to do so might make a lump-sum payment which would provide for all future dues.

Richard L. Humphrey was the first person to avail himself of the plan and, singularly enough, of those who did so, was the first to die. The plan appealed to him of itself, but he saw in it too an opportunity to do a nice thing for a fellow member.

He collected sums of a dollar or so each from a number of the friends of a certain revered member of the Society and purchased for him the paid-up plan. The dues of this member are thus paid for his life, but Mr. Humphrey planned to make the presentation to him of this mark of esteem a formal occasion and to leave with him an illumined memento containing the signatures of the donors. Mr. Humphrey's death undoubtedly will prevent the carrying out of this part of the program. The will must be taken for the deed and yet the deed has been done although not with the delicacy which Mr. Humphrey planned.

Annual Meeting

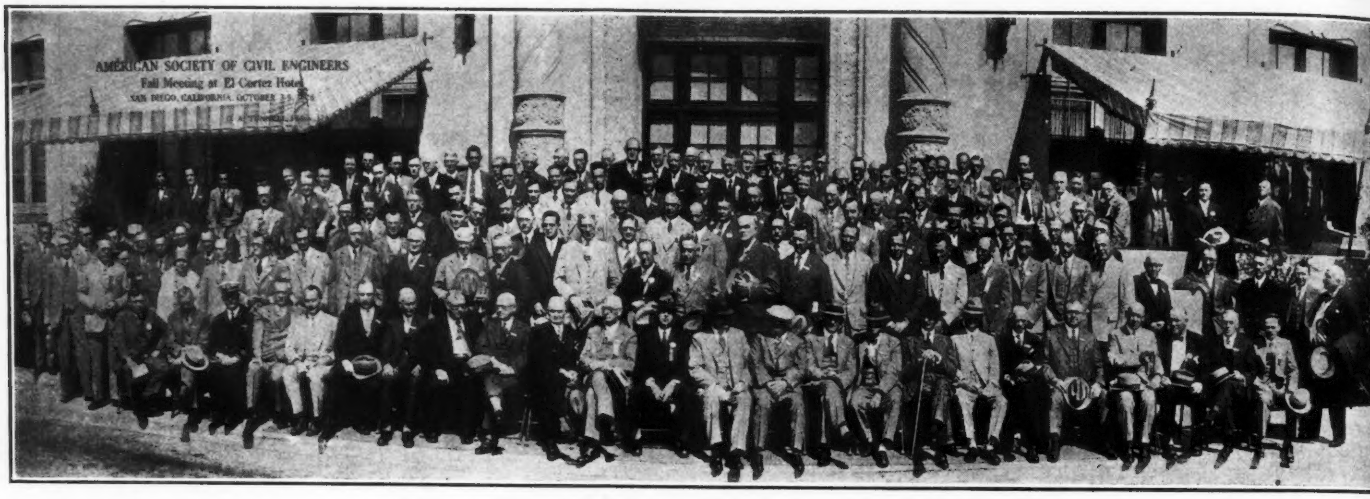
OF the four big Society meetings held each year the Annual Meeting has preeminently the characteristic of a re-union. To it come year after year those who have had the burden, and pleasure, of Society management in years gone by.

To it come also the newer and younger members that their acquaintances may be broadened. It provides one of the Society's two opportunities for doing general business, the other being at the Annual Convention. It is the time for Society committees to report on their work of the year. The Society's prizes for meritorious papers are awarded, and Honorary Memberships are conferred.

At the afternoon session of Wednesday and under the direction of the Technical Divisions, there will be this year, as an innovation, reports which will relate the newest developments in the several branches of civil engineering—the trend which these are likely to give to engineering design and practice, and the modifications of present practice which may be expected or should be carefully considered.

Such reports will be published undoubtedly, but to hear them presented and to be able to discuss their details with the many other engineers present will be an opportunity it will be unfortunate to miss.

Thus in the diverse aspects of business of technical matters and on fraternal grounds the Annual Meeting makes appeal to young and old. Fixed by the Constitution it occurs always in New York on the third Wednesday of January and continues throughout Thursday and Friday. The dates are January 16, 17, and 18, 1929.



It was this happy group of members and guests that gathered in San Diego, Cal., for the Society Meeting held October 3-5, 1928, at El Cortez Hotel. You just know they were having a good time

In Memoriam

MEN of ability and willingness to serve are a boon to an organization. This or any other Society can ill afford to lose such men, typified by Charles B. Ball and Richard L. Humphrey. The death of these two, within a few days of each other, must be a source of sincere regret to the many who are familar with their accomplishments.

If the credit for being "Daddy" of a Technical Division should go to any one member, Mr. Ball is that man. He was the inspiration of the City Planning Division, the guiding hand since its inception. As Secretary he stimulated membership, kept complete records, arranged meetings. Few of its activities were begun without his help; few of its programs that he personally did not lay out.

He was its only Secretary. Not that he did the entire work—far from that; but he got other people to work and kept them at it until the job was done. And invariably it was a good job. Such was the genius of Charles B. Ball. He was efficient and faithful—a model executive for any Technical Division.

Richard L. Humphrey was an entirely different type. It is hard to pick out the particular spot where he shone most, for he was everywhere, in everything. Usually, it was in general Society work. His name immediately recalls the Development Committee, Registration, Local Sections, the "Concrete Report" and the Philadelphia Convention.

He was a tireless worker, whether as an officer of the Society or as a private in the ranks. His passing will leave vacancies in several lines of Society work. This is the more remarkable, remembering that he held no elective office at the time of his death.

In Texas

A COPY of *The Texas Engineer*, "published from time to time at the office of the Secretary, Texas Section, American Society of Civil Engineer", comes to hand giving the program for the "Fall Meeting" of the Section at Laredo, Tex., on November 16 and 17, 1928.

Affairs were scheduled to begin at 8:00 A.M. on Friday and to end with the banquet on Saturday night. Highways formed the principal technical topic and a proposed licensing bill the chief professional topic.

A copy of the proposed bill was inclosed and to those familiar with the registration laws of other States two significant details appear at once. First, the act specifically sets out to regulate the practice of, not general engineering, but *Civil Engineering;* and, second, in so doing the licensees become *certified* civil engineers.

The words, "certified civil engineer", recall the English term, "Chartered Civil Engineer", which, by governmental action, is a title that may be used only by members of the Institution of Civil Engineers.

Credit for drafting the bill is given to a committee composed of Messrs. J. M. Howe, John A. Norris, J. C. McVea, A. J. McKenzie, O. H. Koch, and J. B. Hawley, all members of the Society, of course.

Year Book Changes

THE Year Book went to press last on March 1, 1928. Then the total membership of the Society, in all grades, was 12,903; and just to list the names and addresses, in alphabetical order, required 262 closely printed pages.

The next Year Book will be issued as of March 1st, three months away, but the data for the revision are kept up to date in the office of the Society throughout the year. When the time comes for printing the 1929 Year Book, the copy sent to the printer will be strictly up to the minute and will need only to be revised to take care of such changes as come in while it is in the printer's hands.

The method is to take each page of last year's book and paste it in the center of a brown paper sheet approximately 12 in. in width by 18 in. in length, the borders of which provide room for the necessary notations.

As typical of any, page 190, picked at random, already shows one name crossed off—the member has died; seven new names added, additions to the membership which fall in this part of the alphabet; and changes of address or title for six members.

On December 1st postal cards will be sent to all members and these always bring in a very large number of additional requests for change of address. After December 1st as these cards are returned, sometimes as many as 100 in a day, the data sheets will begin to look less tidy as the date for release to the printer approaches.

December Proceedings

IS Surveying about to recover its lost estate; to find itself again among the popular branches of civil engineering work? Such indeed would be a logical result of papers like the one opening the December Proceedings.

The title taken by W. H. Rayner, Associate Member, is "Specifications for Transit Traversing and Stadia Leveling". These he applies to surveys of just ordinary precision, showing that it is well worth while always to give a job some preliminary study, determining the desirable limit of accuracy and conducting the steps properly to attain the corresponding results. Land Surveyors in general will profit by Professor Rayner's paper.

"Comments on the Design of Sludge Digestion Tanks", by Richard H. Gould, Member, contains, quite logically, something of history and something of analysis, not forgetting a bit of forecast. From his review it is quite evident why this more or less new mode of sewage treatment is growing in popularity.

That many technicians co-operate to evolve a complete City Plan is well recognized. Just why this is so is explained by Arthur A. Shurtleff, Esq., in his paper "Relation of the Landscape Architect to the Allied Professions Engaged in City Planning". Parks, once developed, had to be connected with parkways. Then came boulevards and public plazas, until the whole city was encompassed. Similarly, every phase of technical and artistic work was encountered so that the real functions of the City Planner became those of co-ordinating and harmonizing.

In his "Engineering Applied to National Parks", the Hon. Stephen T. Mather, develops the successive steps by which the present National Park organization was brought about. Engineers' problems and their successes have been due in large part to recognition of artistic and scenic qualities as well as technical details, combining to make these parks such delightful National playgrounds.

The usual printed discussions on open papers are included, numbering 20 on 8 topics. Memoirs, 8 in all, will also be found.

Of great value, also, are the indexes covering all material issued in Proceedings during the calendar year 1928. These appear in two groups treating separately Society Affairs, and Papers and Discussions, the latter divided into subject and author groupings. All told these issues total by far the largest yearly publication in Society history.

Getting It Across

TOO often it is said "He knows his stuff all right, but can't put it across." Frequently the reference is to engineers, and to their attempts at presenting their technical knowledge to the public. Perhaps it is in some civic enterprise, possibly a public hearing on engineering projects, or more likely than not an engineering meeting.

Whatever the occasion, a few basic ideas are important to remember. To be successful the speaker must accommodate himself to the peculiar demands of the moment. If the atmosphere is largely social, the treatment may be personal, conversational, with something of the spirit of "camaraderie". But in a strictly technical meeting, anything less than a formal attitude would be out of place. Then there is the question of manner. Not every one can be a finished speaker, but at least there can be adopted the habits which make a talk attractive and tend to win the hearer. Usually this means that the address should not be read. In general, engineers are familiar enough with their work so that they can speak extemporaneously. Occasionally, however, the matter is more intricate and requires a measured deliberate treatment which only the written form can satisfy; but this should be the exception.

It goes without saying that enunciation should be clear. This perhaps is not so important a failing as that of speaking too low. In picking an objective listener, he should be well toward the rear of the hall. Make sure that he can hear and hear comfortably, and if there is any doubt, speak too loudly rather than too quietly.

Of course diagrams are important aids, but at best they can be only aids. Look away from the screen as much as possible; talk to the audience and not to the blackboard, and don't get absorbed in details.

Nothing estranges an audience more than a disregard of the rights of other speakers. If the time is limited, or set in advance, the attention to the prescribed limit should be meticulous. Even an orator cannot overstep the bounds or appear to "hog" a situation without disaster.

Practice makes perfect. Many engineers profess to be poor speakers and even make this an excuse for not taking part in a meeting, when as a matter of fact, they have never tried to become better. On the other hand, some of the most prominent and successful engineers are marked by an engaging, forceful manner before the public. As the advertisement says: "It can be done."

To sum up: Remember the old question "Does a tree falling in the forest make a sound when there is no ear to hear?" Is a talk to an audience worth anything if the audience because of poor presentation is unable to know what is said?

To the Fore

NEVER before have the qualities of engineers for public office been so signally recognized. Immediately one thinks of Hoover. Throughout the presidential campaign he typified to the public the engineering mind — made synonymous with ability to organize and to accomplish.

Whether or not this was clever political characterizing, at least it seems to have struck a responsive chord. The engineer thus stands in an enviable light before the public.

Perhaps it is not strange, then, that a strictly technical training should be deemed acceptable for a lesser, although still great, political office. Already the civil engineering profession enjoys the proud distinction of having one of its members, Frank C. Emerson, as Governor of the State of Wyoming. The habit seems to grow, and with the recent election comes still another to an equally great office, in the person of Morgan F. Larson, chosen Governor of the sovereign State of New Jersey.

These men are not pseudo- nor "has-been" engineers. Each is in his prime, with a career of active engineering in back of him—in other words, his life work. To each the profession has been a stepping stone to high honor.

Automatic Student Membership

AT the beginning of the current school year the enrollment of the Society's Student Chapter at the Brooklyn Polytechnic Institute suddenly doubled—164 to 324 members. It seemed too good to be true. Investigation showed it was better than good—it was perfect, 100% in fact, with not an eligible student missing.

How come? you will ask. The answer is easy—they had to join. Better perhaps, they made themselves do it, for after all the plan had the sanction, the authority, of the students themselves.

Every student in the Institute pays a Blanket Tax. In this they are like most other colleges, as it includes athletics, publications, student organization, and year book. The point of advance beyond ordinary procedure, however, lies in the inclusion of Student Chapter Dues—this, mind you, by student vote. Thus, every man is a member of one or another Student branch of National Engineering Societies.

Other features of this Chapter's work are progressive notably its systematic planning for the full year's technical activities with a definite aim or goal. The membership provisions, however, are really distinctive. They have this positive advantage, no difficulties arise in getting full membership or in collecting full dues.

M. Am. Soc. C. E.

AN editorial in the "New York Sun" recently commented on scholastic degrees, asking if they were "really worth having"; going on it replied, "undoubtedly they are, if license to use them comes from some notable institution, but so many licenses of the kind come from institutions by no means notable that the value in all cases may be impaired."

Continuing, the editorial elaborated upon its point, more particularly with respect to legal degrees and concluded with the following:

"A far more hopeful suggestion is that bar associations can place their own qualifications for membership so high that the fact of membership itself might be esteemed of greater value than a collegiate degree. It might, for instance, become as good form for a lawyer to write after his name A.B.A., signifying membership in the American Bar Association, as it is for a Fellow of the Royal Society of England to write F.R.S. after his name or a member of the American Society of Civil Engineers to write M. Am. Soc. C. E. after his name."

Proceedings—Vol. 54

THE December issue of Proceedings completes the work of 1928. It is more than the ordinary year. So was 1927. In both these years a determined effort was made to catch up on the papers in hand. To an extent that was accomplished, although, as might be expected from an enlarged membership, papers are coming in at an ever-increasing rate.

Pages of printed matter seem to afford the only practical unit of comparison, and if it be taken for granted that there has been no diminution in quality, represent the increased service to members.

The following table of the past 10 years is illuminating:

Year	*Proceedings (Thousands of Pages)*	*Transactions (Thousands of Pages)*
1919	19 075	15 980
1920	20 440	35 212
1921	19 450	
1922	30 400	19 900
1923	36 915	20 250
1924	30 700	17 440
1925	34 338	17 533
1926	37 161	21 610
1927	48 546	15 919 16 223
1928	57 056	26 040

The edition in the same interval, i.e., ten years, has increased from 9000 to 14,400 copies.

Surveying In Texas

(*Continued from page 1*)

"On the flats was the finest virgin long leaf yellow pine timber that I ever saw. It measured, across the butt, five and a half feet; and was sixty feet to the first big knot, and at least three feet in diameter there."

"In Newton County, I saw only two women wearing shoes—one was the sheriff's wife and the other was the wife of the storekeeper. All the others went barefoot. The school house was a thing of beauty—of unhewn logs, without a south end; the desks were split logs and the seats were the same. The floor was of original dirt. The courtesies of the 'City' were accorded us—the Court House was given us to camp in during a three days' rain.

"On the second night of our stay, the society people gave a dance at the school house in honor of the surveyors. The men came wearing brogans and the ladies barefoot; the men took off their shoes and set them in a row along the side of the wall. I was several times importuned to join the festive crew—but for reasons of innate modesty, I was constrained to refuse. The fiddler was of the old-fashioned kind; he sat in the corner on a log, crossed his legs and swung one in unison with the tune, while he called the 'figures' of the dance—mostly 'swing corners' and 'Gents to the right'. But, believe me, it was fast and furious fun. The dust that was raised was fearful—after each dance."

"One day I looked into the muzzle of a .45 Colt, but to me it seemed a full grown HORSE. I was paying off my men, in the store of my friend, Capt. Ed. Clough, at Woodville in Tyler County. I paid them their salaries and enough additional money to give them railroad fare to Huntsville, and meals on the way. When I came to a man named Bridges, whom I had hired right there at Woodville, I did not give him any extra money. He stood in front of me and demanded the same 'extra' as the other boys; without looking up I refused, and continued to write out his check. When I did look up my face was about a foot from the muzzle of his .45 Colt. I felt little lizards running up and down my spinal column.

"I heard some one cough and looked around. Capt. Clough had Bridges covered with a double barreled shotgun and two of my boys had him covered with pistols. I remarked CALMLY to Bridges that if he would casually look around he would probably remove the 'cannon'. He looked—and turned whiter than a sheet. He put up his gun, saying with a sickly smile, that it was only a bluff.

"Capt. Clough said that it would have cost him his life, if he had not been in line with me. It was all I could do to keep my two boys from beating him. He took his check and went out of our lives forever—we never saw him again."

ie
rt
r-

y,
ne
y-
ns
ok
ow
as
he
in-
to
ld-
ner
ing
he
e—
to
vas
hat
ach

zzle
ned
ing
my
od-
nem
onal
e to
vay.
ned
ght
give
l in
ame
nout
ued
did
foot
. I
and

and
had
har-
boys
re-
that
l he
can-
niter
gun,
t it

ould
not
all I
rom
and
—we

Stevenson Creek Experimental Test Dam, Near Fresno, Calif., Under Test with Reservoir Full, Water 60 Feet Deep at Middle of Dam.

AMERICAN SOCIETY OF CIVIL ENGINEERS

INSTITUTED 1852

PAPERS AND DISCUSSIONS

This Society is not responsible for any statement made or opinion expressed in its publications.

ENGINEERING FOUNDATION
COMMITTEE ON ARCH DAM INVESTIGATION

ARCH DAM INVESTIGATION

VOL. I

COMMITTEE

CHARLES D. MARX, *Chairman,*
FRED A. NOETZLI, *Secretary.*

PAUL BAILEY,	D. C. HENNY,
C. DERLETH, JR.,	M. M. O'SHAUGHNESSY,
H. W. DENNIS,	H. HOBART PORTER,
ALFRED D. FLINN,	JOHN L. SAVAGE,
H. HAWGOOD,	F. E. WEYMOUTH,

SILAS H. WOODARD.

November, 1927

FOREWORD

Engineering Foundation presents in the following pages the first instalment of an experimental study of Arch Dams conducted by a Committee composed of members of the American Society of Civil Engineers. Two or three years will be required to finish the work in hand. Then the Committee will complete its report. Since the investigation is still in the stage of research, reduction of the partial results to the form of guides for practice in design, construction, or examination of Dams would be premature. Nevertheless, the practising Engineer will find in this first volume of the report much usable information. The attention of Corporation Executives and Public Officials is directed particularly to the General Statement by the Committee in Part I.

It seems almost unnecessary to remark that the extreme thinness of the large experimental dam built by the Committee was not intended as a pattern for dams to be built for service. The dimensions were selected with a view to getting as large deformations as practicable in order to reduce the difficulties of their measurement and to increase the accuracy of the observations.

The Arch Dam Investigation is a good example of one kind of co-operative research for which Engineering Foundation was created by the American Societies of Civil, Mining and Metallurgical, Mechanical, and Electrical Engineers. The personal services freely given by the Members of the Committee and its Sub-Committees have been made fruitful by the funds contributed. Collaboration by Universities, Technical Bureaus of the Government, and other organizations have supplemented the Committee's efforts. The support of the American Society of Civil Engineers, one of the Foundation's constituent bodies, has been most helpful and this Society's publications are the natural channel through which to present the results of a Civil Engineering Research.

Engineering Foundation commends the Committee on Arch Dam Investigation and its Sub-Committees for this persistent devotion to the task undertaken in 1922 and expresses appreciation for the varied forms of co-operation given by numerous persons, corporations, and organizations in the United States and other countries.

Lewis B. Stillwell, *Chairman,*
Alfred D. Flinn, *Director.*

CONTENTS

LETTER OF TRANSMITTAL

STANFORD UNIVERSITY, CAL., Nov. 3rd, 1927.

MR. L. B. STILLWELL, *Chairman*,
Engineering Foundation,
New York, N. Y.

SIR:

With this letter I have the honor to transmit to Engineering Foundation a report of the Committee on Arch Dam Investigation. The Committee held its first meeting in San Francisco, January 18, 1923. Although its work is not finished, the major experiments undertaken are so far advanced and the work has been so successful that the Committee feels under obligation to make available to its supporters and others some of the information gained. This report was authorized by the Committee at a meeting in Los Angeles on July 20, 1927. The Committee expressed the hope that Engineering Foundation would print and issue the report at as early a date as may prove convenient.

Respectfully yours,

CHAS. D. MARX, *Chairman*,
Committee on Arch Dam Investigation.

PART I.—GENERAL STATEMENT BY THE COMMITTEE*

1.—Economic Considerations

Locating, designing, and constructing arch dams and the power, irrigation, water supply, or flood control systems connected with them, are tasks for trained engineers. Engineers have also a large share in determining the economic feasibility of such projects. Consequently the major portion of this report on an experimental investigation of arch dams has been written in the engineer's language. The subject concerns also investors, bankers, contractors, corporation executives, regulatory authorities, and other non-technical men. In a report before completion of the project, statements about conclusions and benefits necessarily are incomplete and subject to revision. Nevertheless, a few tentative statements are made here for the groups just named, because the information is needed and the progress achieved justifies them.

Dams are key structures in the systems of which they are parts. On their security commonly depends, not only the integrity of the investments in the dams and appurtenant works, but also, both directly and indirectly, the safety of other properties of great value and sometimes of many lives. Naturally, therefore, most dams built by engineers have been very conservatively designed. Ultra-conservatism in such structures often adds much to their cost and sometimes renders the projects financially infeasible.

Hitherto, conservatism in the design of arch dams has been compounded of ignorance and justifiable prudence, and the latter has been clouded by the former. Ignorance was due to lack of experimental knowledge which could be gotten only by an expenditure of funds and an assemblage of special talents which could not be commanded by any engineer single-handed and by few, if any, companies alone. Investors and the community have a vital interest in these aspects of many engineering problems. It is a good investment to assist engineers to acquire new experimental knowledge that will lead to reduction of cost, increase of productiveness, or enhancement of physical security of engineering works.

Such acquisition of new knowledge is often possible only through co-operation. The Arch Dam Investigation is a good example of co-operative engineering research, as the list of affiliated agencies (see Acknowledgments) shows. In this instance the needed assemblage of funds, talents, and other resources was brought about by use of the Engineering Foundation of the American Societies of Civil, Mining and Metallurgical, Mechanical, and Electrical Engineers. Engineers in the Far Western States, where the arch dam problem was pressing for solution, appealed to Engineering Foundation for aid, and the Committee on Arch Dam Investigation was organized.

* For complete statement of membership of Committee and Sub-Committees and of contributors and co-operators, see Acknowledgments, etc., following Part IX.

There were three general methods by which experimental knowledge might be sought:

1.—Tests and observations on existing dams or dams being built;
2.—Tests of small models in laboratories;
3.—Tests on one or more dams built especially for experimental purposes, but of sizes approximating dams built for service.

Much skepticism existed as to the dependability of results of tests of small models; conditions obtaining in such tests seemed very different from those of real dams. Funds and facilities for building and testing a full-sized model were not in sight when the Committee began its work. Attempts were made, therefore, to get information by studying existing arch dams. Other chapters tell of the remarkable resources that became available to this Committee and the use made of them.

The Committee ventures the following conclusions, subject to modification in a second volume of the report, which it expects to submit upon the completion of its work.

2.—Tentative Conclusions

1.—Tests of the Stevenson Creek full-sized, experimental arch dam have demonstrated the great strength of a thin, unreinforced concrete arch dam,* if well built, on proper foundations, and experience has shown the ability of such a dam to resist floods.

2.—These tests have yielded positive information as to how this arch dam carries its load of water pressure and resists other forces acting upon it.

3.—The tests have supplied data which appear to be suitable for use in devising rational methods for the intelligent design of arch dams and for examining the stability of existing arch dams, within some limits not yet determined.

4.—Indications of agreement with the Stevenson Creek results (gotten by independent investigators on one large dam in service and by another group of experimenters at Princeton University, using a small model of very different material) gives assurances that some dependable experimental knowledge has been gotten to confirm or modify assumptions heretofore guiding the design of arch dams.†

5.—Agreement between the Princeton model and the full-sized dam indicates that models may be used by competent engineers, with confidence, for seeking further general information about arch dams or for determining the strength of proposed or existing dams, at a small fraction of the cost of tests of a full-sized dam or a model large enough to use the ordinary materials of construction.

* Single arch concrete dams are commonly built with no steel reinforcement, as little if any advantage could be gained by the use of steel, except that special conditions in some dams may make reinforcement advantageous in particular parts.

† Other independent experiments with models of different materials, at different scales, by different methods, are being made near Denver, Colo., to test further the usefulness of models.

6.—With the aid of information being gotten by the Committee, arch dams may hereafter be built on some sites of less thickness than would formerly have been considered necessary.

7.—Economic benefits may be expected, therefore, in lower costs of some dams. For some projects this reduction in the cost of dams may be sufficient to make financing possible, thus permitting the creation of productive properties which otherwise would be impracticable.

8.—Dependable means for determining the safety of proposed arch dams will make it possible for regulatory authorities to approve less costly designs for some sites than would now be demanded.

3.—The Technical Reports

With members widely scattered, whose primary responsibilities were for other duties, it was necessary for the Committee to obtain the assistance of competent investigators who could devote the required time. Various portions of the investigation were assigned to these investigators, who carried out their several assignments under the general supervision of the Committee and its appropriate sub-committees. Correspondingly, "parts" of this report have been prepared by these specialists. The Committee has examined the drafts of these chapters, has accepted them, and has assembled them into this volume for publication.

Some of the purposes of the Committee in preparing this report before completion of its work are:

(1) To make information now in hand available;
(2) To submit its work at this stage to examination and discussion;
(3) To invite suggestions and contributions of additional information;
(4) To detect and correct errors;
(5) To give its supporters an earlier, although partial, return for their contributions; and,
(6) To disseminate information of the success of this co-operative research, in order to enlist the support of former and new contributors, thus making possible the continuation of the Committee's work.

By these means the Committee hopes to make its final statement of greater value. Besides specific contributions to the knowledge of arch dams, this investigation is obtaining information of general application about the physical properties of Portland cement and concrete, about instruments and methods, and about the use of models.

Interest in the investigation has been world wide, as attested by correspondence, visits, and papers in the technical and popular press. In spite of difficulty of access, the Stevenson Creek Dam has been visited by an unexpectedly large number of persons other than members of the construction and test forces. There have been visitors also to offices of members of the Committee, and the office of Engineering Foundation, seeking information. Among

these visitors have been engineers from Argentina, Czechoslovakia, Holland, India, Japan, Norway, and Mexico.

Discussion of the report is invited and should be sent to the American Society of Civil Engineers, at 33 West 39th Street, New York, N. Y.

CHARLES D. MARX, *Chairman,*
FRED A. NOETZLI, *Secretary,*
PAUL BAILEY,
C. DERLETH, JR., alternate, R. E. DAVIS,
H. W. DENNIS,
ALFRED D. FLINN,
H. HAWGOOD,
D. C. HENNY,
M. M. O'SHAUGHNESSY,
H. HOBART PORTER, alternate, WYNNE MEREDITH,
JOHN L. SAVAGE, alternate, IVAN E. HOUK,
F. E. WEYMOUTH, alternate, JULIAN HINDS,
SILAS H. WOODARD,
Committee on Arch Dam Investigation.

PART II.—ARCH DAMS: AN EXPERIMENTAL INVESTIGATION

By Fred A. Noetzli,* M. Am. Soc. C. E.

1.—The Problem

There has been demand for lower cost of dams while conserving safety and permanence, for power development, water supply, irrigation, and flood control. To meet this need types of concrete dams using less material than "gravity" dams have been devised. The arch type has been used for centuries for impounding water. Many arch dams of great heights have been built, especially in the western parts of the United States. None has failed for reason of structural weakness. Some arch dams are very thick, and others very thin. Either there has been a waste of material in the thick dams, or the limit of safety has been closely approached in the thinnest.

The exact determination of stresses in an arch dam is a complicated problem. At first glance, the design of a horizontal circular arch under water loads appears to be simple; but the effects of complete or partial restraint at the base and along the sides, temperature changes, and other variable factors, render the intelligent designing of an arch dam difficult.

In the past most arch dams were designed by the so-called "cylinder" formula, according to which a dam was considered as made up of a series of horizontal arches, and the stresses in each elementary arch were calculated according to the relation of the wall thickness to the radius of the arch.

Thus,

$$\sigma = \frac{p\,y\,r_u}{t}$$

in which,

σ = arch stress;
p = weight per unit of volume of water;
y = depth of water;
r_u = up-stream radius of arch;
t = thickness of arch.

The cylinder formula is based on the assumption that when the load is applied, the perimeter of the arch is shortened, due to the elastic deformation of the concrete, all points moving a small distance toward the center of the cylinder. However, if the ends of the arch are rigidly held in place, the points of the ring are not free to move inward. Thus, the shortening of the arch rib introduces stresses which generally are referred to as "rib-shortening" stresses. The cylinder formula takes no account of these rib-shortening stresses, which, under certain conditions, may attain high values.

* Cons. Hydr. Engr., Los Angeles, Calif.

Again, if the bottom of an arch dam is set, for instance, in a groove, so as to be rigidly held against displacement, the lowest arch ring will not be subject to arch stress, but will simply serve to transmit the load coming upon it to the supporting groove. Successive arch rings above the bottom will be partly restrained, with the result that they will be only partly stressed as arches, the remaining load being carried by vertical cantilever, beam, or other action.

For facility of mathematical analysis, some engineers now consider an arch dam as made up of a series of elementary horizontal arches and a series of elementary vertical cantilevers or beams. As a first approximation, the elementary horizontal arches and one elementary vertical beam at the center of the dam may be considered. The water load is then divided between horizontal and vertical elements in such a way that the deflections at the points of intersection of these elements are identical. A greater degree of refinement is obtained by considering, instead of only one vertical element, a series, usually of three, five, or seven, such vertical elements. The problem is then to determine the partial loads for all the assumed elementary horizontal arches and all elementary vertical beams such that the deflections of the two systems at all points of intersection are identical.

Computation of stresses in an arch dam is further complicated by the various changes of temperature. If the temperature rises uniformly throughout the concrete, the dam will deflect up stream; if the temperature of the concrete is lowered, the dam will tend to deflect down stream. Such movements introduce bending stresses, which are added to the stresses produced by the water load. The determination of temperature stresses in an arch dam is difficult, even if the temperature change is uniform throughout the mass. Generally, there will be a difference between the temperature on the water, and on the air, side of the dam, which fact complicates the problem still more.

There are additional influences which affect the stresses in an arch dam: Shrinkage of the concrete due to setting of cement, swelling due to moisture in concrete, lateral deformation (Poisson's law), plastic flow of concrete, yielding of foundation, and various minor influences.

2.—Summary of Project

The purpose of the arch dam investigation was to secure experimental data and other information on arch and multiple-arch dams. Before this investigation, there were few, if any, experimental data available on the way an arch dam of any shape supported its water load and resisted other forces. Hitherto design and such mathematical analysis as was undertaken were based on assumptions, general knowledge of engineering materials and structures, and accumulating experience with the increasing number of dams of various shapes and dimensions.

As a beginning of experimental work several service dams were equipped with instruments for measuring strains, deflections, and temperature changes. In some other dams, instruments were placed in the concrete during construction. Observations made periodically on these dams have included measurements when the dams were completely and partly loaded (reservoirs filled or partly emptied).

An arch dam on Stevenson Creek was constructed by this Committee in the spring of 1926 purely for experimental purposes. It was equipped during construction with suitable instruments, many of special design, in order to obtain as complete information as possible on strains, deflections, and other phenomena. The dam was tested during the summer of 1926 under many different load conditions. Information on its behavior was also obtained for changes of temperature in the dam when the reservoir was empty.

Tests were made in a field laboratory at the dam site for determining the grading and physical properties of the concrete aggregate used and the strength and modulus of elasticity of concrete specimens at different ages, during the construction and testing of the dam. An extensive program of laboratory tests on concrete of the same kind of cement and aggregate is being carried out in the Materials Testing Laboratories of the University of California. Experiments on models of the Stevenson Creek and other dams are being made at the University of Colorado. At Princeton University, tests on a small celluloid model have been completed.

3.—Design of Experimental Arch Dam

The design of the Stevenson Creek Test Dam was the subject of much study. Several designs were prepared for different sites, different arch radii, thicknesses, and corresponding unit stresses. The following conditions guided the preparation of the plans for the dam as constructed:

1.—The dam was to be of the simple arch type;

2.—It was to be relatively safe at an initial height of 60 ft., but it was also to be reasonably certain to break under load, if and when raised to a height of 100 ft. or thereabouts.

The up-stream face was made vertical with a constant radius of 100 ft. from top to bottom. For the foundation the bed-rock was excavated to such lines as to make the profile along the up-stream face practically symmetrical, of V-shape, with a slight rounding at the bottom. The second condition controlled the thickness, which is 7 ft. 6 in. at the base and diminishes to 2 ft. at a height of 30 ft. From there to the top it is 2 ft. The length measured along the crest is 140 ft. The ratio between thickness at crest and length is, therefore, 1 : 70.

Combined cantilever and arch action was assumed. The computations of the amounts of water load carried by the elementary vertical cantilevers and by the elementary horizontal arches, respectively, were made according to a tentative method.* This consists in assuming one or several vertical slices of the dam, so-called cantilevers, each 1 ft. wide; in assuming further a series of horizontal arch slices; and then in determining by a semi-graphical method the division of loads upon cantilevers and arch slices in such a manner that the deflections of the points cammon to both systems are as nearly the same as practicable. The essential data are given in Tables 1 and 2. The analysis was made for six elementary horizontal arches, 10 ft. apart, and for one vertical cantilever slice at the center of the dam. Table 1 gives the dimensions

* "Gravity and Arch Action in Curved Dams," by Fred A. Noetzli, M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. LXXXIV (1921), p. 1.

of the arch elements at the various elevations; also the arch loads and deflections for full and for partial water pressure.

For the purpose of making the down-stream face circular in vertical radial sections, the dam between the base and the 30-ft. elevation was constructed with a thickness which differs slightly from that used in the design, as this facilitated the erection of forms true to prescribed lines. Thus, at the 10-ft. elevation the thickness assumed in the design was 4.25 ft., but the thickness as constructed is 4.40 ft. At the 20-ft. elevation the thickness was changed from 2.75 ft. to 2.58 ft. These differences are relatively small and may be neglected when comparing the loads and stresses determined in the design with those obtained from the experimental data.

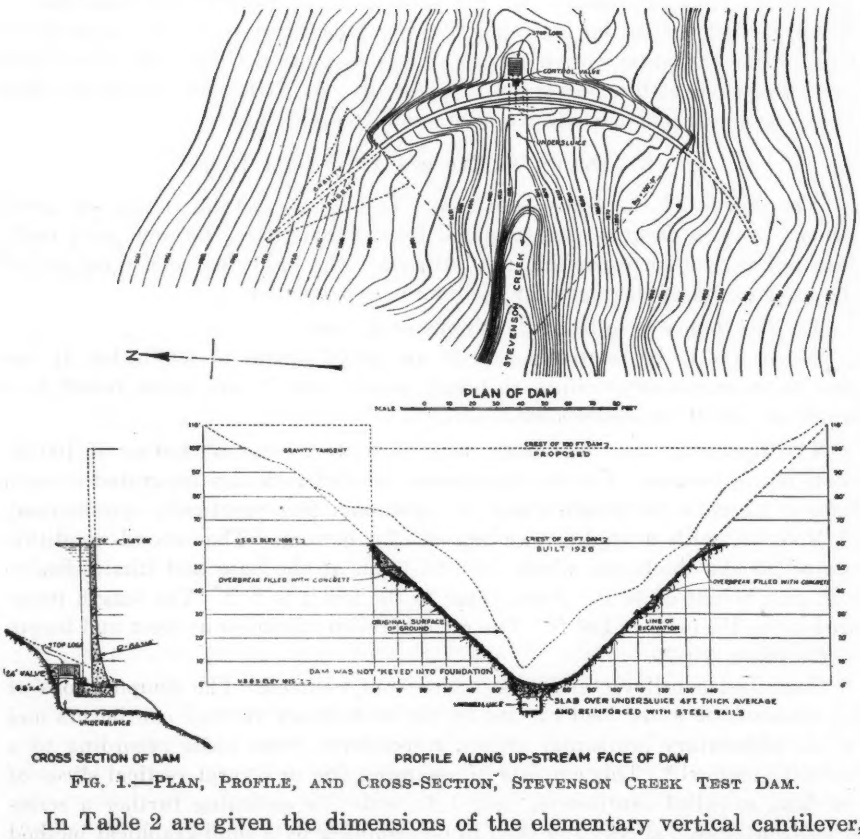


FIG. 1.—PLAN, PROFILE, AND CROSS-SECTION, STEVENSON CREEK TEST DAM.

In Table 2 are given the dimensions of the elementary vertical cantilever, and its moment areas and deflections for partial load. The division of the load between cantilever and arches was made under the assumption of a modulus of elasticity, $E = 2\,500\,000$ lb. per sq. in. for the arches, and $E = 2\,000\,000$ lb. per sq. in. for the cantilever. It seemed proper to assume a smaller modulus for the cantilever than for the arches, inasmuch as some portions of the cantilever are in tension and, further, in order to make some allowance for the deflection of the cantilever due to shear.

In Fig. 2 are shown the graphical constructions by which, in combination with the data in Tables 1 and 2, the approximate division of water load was obtained by the so-called tentative, or "combined cantilever and arch", method. The reservoir is assumed to be filled to the crest of the dam. The temperature is taken as constant. Fig. 2(*a*) is a vertical section through the dam at the arch crowns. Fig. 2(*b*) gives the deflections of the elementary horizontal arches for full water load, as computed from Cain's formula for arches with fixed ends.* The division of water load as arrived at after several trial computations, is shown in Fig. 2(*c*). The deflection curves for the cantilever and arches, respectively, computed for the divided loads (Fig. 2(*f*)) coincide closely with each other, which is the criterion for proper division of the water pressure under the assumptions. The computation of the stresses in the cantilever and in the arches made for the divided loads shown in Fig. 2(*c*) furnished the results given in Table 3. These values of stresses are also shown graphically in Fig. 3.

TABLE 1.—DESIGN OF STEVENSON CREEK DAM: DATA FOR CALCULATING ARCH DEFLECTIONS.

(Conditions:
Load: Water surface at Elevation 60.
Temperature: No change.)

Elevation, in feet.	Arch thickness, t, in feet.	Radius of arch center line, r, in feet.	$\frac{t}{r}$	Length of arch, in feet.	Central angle, 2ϕ.	Deflection coefficient, C_s	$\frac{r\,r_u}{Et}$	Deflection of arches for: Full load on arches. Load, in pounds per square foot.	Full load on arches. Deflection. Feet.	Full load on arches. Deflection. Inches.	Partial load on arches. Load, in pounds per square foot.	Partial load on arches. Deflection, in inches.
60	2.00	99.0	0.0202	139	79°40′	1.85	1 375	0	0	0	50	0.015
50	2.00	99.0	0.0202	118	67°40′	1.84	1 375	625	0.0158	0.190	775	0.236
40	2.00	99.0	0.0202	97	55°30′	1.81	1 375	1 250	0.0312	0.374	1 210	0.362
30	2.00	99.0	0.0202	76	43°30′	1.74	1 375	1 875	0.0449	0.539	1 335	0.383
20	2.75	98.63	0.0279	55	31°30′	1.27	996	2 500	0.0316	0.379	1 250	0.190
10	4.25	97.88	0.0434	34	19°30′	0.23	638	3 125	0.0046	0.055	875	0.015
0	7.50	96.25	0.0780	....	....	....	357	3 750	0	0	0	0

The stresses at the base of the cantilever were computed at 429 lb. per sq. in. compression at the down-stream face, and 380 lb. per sq. in. tension at the up-stream face between the dam and bed-rock. According to these computations there was, therefore, probability of a crack developing at the up-stream face between the dam and bed-rock. Such a crack did occur when the dam was loaded, and, with the reservoir full to the crest, an opening of the crack of as much as 0.05 in. was measured.

* "The Circular Arch under Normal Loads," by William Cain, M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. LXXXV (1922), p. 270.

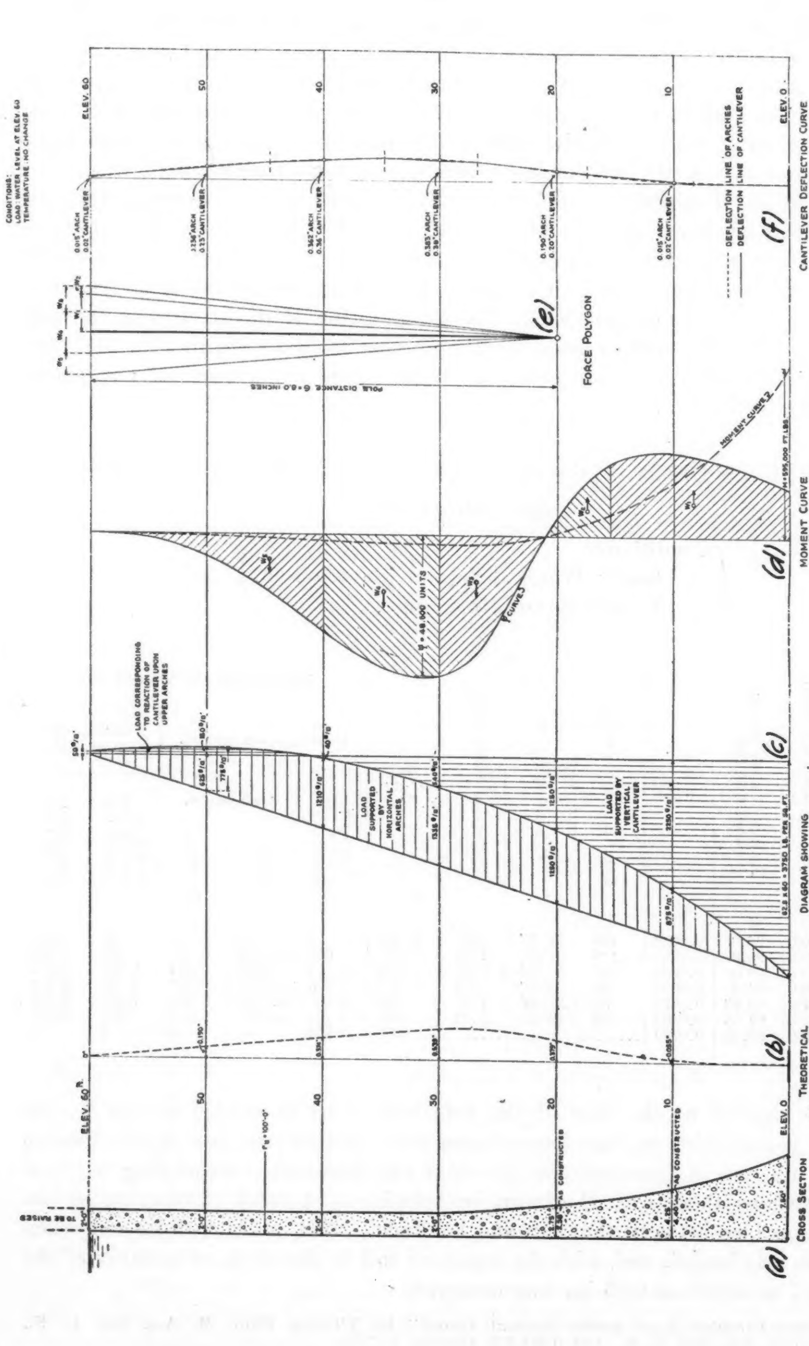


FIG. 2.—DIVISION OF WATER LOAD BETWEEN CANTILEVER AND ARCHES FOR FULL RESERVOIR AND NO CHANGE IN TEMPERATURE, AS DETERMINED IN THE DESIGN OF THE TEST DAM.

TABLE 2.—DESIGN OF STEVENSON CREEK DAM: DATA FOR CALCULATING CANTILEVER DEFLECTIONS.*

$$\left(\text{Pole distance, } G = \frac{288\ 000\ 000}{7\ 200\ 000 \times 60} = 0.667 \text{ ft., or } 8.0 \text{ in.}\right)$$

Elevation, in feet.	Thickness of cantilever, t, in feet.	Partial load, in pounds per square foot.	Bending moments, M, in foot-pounds.	Moment of inertia, I.	$\frac{M}{I}$.	$\frac{M}{I}$-areas = W.	Deflection, in inches.	Elevation, in feet.
60	2.00	− 50	0	0.667	0	....	0.02	60
50	2.00	− 150	− 5 010	0.667	− 7 520	$W_5 = -224\ 000$	0.23	50
40	2.00	+ 40	− 22 300	0.667	−33 500	$W_4 = -435\ 500$	0.36	40
30	2.00	+ 540	− 32 400	0.667	−48 600	$W_3 = -268\ 000$	0.38	30
20	2.75	+1 250	+ 12 450	1.73	+ 7 200	$W_2 = +\ 86\ 000$	0.20	20
10	4.25	+2 250	+187 500	6.39	+29 400	$W_1 = +390\ 000$	0.02	10
0	7.50	+3 750	+595 000	35.2	+16 900		0	0

* Moduli of elasticity assumed: For arches, $E = 2\ 500\ 000$ lb. per sq. in.; for cantilever, $E = 2\ 000\ 000$ lb. per sq. in.

TABLE 3.—DESIGN OF STEVENSON CREEK DAM: STRESSES IN ARCHES AND CANTILEVERS.

(Conditions:
Load: Water surface at Elevation 60.
Temperature: No change.)

Elevation, in feet.	Arch sections.		Arch stresses from partial water load, in pounds per square inch.	Cantilever section.	Cantilever stresses from partial water load, in pounds per square inch.
Crest = 60	Crown:	Extrados........	20		
		Intrados........	15	Up stream.	0
	Abutment:	Extrados........	12		
		Intrados........	22	Down stream.	0
50	Crown:	Extrados........	321		
		Intrados........	210	Up stream.	+ 63
	Abutment:	Extrados........	156		
		Intrados........	376	Down stream.	− 42
40	Crown:	Extrados........	540		
		Intrados........	277	Up stream.	+253
	Abutment:	Extrados........	151		
		Intrados........	669	Down stream.	−212
30	Crown:	Extrados........	665		
		Intrados........	207	Up stream.	+369
	Abutment:	Extrados........	− 11		
		Intrados........	+879	Down stream.	−307
20	Crown:	Extrados........	+491		
		Intrados........	− 67	Up stream.	− 15
	Abutment:	Extrados........	− 341		
		Intrados........	+771	Down stream.	+ 78
10	Crown:	Extrados........	+100		
		Intrados........	− 70	Up stream.	−367
	Abutment:	Extrados........	−153		
		Intrados........	+187	Down stream.	+425
Base = 0	....		0	Up stream.	−380
	....		......	Down stream.	+429

FIG. 2.—DIVISION OF WATER LOAD BETWEEN CANTILEVER AND ARCHES FOR FULL RESERVOIR AND NO CHANGE IN TEMPERATURE, AS DETERMINED IN THE DESIGN OF THE TEST DAM.

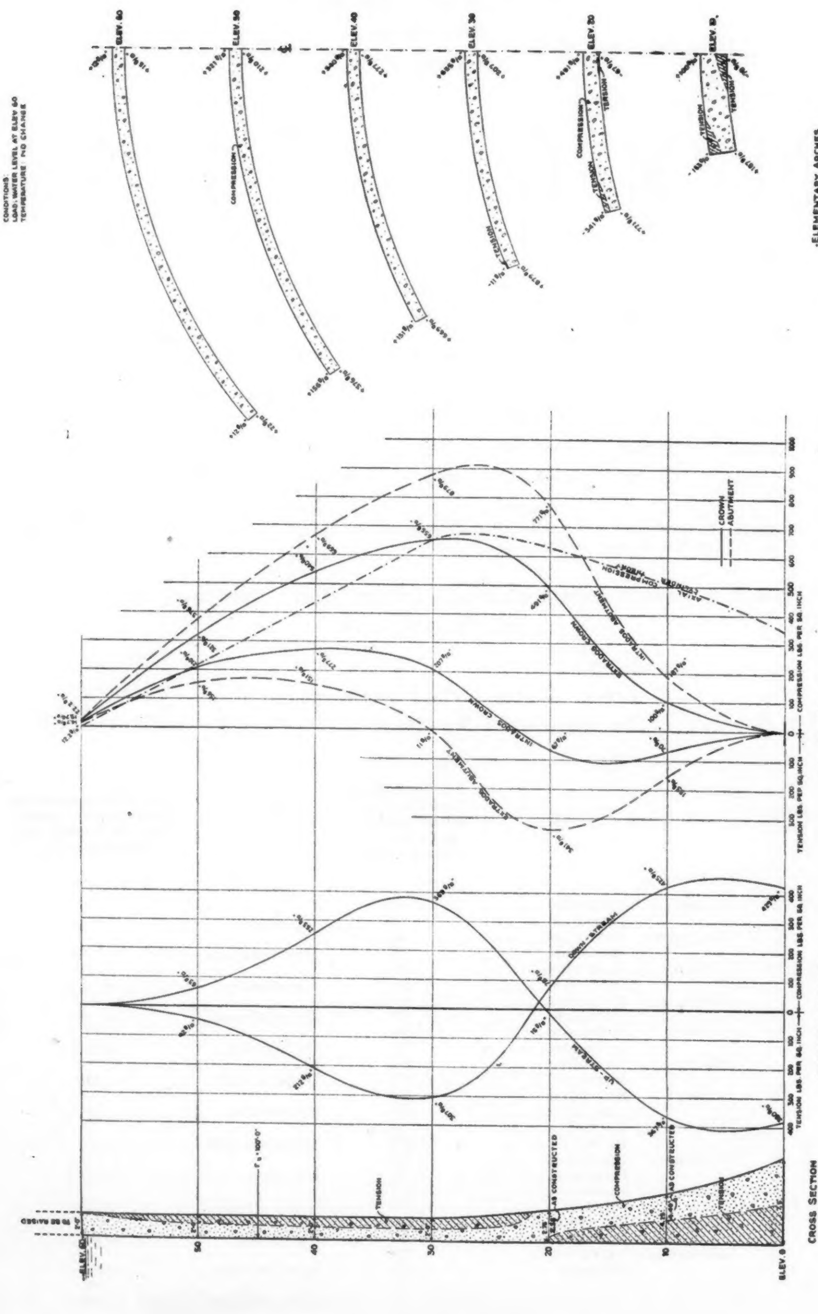


FIG. 3.—STRESSES IN CANTILEVER AND ARCHES AS DETERMINED IN THE DESIGN.

The stresses in the arches due to the divided loading were computed by the elastic theory, assuming the arches as fixed at the abutments, and loaded by a uniform radial load of the intensity determined at the crown by the "combined cantilever and arch" method. (See Table 1.)

Similarly, an analysis of the stresses was made for the conditions of a full reservoir and a uniform drop in temperature of 20° Fahr. The data and graphical constructions are given in Tables 4 and 5, and by the diagrams of Fig. 4. It should be noted that a variation of the arch temperature may be considered as equivalent to a load acting similarly to the water pressure. After the equivalent temperature load has been determined and added algebraically to the water pressure, the division of the total load may be effected by the tentative cantilever and arch method in a manner similar to that described previously for the water pressure.

TABLE 4.—DESIGN OF STEVENSON CREEK DAM: DATA FOR CALCULATING ARCH DEFLECTIONS.

(Arch Deflection, $D = - C_s\ (r e t_0)$. In this Table, $t_0 = 20°$ and e, the Coefficient of Linear Expansion of Concrete = 0.0000055 per degree Fahrenheit.)

Conditions:
Load: Water surface at Elevation 60.
Temperature: Drop of 20° Fahr.

Elevation, in feet.	Arch thickness, t, in feet.	Radius of arch center line, r, in feet.	Deflection coefficient.	Arch Deflection, D, Due to Drop of Temperature.		Equivalent temperature load, in pounds per square foot.	Deflection of Arches for			
							Full Load on Arches, Water Pressure Plus Equivalent Temperature Load.		Partial Load on Arches.	
				Feet.	Inches.		Load, in pounds per square foot.	Deflection, in inches.	Load, in pounds per square foot.	Deflection, in inches.
60	2.00	99.0	1.85	0.0201	0.242	789	789	0.242	859	0.254
50	2.00	99.0	1.84	0.0200	0.240	789	1 414	0.430	1 614	0.490
40	2.00	99.0	1.81	0.0197	0.236	789	2 039	0.610	1 929	0.578
30	2.00	99.0	1.74	0.0189	0.277	789	2 664	0.766	1 944	0.559
20	2.75	98.63	1.27	0.0138	0.165	1 090	3 590	0.544	1 910	0.290
10	4.25	97.88	0.23	0.0025	0.030	1 705	4 830	0.085	1 730	0.030
0	7.50	96.25	....	......	.....	3 000	6 750	......	0	......

The theoretical arch stresses for this condition are given in Table 6. Even if the cantilever should be strong enough to support its theoretical proportion of the load, the arch stresses for the remaining load would be high, especially at Elevation 30.

Division of the water pressure between cantilevers and arches with water in the reservoir to Elevation 40, by the tentative method, was determined as shown in Fig. 5. The corresponding data are given in Tables 7 and 8. The design was prepared by the writer.

TABLE 5.—DESIGN OF STEVENSON CREEK DAM: DATA FOR CALCULATING CANTILEVER DEFLECTIONS.*

$$\left(\text{Pole distance, } G = \frac{288\ 000\ 000}{7\ 200\ 000 \times 60} = 0.667 \text{ ft., or } 8.0 \text{ in.}\right)$$

Elevation, in feet.	Thickness of cantilever, t, in feet.	Partial load, in pounds per square foot.	Bending moments, M, in foot-pounds.	Moment of inertia, I.	$\frac{M}{I}$.	$\frac{M}{I}$-areas = W.	Deflection, in inches.	Elevation, in feet.
60	2.00	− 70	0	0.667	0	....	0.27	60
50	2.00	− 200	− 6 900	0.667	−10 350	$W_5 = -302\ 000$	0.48	50
40	2.00	+ 110	− 29 550	0.667	−44 200	$W_4 = -553\ 500$	0.59	40
30	2.00	+ 720	− 38 300	0.667	−57 400	$W_3 = -289\ 000$	0.55	30
20	2.75	1 680	+ 28 300	1.73	+16 350	$W_2 = +170\ 000$	0.29	20
10	4.25	3 100	+267 900	6.39	+42 000	$W_1 = +526\ 000$	0.04	10
0	7.50	6 750	+838 000	35.2	+23 800		....	0

* Moduli of elasticity assumed: For arches, $E = 2\ 500\ 000$ lb. per sq. in.; for cantilever, $E = 2\ 000\ 000$ lb. per sq. in.

TABLE 6.—DESIGN OF STEVENSON CREEK DAM: STRESSES IN ARCHES AND CANTILEVERS.

(Conditions:
Load: Water surface at Elevation 60.
Temperature: Drop of 20° Fahr.)

Elevation, in feet.	Arch sections.		Arch stresses from partial water load, in pounds per square inch.	Cantilever section.	Cantilever stresses from partial water load, in pounds per square inch.
Crest = 60	Crown:	Extrados........	+ 65		
		Intrados........	− 21	Up stream.	0
	Abutment:	Extrados........	− 64		
		Intrados........	+ 102	Down stream.	0
50	Crown:	Extrados.....	+ 396		
		Intrados........	+ 164	Up stream.	+ 82
	Abutment:	Extrados........	+ 52		
		Intrados........	+ 474	Down stream.	− 62
40	Crown:	Extrados........	+ 587		
		Intrados........	+ 169	Up stream.	+329
	Abutment:	Extrados........	− 34		
		Intrados........	+ 794	Down stream.	−287
30	Crown:	Extrados........	+ 683		
		Intrados........	+ 29	Up stream.	+431
	Abutment:	Extrados........	− 292		
		Intrados........	+1 006	Down stream.	−386
20	Crown:	Extrados........	+ 471		
		Intrados........	− 378	Up stream.	−102
	Abutment;	Extrados........	− 794		
		Intrados........	+ 902	Down stream.	+165
10	Crown:	Extrados........	− 80		
		Intrados........	− 417	Up stream.	−553
	Abutment:	Extrados........	− 582		
		Intrados........	+ 92	Down stream.	+611
0	....	....	......	Up stream.	−560
	....	....	......	Down stream.	+609

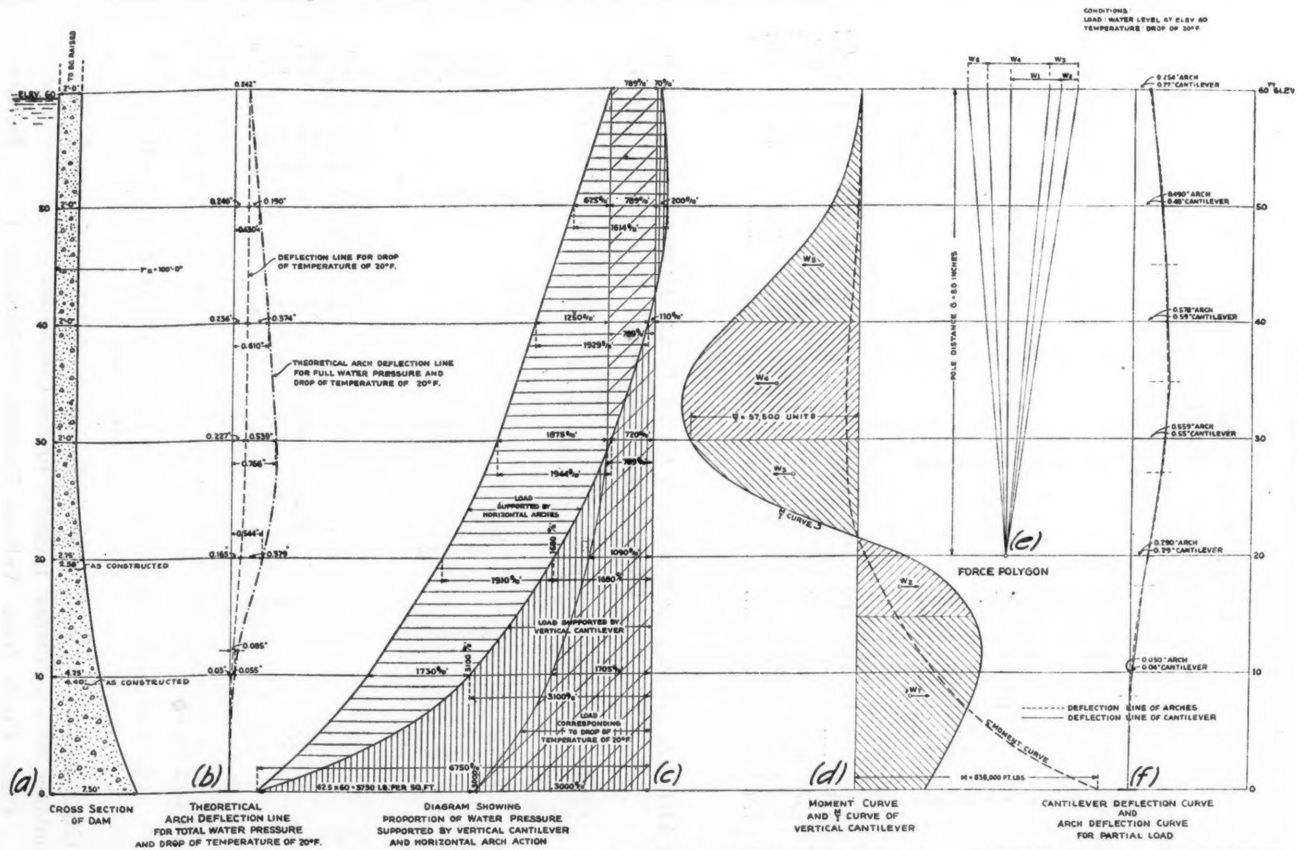


FIG. 4.—DIVISION OF WATER LOAD BETWEEN CANTILEVER AND ARCHES FOR FULL RESERVOIR AND A UNIFORM DROP IN TEMPERATURE OF 20 DEGREES, FAHRENHEIT.

TABLE 7.—DESIGN OF STEVENSON CREEK DAM: DATA FOR CALCULATING ARCH DEFLECTIONS.

(Condition:
Load: Water surface at Elevation 40.
Temperature: No change.)

Elevation, in feet.	Arch thickness, t, in feet.	Radius of arch center line, r, in feet.	$\frac{t}{r}$.	Length of arch, in feet.	Central angle, 2ϕ.	Deflection coefficient, C_δ.	$\frac{r\,r_u}{E\,t}$.	Deflection of Arches, for: Full Load on Arches. Load.	Full Load on Arches. Deflection.	Partial Load on Arches. Load.	Partial Load on Arches. Deflection.
								Pounds per square foot.	Inches.	Pounds per square foot.	Inches.
60	2.00	99.0	0.0202	139	79°40′	1.85	1 375	0	0	— 85	—0.026
50	2.00	....	......	118	67°40′	1.84	.....	0	0	+ 30	+0.009
40	....	....	......	97	55°30′	1.81	.....	0	0	210	0.063
30	2.00	99.0	0.0202	76	43°30′	1.74	1 375	625	0.180	400	0.114
20	2.75	98.63	0.0279	55	31°30′	1.27	996	1 250	0.190	550	0.083
10	4.25	97.88	0.0434	34	19°30′	0.23	638	1 875	0.033	975	0.017
0	7.50	96.25	0.0780	...	......	....	357	2 500	.....	0	0

TABLE 8.—DESIGN OF STEVENSON CREEK DAM: DATA FOR CALCULATING CANTILEVER DEFLECTIONS.*

$$\left(\text{Pole distance, } G = \frac{1}{2} \times \frac{288\,000\,000}{3\,600\,000 \times 60} = 0.667 \text{ ft., or } 8.0 \text{ in.}\right)$$

Elevation, in feet.	Thickness of cantilever, t, in feet.	Partial load, in pounds per square foot.	Bending moments, M, in foot-pounds.	Moment of inertia, I.	$\frac{M}{I}$.	$\frac{M}{I}$-areas = W.	Deflection, in inches.
60	2.00	85	0	0.667	0	$W_7 = +\ 9\,500$	—0.02
50	2.00	— 30	+ 1 425	......	+ 2 140	$W_6 = +\ 27\,000$	+0.02
40	2.00	— 210	+ 700	......	+ 950	$W_5 = -115\,000$	0.07
30	2.00	+ 225	— 14 580	0.667	—21 900	$W_4 = -175\,000$	0.11
20	2.75	700	— 7 980	1.73	— 4 610	$W_3 = -\ 2\,500$	0.07
10	4.25	900	+ 66 820	6.39	+10 450	$W_2 = +\ 73\,000$	0.02
0	7.50	2 500	+237 300	35.2	+ 6 740	$W_1 = +\ 86\,500$	0

* Moduli of elasticity assumed: For arches, $E = 2\,500\,000$ lb. per sq. in.; for cantilever, $E = 2\,000\,000$ lb. per sq. in.

4.—EXPERIMENTAL INFORMATION FROM DAMS IN SERVICE AND UNDER CONSTRUCTION

Experiments were made periodically on the following four arch dams in accordance with a program prepared by the Committee:

Clear Creek Dam, Yakima Project, Washington, U. S. Bureau of Reclamation;

Gerber Dam, Klamath Project, Oregon, U. S. Bureau of Reclamation;
Emigrant Creek Dam (constructed in the summer of 1924), Talent Irrigation District, Oregon;
Dam No. 6, Big Creek Development, Southern California Edison Company.

Tests were recently initiated on three dams, construction of which was started in the early part of 1927, namely:

Gibson Dam, Sun River Project, Montana, U. S. Bureau of Reclamation;
Bull Run Storage Dam, City of Portland, Ore.;
Coolidge Dam, Gila River, Arizona.

Some experimental work is being conducted on several other dams, for instance, on the Shaver Lake Dam, Big Creek Development, Southern California Edison Company, and on the Cutler Dam, Utah Power and Light Company. Attempts were made at various times to test the Lake Spaulding Arch Dam, and the Lake Eleanor and Lake Hodges Multiple-Arch Dams; but remoteness, lack of funds, and other considerations prevented tests from being made on a useful scale.

In general, it was found extremely difficult to obtain satisfactory measurements of strains and deflections on service dams. Usually weeks or months elapse before the reservoir level changes sufficiently to produce substantial changes in deflections and strains due to loads. During the same period, the deformations of the concrete, due to changes of temperature, or of moisture content of the concrete, and to yielding under continuous load, may be of such magnitude as almost completely to obliterate the strains due to changes of the water load. For some dams the reservoirs do not become full every year, and for others they are never quite emptied, so that "full-load" and "no-load" readings within a reasonably short time cannot be obtained. Furthermore, it is difficult to obtain sufficient information on the variation of the temperature unless a large number of thermometers be placed in the dam. For service dams this is seldom practicable, especially for those from 50 to 100 ft. thick.

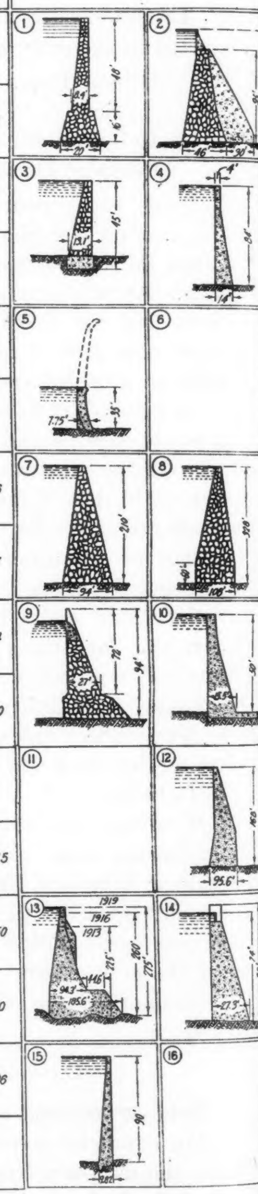
Conditions for tests are naturally much more favorable for dams in which instruments were placed during construction. Interesting test data were obtained from the Emigrant Creek Dam, in Oregon. The deflection measurements revealed the fact that with the water level rising in the reservoir, the arch crest deflected up stream near the quarter-points, and down stream near the arch crown. The tests on most service dams under investigation by the Committee are still in progress. No definite conclusions have yet been drawn.

5.—Data of Existing Arch and Multiple-Arch Dams

Data on existing arch and multiple-arch dams in the United States and foreign countries were collected and condensed in Tables 9 to 14. On some dams the data are fragmentary, and for other dams they may not have been included. Information will be welcomed by the Committee for use in subsequent reports.

TABLE 9.—PRINCIPAL DATA ON ARCH DAM IN THE

NUMBER	NAME OF DAM	LOCATION AND STREAM	OWNER	WHEN BUILT	PURPOSE	MAXIMUM HEIGHT FEET	ARCH SPAN AT CREST FEET	TOTAL LENGTH OF DAM AT CREST, FEET	UP-STREAM RADIUS AT CREST FEET	UP-STREAM RADIUS AT BASE FEET	TYPICAL SECTIONS	ARCH THICKNESS AT BASE FEET
1	Bear Valley	San Bernardino Mountains California	Bear Valley Mutual Water Co.	1883-1884	Irrigation	64		300	335	335	(1)	8.4
2	Sweetwater	Sweetwater River San Diego Co. California	San Diego Land and Town Company	1886-1888	Irrigation	1st stage 95 Ultimate 115	300	380	222	222	(2)	46
3	Rio Grande	Rio Grande Isthmus of Panama		1888	Storage	45		108	53	53	(3)	13.1
4	Upper Otay	Otay Creek San Diego County California	City of San Diego California	1900	Irrigation and Water Supply	84		350	359	359	(4)	14
5	Six Mile Creek (Ithaca)	Six Mile Creek New York	Ithica Water Works Company	1903	Water Supply of Ithaca New York	35			60	60	(5)	7.75
6	Campbell's	Cascadilla Creek Ithaca New York		1904		25		88	70	70	(6)	4
7	Pathfinder	North Platte River Wyoming	United States Reclamation Service	1905-1910	Irrigation	210	425		155	186	(7)	94
8	Shoshone	Shoshone River Wyoming	United States Reclamation Service	1905-1910	Irrigation	328			150	197	(8)	108
9	Halligan	Cache la Poudre River Colorado	North Poudre Irrigation Company Ft. Collins Colorado	1910	Irrigation	94		350	324	324	(9)	27
10	Las Vegas	Las Vegas New Mexico	Agua Pura Co. Las Vegas New Mexico	1910	Storage	50		210	250	250	(10)	15.5
11	Salmon River	Salmon River Idaho		1912	Irrigation	220		490			(11)	
12	Salmon Creek	Salmon Creek Alaska	Alaska Gastineau Mining Company. Juneau, Alaska	1912-1913	Power	168	545	640	331	147.5	(12)	37.5
13	Spaulding	South Fork Yuba River California	Pacific Gas and Electric Company. San Fransisco California	1912-1913 Raised 1916 and 1919	Power	275	500	780	440	250	(13)	141
14	Big Creek No 4	Big Creek California	Southern Calif. Edison Company Los Angeles California	1913	Power	80	175	280	150	150	(14)	27.3
15	Clear Creek	Yakima Project Washington Tieton River	United States Reclamation Service	1913	Irrigation	90	180	400	123	106	(15)	9.8
16	Butte Creek	Butte Creek California	Pacific Gas and Electric Company San Fransisco California	1915	Power	45	85				(16)	



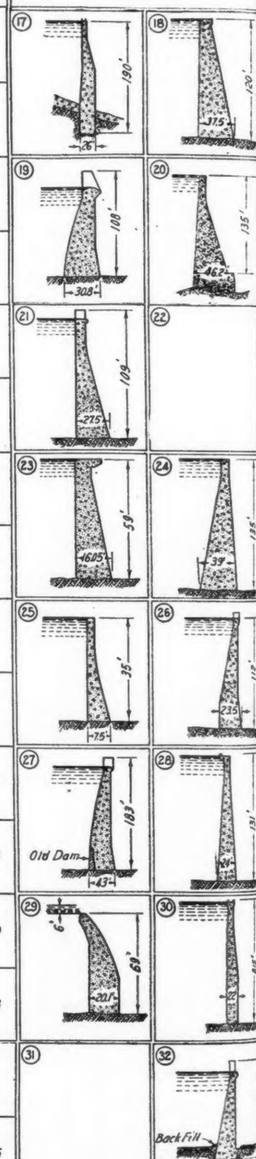
DAM[S] IN THE UNITED STATES.

ARCH THICKNESS AT BASE FEET	MAXIMUM AXIAL CYLINDER STRESS lb./sq. in.	WING WALLS	CONCRETE VOLUME CU. YDS.	CONCRETE MIXTURE	FOUNDATION	RESERVOIR CAPACITY ACRE FT.	TYPE OF DAM	DESIGNER	LIST OF LITERATURE	REMARKS
8.4	835	None		Masonry	Granite	5360	Constant Radius	F.E. Brown	E. Wegmann, The Design and Construction of Dams. 8th ed. p.135 W. G. Bligh, Dams & Weirs. p.104	Submerged by a multiple arch dam built in 1910-11
46	200	None	20507	Masonry up to 95' Cement mortar 1:3 and 1:2	Solid Rock	18100	Constant Radius	F.E. Brown	E. Wegmann, The Design and Construction of Dams. p. 136-40 W. G. Bligh, Dams & Weirs p.109-11	Dam was raised 20' and thickness increased to full gravity section. Stress is given for first stage only.
13.1	680		Masonry 1035 Concrete 187			405	Constant Radius		P. Ziegler, der Talsperrenbau 1911, p. 235	
14	940				Porphyry	2000	Constant Radius	E.S. Babcock	E. Wegmann, The Design and Construction of Dams. p. 220	Upper part reinforced with steel plates and railroad cables
7.75	118	None		1:2:2	Bluish grey Shale		Dome	G.S. Williams	E. Wegmann, The Design and Construction of Dams. p. 200-202	Built up to a height of 35 feet.
4	190	Two gravity wings					Constant Radius	G.S. Williams	Trans. Am. Soc. C.E. 1924 p.345	Reinforced with miscellaneous iron and steel rods.
94	181	Two		Cyclopean rubble masonry $1:2\frac{1}{2}:4$	Granite	100000	Constant Radius	United States Reclamation Service	E. Wegmann, The Design and Construction of Dams. p. 427-30 W.G. Bligh, Dams & Weirs. p. 105 Davis and Wilson, Irrigation Engineering. p. 601 A.P. Davis, Irrigation Works Constructed by U.S. Government. p. 177.	
108	266		65000	Cyclopean rubble masonry	Granite	455000	Constant Radius	United States Reclamation Service	E. Wegmann, The Design and Construction of Dams. p. 430-31 W.G. Bligh, Dams & Weirs. p. 105-7. A.P. Davis, Irrigation Works. p. 379. Davis and Wilson, Irrigation Engineering. 7th ed. p. 474.	
27	375	None	15634 incl. masonry	1:3:5 1:3:6 $1:2\frac{1}{2}:4$	Rock	6408	Constant Radius	G.N. Houston	Trans. Am. Soc. C.E. 1912.	Height of arch 72 ft. Top of arch reinforced
15.5	350		2700					C.W. Sherman	Trans. Am. Soc. C.E. 1915.	
						228000	Constant Radius			
37.5	330	None	52000				Variable Radius	L.R. Jorgensen	E. Wegmann, The Design and Construction of Dams. p. 436g.	Deflections of dam were measured
141		Two gravity wings	192000	$1:2\frac{1}{2}:5$			Variable Radius	L.R. Jorgensen	E. Wegmann, The Design and Construction of Dams. p. 436g. Trans. Am. Soc. C.E. 1915. p. 685	The dam is raised. Base was started as gravity section.
27.3	170	Two gravity wings			Granite		Constant Radius	Southern California Edison Co.		
9.82	400	Two gravity wings			Rock		Variable Radius	United States Reclamation Service		Dam was constructed in two stages. Investigated 1924-1927 by Committee on Arch Dam Investigation.
							Variable Radius			

TABLE 9.

NUMBER	NAME OF DAM	LOCATION AND STREAM	OWNER	WHEN BUILT	PURPOSE	MAXIMUM HEIGHT FEET	ARCH SPAN AT CREST FEET	TOTAL LENGTH OF DAM AT CREST, FEET	UP-STREAM RADIUS AT CREST FEET	UP-STREAM RADIUS AT BASE FEET
17	East Canyon Creek	East Canyon Creek Utah	Davis and Webber Counties Canal Company	1916	Irrigation	190	115	290	98.8	78
18	Sun River Diversion	Sun River Project Idaho	United States Reclamation Service	1917	Irrigation	120	125	250	80	50
19	Kerckhoff	San Joaquin River California	San Joaquin Light and Power Corp. Fresno, Calif.	1919	Power	108	450	570	205	136
20	Gibraltar	Santa Ynez River California	City of Santa Barbara California	1919	Domestic Water Supply	165	460	950	243.5	250
21	Warm Springs	Malheur River Oregon	Warm Springs Irrigation District	1920	Irrigation	109		549	190	190
22	Carmel River	Monterey California		1920		90		270	135	80
23	Big Creek No 5	Big Creek California	Southern California Edison Company Los Angeles California	1921	Power	59		219	150	150
24	Big Creek No 6	San Joaquin River California	Southern California Edison Company Los Angeles California	1922	Power	135	300	380	180	180
25	Malibu Lake	Malibu Lake California	Malibu Lake Club	1922	Pleasure	35	140	160	135	135
26	Lost Creek	Lost Creek Butte County California	Oroville-Wyandotte Irrigation District	1923-1924	Irrigation	112	360		200	90
27	Bullards Bar	Yuba River California	Yuba River Power Company San Fransisco California	1922-1924	Power and Irrigation	183	440		240	94
28	Upper Hubbart	Flathead Project Montana	United States Reclamation Service	1923	Irrigation	131	400	503	220	188
29	Oak Grove	Oak Grove Creek Clackamas County Oregon	Portland Railway Light and Power Company	1923-1924	Power	69			127.5	120
30	Mormon Flat	Salt River Arizona	Salt River Valley Water Users Association	1923-1925		216	320	416	193	108
31	Copco No 1	Klamath Oregon	California Oregon Power Company	1922	Power	257		420		
32	Gerber	Klamath Project Oregon	United States Reclamation Service	1923-1925	Irrigation and Flood Control	82	360	478	197	155

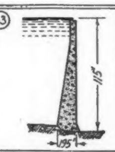
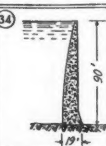
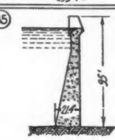
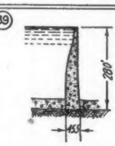
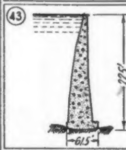
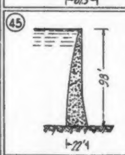
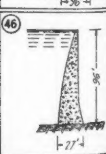
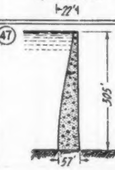
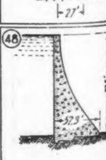
TYPICAL SECTIONS



(*Continued*).

ARCH THICKNESS AT BASE FEET	MAXIMUM AXIAL CYLINDER STRESS lb./sq. in.	WING WALLS	CONCRETE		FOUNDATION	RESERVOIR CAPACITY ACRE FT.	TYPE OF DAM	DESIGNER	LIST OF LITERATURE	REMARKS
			VOLUME CU. YDS.	MIXTURE						
26.0	174	Two gravity wings	16000	1:3:5	Limestone Conglomerate	28000	Variable Radius	A. F. Parker	Trans. Am. Soc. C.E. Vol. 83 (1919-1920) p. 574	
37.5	76	One gravity wing					Variable Radius	United States Reclamation Service		
30.8	225	Two gravity wings	23000	1:2½:5	Granite		Variable Radius	B. F. Jakobsen	Trans. Am. Soc. C.E. 1921 p. 107 Electrical World. Feb. 1921 p. 471	Water level raised 24 feet by radial gates.
46.2	318	One	50000	1:2½:5	Sandstone	16000	Constant Radius	Quinton, Code & Hill	Journal of Electricity Oct. 1, 1920 p. 325 E. Wegmann The Design and Construction of Dams 1927, p. 641	
27.5	313	None	19400	1:2½:5	Close-grained basalt with fine seams filled with putty like clay	170000	Constant Radius	A. J. Wiley	Engineering News-Record. March 1924	
							Variable Radius			
16.1	238	None		1:3:6	Granite		Constant Radius	Southern California Edison Company		Spillway elevation 11 feet below crest.
39	270	Two gravity wings		1:2½:5	Granite		Constant Radius	Southern California Edison Company		
7.5	275	None	1600	1:2½:5	Granite		Constant Radius	F. A. Noetzli		
23.5	312	None	11000	1:2¼:6½	Hard serpentine and diorite	5780	Variable Radius	L. R. Jorgensen	Western Construction News. June 1927	The dam is to be raised.
43	337			1:3:6		11000	Variable Radius	L. R. Jorgensen	Western Construction News April 1927	Designed for storage debris. Specific gravity 90 lb./sq. ft. equivalent to hydrostatic pressure.
24	400	None	17000	1:2½:5	Quartzite	12000	Variable Radius		Water Works June 1925	Upper part reinforced. Investigated by committee on arch dam investigation
20	195	None			Rock		Constant Radius			The dam is reinforced.
22	458	None	43000	1:2:4	Rock	90000	Variable Radius	F. J. O'Hara	Engineering News-Record. May 13, 1926.	
95							Variable Radius	L. R. Jorgensen	Modern Irrigation June 1927	
17.8	310	None	11900	1:2:4 and 1:2¼:4¼	Stratified hard and soft Lava	94000	Variable Radius	United States Reclamation Service	Western Construction News. Feb. 1926 New Reclamation Era. Dec. 1925 Engineering News-Record. Feb. 25, 1926	Investigated by Committee on Arch Dam Investigation.

TABLE 9.—

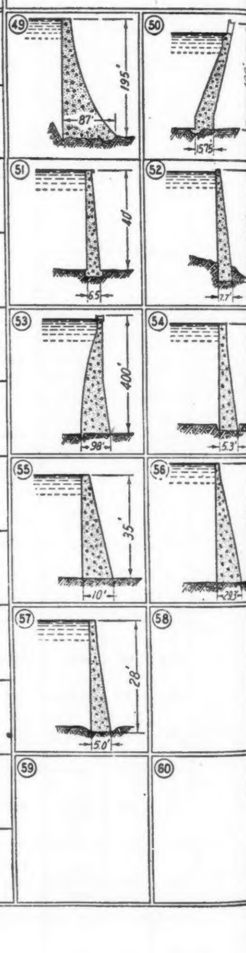
NUMBER	NAME OF DAM	LOCATION AND STREAM	OWNER	WHEN BUILT	PURPOSE	MAXIMUM HEIGHT FEET	ARCH SPAN AT CREST FEET	TOTAL LENGTH OF DAM AT CREST, FEET	UP-STREAM RADIUS AT CREST FEET	UP-STREAM RADIUS AT BASE FEET	TYPICAL SECTIONS
33	Emigrant Creek	Emigrant Creek Talent Oregon	Talent Irrigation District	1924	Irrigation	127	245	430	165	132	(33)  (34) 
34	Conco	Concow Creek California	Thermalito Irrigation District	1924	Irrigation	90		275	181	78	
35	Drum Afterbay	Bear River California	Pacific Gas and Electric Company San Fransisco California	1924	Power	97	255	322	175	190.2	(35)  (36)
36	Malibu or Rindge	Malibu Creek California	Rindge Ranch Company	1924	Irrigation	100					
37	Moyie River	Moyie River Near Bonners Ferry Idaho	Cyanide Gold Mining Company			53		154	65		(37) (38)
38	Lanier Lake	Lanier Lake at Tryon North Carolina	Tryon Development Company Tryon North Carolina	1925		62		236			
39	Cushman Lake	Skokomish River Washington	City of Tacoma Washington	1925	Power	280	340	1110	210	163	(39)  (40) 
40	Melones	Stanislaus River California	Oakdale and South San Joaquin Irrigation District	1926-1927	Irrigation	191	390	591	238	238	
41	Bluewater-Toltec	Bluewater Canyon New Mexico	Bluewater-Toltec Irrigation District New Mexico	1927	Irrigation	97		500	175	175	(41) (42)
42	Santeetlah	Tallassee River Pennsylvania	Tallassee Power Company Pennsylvania	1926-1927	Power	202		340	300	207	
43	Big Santa Anita	Big Santa Anita Creek California	Los Angeles County Flood Control California	1926-1927	Flood Control	225	520	590	305	129	(43)  (44) 
44	Pacoima	Pacoima Creek California	Los Angeles County Flood Control California	1926-1928	Flood Control	380	530	600	330	171	
45	Grizzly Creek	Grizzly Creek California	Hydro-Electric Feather River Power Company	1927	Power	98		500	205	140	(45)  (46) 
46	Bucks Creek	Bucks Creek California	Hydro-Electric Feather River Power Company	1927	Power	96		420	213	158	
47	Horse Mesa	Salt River Arizona	Salt River Irrigation Project	1926-1927	Irrigation	305	425	810	Arches are not circular See Remarks		(47)  (48) 
48	Bowman, South	Nevada County California	Nevada Irrigation District California	1928	Irrigation	103		373	175	175	

(*Continued*).

ARCH THICKNESS AT BASE FEET	MAXIMUM AXIAL CYLINDER STRESS lb./sq. in.	WING WALLS	CONCRETE		FOUNDATION	RESERVOIR CAPACITY ACRE FT.	TYPE OF DAM	DESIGNER	LIST OF LITERATURE	REMARKS
			VOLUME CU. YDS.	MIXTURE						
19.5	340	One gravity wall and one auxiliary arch	15200	1:2½:5	Tertiary Sandstone	8000	Variable Radius	F.C. Dillard		The arch is reinforced. Investigated by Committee on Arch Dam Investigation.
19	300		6300	1:3:6		8200	Variable Radius	Constant Angle Arch Dam Company		
21.4	365	None	5813	1:2½:5	Granite	275	Constant Radius	I.C. Steele		
							Constant Radius	Wayne Loel		
					Stratified Rock			O. Jones	Engineering News-Record Oct. 14, 1926	Dam damaged due to washing out of soft rock of one abutment. Dam was reinforced with railroad rails.
								G. Kershaw	Engineering News-Record Oct. 14, 1926	Dam failed Jan. 21, 1926 due to washing out of decomposed rock under West abutment.
52	250	One earth fill & two gravity wings	90000	1:2½:5	Basalt	460000	Variable Radius	B.E. Torpen	Engineering News-Record. April 4, 1927	
74.2	266	Two gravity wings	91000	1:3:4:3 cobbles	Dolerite and Diorite	112500	Constant Radius	A.J. Wiley	Modern Irrigation. June 1927 Western Construction News. May 1927	
		One gravity wing		1:4:4½ and cobbles		53000	Constant Radius	V.L. Sullivan	Modern Irrigation. June 1927	Dam is reinforced.
							Variable Radius			
61.5	350	None	76000	1:2.3:6.4	Granite	1500	Variable Radius	Los Angeles County Flood Control	Western Construction News. Nov. 25, 1927	
98	300	None	151000	1:2.8:6	Granite	12000	Variable Radius	Los Angeles County Flood Control		
22	300	One gravity wing	11000	1:3:6 with 6" cobbles minimum size	Granite	1200	Variable Radius	L.R. Jorgensen	Western Construction News. Feb. 1926	
27	300	Two gravity wings		1:3:6 with 6" cobbles minimum size	Granite	13700	Variable Radius	L.R. Jorgensen	Western Construction News. Feb. 1926	
57	400			1:2½:5	Quartzite	245000	Variable Radius	C.C. Cragin		Curvature of arches at all elevations determined according to shape of pressure line.
52.3	150	One gravity wing and one Ambursen spillway	3000			65000		F.H. Tibbetts	Western Construction News. Oct 1927	

TABLE 9

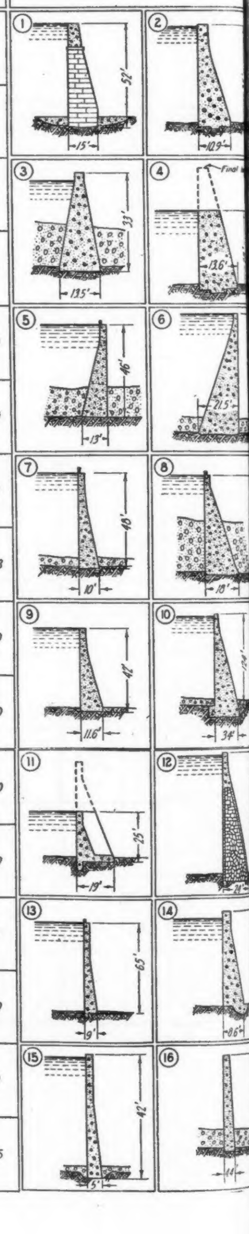
NUMBER	NAME OF DAM	LOCATION AND STREAM	OWNER	WHEN BUILT	PURPOSE	MAXIMUM HEIGHT FEET	ARCH SPAN AT CREST FEET	TOTAL LENGTH OF DAM AT CREST, FEET	UP-STREAM RADIUS AT CREST FEET	UP-STREAM RADIUS AT BASE FEET	TYPICAL SECTIONS
49	Gibson	North Fork of Sun River Great Falls Montana	United States Bureau of Reclamation	1926-1928	Irrigation	195	700	850	405	405	
50	Railroad Canyon	San Jacinto River California	Tamescal Water Company California	1927-1928	Irrigation	100	325	625	205	134	
51	Morris	James Creek Near Willits California	Central Mendocino County Power Company	1924	Irrigation and Power	40		110	90	90	
52	Fruita Canyon		Town of Fruita Colorado	1924	Water Supply	56.5	110	117	60.6	60.6	
53	Diablo Canyon	Skagit River Washington	City of Seattle Washington		Power	400	532	1142	390	163.5	
54	Buffalo Creek	Buffalo Creek North Carolina	Ivey Power Company	1916	Power	27	150	170	100	100	
55	Moretz Mill	Little River North Carolina	Dudley Shoals Cotton Mill Company	1917	Power	35	220	320	145	145	
56	Tuxedo	Green River North Carolina	Blue Ridge Power Company	1917	Power	121	200	370	150	150	
57	Yadkin River	Yadkin River North Carolina	Watts Cotton Mills	1919	Power	28	120	160	76	76	
58											
59											
60											



(Continued).

ARCH THICKNESS AT BASE FEET	MAXIMUM AXIAL CYLINDER STRESS lb./sq. in.	WING WALLS	CONCRETE		FOUNDATION	RESERVOIR CAPACITY ACRE FT.	TYPE OF DAM	DESIGNER	LIST OF LITERATURE	REMARKS
			VOLUME CU. YDS.	MIXTURE						
•87	395	None			Fine-grained Limestone	105000	Constant Radius	United States Bureau of Reclamation	Modern Irrigation. June 1927	
15.75	360	Two gravity wings	12500	1800 lb./sq. in. at 28 days	Slate	11000	Variable Radius Cone Type	F.A. Noetzli		Upper portion of dam is reinforced.
6.5	250		800	1:3:6		1400	Constant Radius	C.E. Grunsky	Modern Irrigation. Jan. 1926 Aqua Fax May 1925	Dam is reinforced.
7.7	300	None			Hard Igneous Rock		Constant Radius			
98		Two gravity wings	250000	1:7½	Granite		Variable Radius	Constant Angle Arch Dam Company		
5.3	270	None					Constant Radius	C.A. Mees		
10.0	280	One gravity wing					Constant Radius	C.A. Mees		Designed for 10 ft. overflow.
29.3	280	One gravity wing					Constant Radius	C.A. Mees		Designed for 4 ft. overflow.
5.0	225	One gravity wing					Constant Radius	C.A. Mees		Designed for 6 ft. overflow.

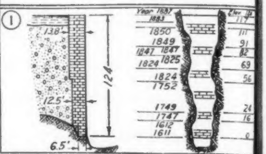
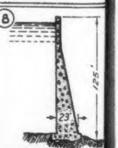
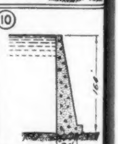
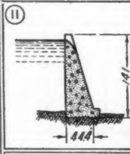
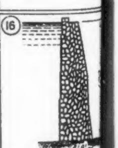
TABLE 10.—PRINCIPAL D ON A

NUMBER	NAME OF DAM	LOCATION AND STREAM	OWNER	WHEN BUILT	PURPOSE	MAXIMUM HEIGHT FEET	ARCH SPAN AT CREST FEET	TOTAL LENGTH OF DAM AT CREST, FEET	UP-STREAM RADIUS AT CREST FEET	UP-STREAM RADIUS AT BASE FEET	TYPICAL SECTIONS	ARCH THICKNESS AT BASE FEET
1	Parramatta	New South Wales	Government	1858-1898	Irrigation	52		225	160	160		15
2	Lithgow Nº 1	New South Wales	Government	1896	Irrigation	35		178	100	100		10.9
3	Parkes	New South Wales	Government	1897	Irrigation	33		540	300	300		13.5
4	Picton	New South Wales	Government	1897	Irrigation	28		112	120	120		13.6
5	Cootamundra	New South Wales	Government	1898	Irrigation	46		640	250	250		13
6	Tamworth	New South Wales	Government	1898	Irrigation	61		440	250	250		21.5
7	Wellington	New South Wales	Government	1899	Irrigation	48		350	150	150		10
8	Mudgee	New South Wales	Government	1899	Irrigation	50		498	253	253		18
9	Wollongong	New South Wales	Government		Irrigation	42		535	200	200		11.6
10	Barossa	Gawler		1903	Irrigation and Water Supply	124	370		200	200		34
11	Katoomba	New South Wales	Government	1905	Irrigation	25		320	220	220		20.
12	Lithgow Nº 2	New South Wales	Government	1906	Irrigation	87		221	100	100		24
13	Medlow	New South Wales	Government	1906	Irrigation	65		124	60	60		9
14	Queen Charlotte Vale	New South Wales	Government	1906	Irrigation	32		113	90	90		8.6
15	Barren Jack	New South Wales		1908		42			80	80		5
16	Wooling	Macedon Victoria		1916		33			135	135		4.4

ON ARCH DAMS IN AUSTRALIA.

ARCH THICKNESS AT BASE FEET	MAXIMUM AXIAL CYLINDER STRESS lb./sq. in.	WING WALLS	CONCRETE		FOUNDATION	RESERVOIR CAPACITY ACRE FT.	TYPE OF DAM	DESIGNER	LIST OF LITERATURE	REMARKS
			VOLUME CU. YDS.	MIXTURE						
15	233	None		1:2.5:5	Sandstone	2980	Constant Radius	Upper part by C. W. Darley	E. Wegmann, The Design and Construction of Dams	Original height 41 ft. of masonry in Roman cement, built 1858. Raised to 52 ft. with concrete in 1898.
10.9	155	None		1:2.5:5	Sandstone	340	Constant Radius	C. W. Darley	E. Wegmann, The Design and Construction of Dams.	
13.5	372	One gravity wing		1:2.5:5	Granite	2620	Constant Radius	C. W. Darley	E. Wegmann, The Design and Construction of Dams.	
13.6	186	None		1:2.5:5	Sandstone	320	Constant Radius	C. W. Darley	E. Wegmann, The Design and Construction of Dams.	Constructed to be raised 14 ft. when required.
13	387	None		1:2.5:5	Granite	3120	Constant Radius	C. W. Darley	E. Wegmann, The Design and Construction of Dams.	
21.5	310	One gravity wing		1:2.5:5	Granite	1150	Constant Radius	C. W. Darley	E. Wegmann, The Design and Construction of Dams	
10	310	None		1:2.5:5	Conglomerate	620	Constant Radius	C. W. Darley	E. Wegmann, The Design and Construction of Dams.	
18	310	None		1:2.5:5	Altered Slate	963	Constant Radius	C. W. Darley	E. Wegmann, The Design and Construction of Dams	
11.6	310	One gravity wing		1:2.5:5	Basalt	3670	Constant Radius	C. W. Darley	E. Wegmann, The Design and Construction of Dams	
34	306		18000			3675	Constant Radius		W. G. Bligh, Dams & Weirs. 1918	Crest of dam is reinforced with rails.
20.3	233	None		1:2.5:5	Sandstone	780	Constant Radius	C. W. Darley	E. Wegmann, The Design and Construction of Dams.	Constructed with buttresses. Ultimate height 50 feet.
24	155	None		1:2.5:5	Sandstone	2020	Constant Radius	L. A. B. Wade	E. Wegmann, The Design and Construction of Dams	Arch built in masonry and faced with concrete
9	186	None		1:2.5:5	Sandstone	1530	Constant Radius	L. A. B. Wade	E. Wegmann, The Design and Construction of Dams	
8.6	155	None		1:2.5:5	Quartzite		Constant Radius	L. A. B. Wade	E. Wegmann, The Design and Construction of Dams.	
5	290						Constant Radius		Trans. Am. Soc. C.E. 1915. p. 572 Trans. Am. Soc. C.E. 1922. W. G. Bligh, Dams & Weirs 1918.	Arch is reinforced with rails. Deflections of this dam were measured.
4.4	440						Constant Radius	B. A. Smith	Trans. Am. Soc. C.E. 1919-20	Arch is reinforced.

TABLE 11.—PRINCIPAL DATA

NUMBER	NAME OF DAM	LOCATION AND STREAM	OWNER	WHEN BUILT	PURPOSE	MAXIMUM HEIGHT FEET	ARCH SPAN AT CREST FEET	TOTAL LENGTH OF DAM AT CREST, FEET	UP-STREAM RADIUS AT CREST FEET	UP-STREAM RADIUS AT BASE FEET	TYPICAL SECTIONS
1	Ponte Alto	Aviso Creek Adige Valley Trentino, Italy	District and Government	1611-1884	Flood Control	124		40	54	49	1 
2	Camelli	Gola Creek Adige Valley Trentino, Italy	District and Government	1882-1883	Flood Control	69					
3	Terragnolo	Leno Creek Adige Valley Trentino, Italy	District and Government	1884	Flood Control	56					3 4
4	San Colombano	Leno Creek Adige Valley Trentino, Italy	District and Government	1884	Flood Control	61					
5	Cantang' '	Fersina Creek Adige Valley Trentino, Italy	District and Government	1884	Flood Control	79					5 6
6	San Giorgio	Aviso Creek Adige Valley Trentino, Italy	District and Government	1886	Flood Control	88					
7	Ponte Della Serra	Cismon Creek Brenta River Belluno, Italy	Adriatic Electric Company	1910	Power	144					7 8 
8	Corfino	Corfino Creek Serchio River Massa, Italy	Liguro-Tuscanian Electric Company	1914	Power	125			77	77	
9	Lake Campelli	Goglio Valley Serio River Bergamo, Italy	Crespi Electric Company	1920	Power	45					9 10 
10	Furlo	Candigliano River Pesaro, Italy	Union of Electricity Users	1922	Power	164					
11	Turrite	Turrite di Gallicano Creek Serchio River Massa, Italy	Liguro-Tuscanian Electric Company	1922	Power	141			111	111	11  12
12	Zolezzi	Penna River Genoa, Italy	Monte Ajona Hydroelectric Company	1923	Power	69					
13	Molinara	Molinara Creek Lake Como-Adda-Po Como, Italy	The Barite Company	1923	Power	66					13 14
14	Scuro o di Spigno Lake	Valla Creek Alessandria Italy	Chivasso Iron Works	1923	Power	131					
15	Gurzia	Chiusella Creek Po Valley Turin, Italy	Upper Italian Electric Company	1923	Power	154					15 16 
16	Zola	Near Aix Bouches du Rhone France	City of Aix	1843-1852	Water Supply	120		205	158	158	

ON ARCH DAMS IN EUROPE.

ARCH THICKNESS AT BASE FEET	MAXIMUM AXIAL CYLINDER STRESS lb./sq. in.	WING WALLS	CONCRETE VOLUME CU. YDS.	CONCRETE MIXTURE	FOUNDATION	RESERVOIR CAPACITY ACRE FT.	TYPE OF DAM	DESIGNER	LIST OF LITERATURE	REMARKS
6.5	410	None		Cut stone masonry	Dolomite		Variable Radius		Annali delle Utilizzazioni delle Acque. Anno. 1924 Vol. I. Fasc. 1.	Dam started in year 1611. Raised in the years 1747, 1749, 1752, 1824, 1847, 1849, 1850, 1883 and 1887.
				Cut stone masonry	Dolomite				Annali delle Utilizzazioni delle Acque. Anno. 1924. Vol. I. Fasc. 1.	
				Cut stone masonry	Dolomite				Annali delle Utilizzazioni delle Acque. Anno. 1924. Vol. I. Fasc. 1	
				Cut stone masonry	Dolomite				Annali delle Utilizzazioni delle Acque. Anno. 1924. Vol. I. Fasc. 1	
				Cut stone masonry	Dolomite				Annali delle Utilizzazioni delle Acque. Anno. 1924. Vol. I. Fasc 1	
				Cut stone masonry	Dolomite				Annali delle Utilizzazioni delle Acque. Anno. 1924. Vol. I. Fasc 1	
		None				3730			Annali delle Utilizzazioni delle Acque. Anno. 1924. Vol. I. Fasc. 1	
23	181					650	Constant Radius		Annali delle Utilizzazioni delle Acque. Anno. 1924. Vol. I Fasc. 1 H. Kelen, die Staumauern. 1926. p. 184.	
		None		Concrete		400			Annali delle Utilizzazioni delle Acque. Anno. 1924. Vol. I. Fasc 1	
		None				1620	Constant Radius		Annali delle Utilizzazioni delle Acque. Anno. 1924. Vol. I. Fasc. 1	
44.4	156	None				730	Constant Radius		Annali delle Utilizzazioni delle Acque. Anno. 1924. Vol. I. Fasc. 1. H. Kelen, die Staumauern. 1926. p. 185	
		Two gravity wings				55			Annali delle Utilizzazioni delle Acque. Anno. 1924. Vol. I. Fasc. 1.	Arch is reinforced concrete. Gravity wings are masonry in lime mortar
				Masonry in lime mortar					Annali delle Utilizzazioni delle Acque. Anno. 1924. Vol. I. Fasc. 1.	
						2340			Annali delle Utilizzazioni delle Acque. Anno. 1924. Vol. I. Fasc. 1.	
		Two gravity wings				1470			Annali delle Utilizzazioni delle Acque. Anno. 1924. Vol. I. Fasc. 1	Left gravity wing is masonry in lime mortar.
41.8	197	None		Rubble masonry	Hard conglomerate cemented with Calcite	1140	Constant Radius	M. Zola	E. Wegmann, The Design and Construction of Dams. 1922. p. 64 P. Ziegler, Talsperrenbau. 1911. p. 233 H. Kelen, die Staumauern. 1926 p. 184	

TABLE 1

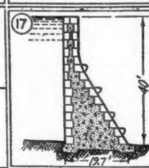
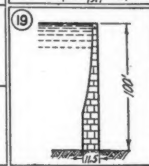
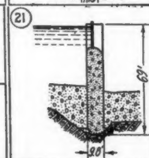
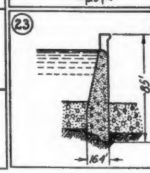
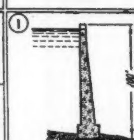
NUMBER	NAME OF DAM	LOCATION AND STREAM	OWNER	WHEN BUILT	PURPOSE	MAXIMUM HEIGHT FEET	ARCH SPAN AT CREST FEET	TOTAL LENGTH OF DAM AT CREST, FEET	UP-STREAM RADIUS AT CREST FEET	UP-STREAM RADIUS AT BASE FEET	TYPICAL SECTIONS
17	Fully	Fully Lake Canton Valais Switzerland	Electro-Chemical Company of Martigny	1909-1917	Power	40		360	492	492	⑰ 
18	La Jogne	La Jogne River Canton Fribourg Switzerland	Electric Company of Fribourg	1918-1921	Power	171	250	530	123		⑱ 
19	Amsteg	Reuss River Canton Uri Switzerland	Swiss Government Railroads	1922	Power	100	96	216	66	46	⑲ 
20	Montejaque	Gaduares River Malaga, Spain	Sevillan Electric Company	1923-1924	Power	273		256	123	72	⑳ 
21	Gullspång	Sweden		1911		69					㉑ 
22	Stenkvill-Klints	Sweden									㉒
23	Gideåbacka	Gideåbacka Sweden	Gidå och Husums Aktiebolags	1918	Power	85	131	165	98	72	㉓ 
24											㉔

TABLE 12.—PRINCIPAL DATA ON

NUMBER	NAME OF DAM	LOCATION AND STREAM	OWNER	WHEN BUILT	PURPOSE	MAXIMUM HEIGHT FEET	ARCH SPAN AT CREST FEET	TOTAL LENGTH OF DAM AT CREST, FEET	UP-STREAM RADIUS AT CREST FEET	UP-STREAM RADIUS AT BASE FEET	TYPICAL SECTIONS
1	Huacal	Huacal Creek Near Nacozari Sonora Mexico	Moctezuma Copper Company Nacozari, Sonora Mexico	1911-1912	Storage	100		140	76	76	① 
2	Manila	Manila Philippine Islands		1913		98	100				②
3	Urayama River	Urayama River Tokio, Japan		1921		45			48		③
4	Hartebeespoort	Crocodile River Pretoria Transvaal	Transvaal Irrigation Department	1920-1923	Irrigation	194			240	148	④

(*Continued*).

ARCH THICKNESS AT BASE FEET	MAXIMUM AXIAL CYLINDER STRESS lb./sq. in.	WING WALLS	CONCRETE VOLUME CU. YDS.	CONCRETE MIXTURE	FOUNDATION	RESERVOIR CAPACITY ACRE FT.	TYPE OF DAM	DESIGNER	LIST OF LITERATURE	REMARKS
19.7	433	None			Rock	3950	Constant Radius	A. Boucher	Schweizerische Bauzeitung 1922 p.247	Arch was raised in 1917. Faced with ashlar masonry.
73.8		One gravity wing					Variable Radius	A. Rohn A. Stucky H. Gruner	Bulletin Technique de la Suisse Romande. Année 1922	
11.5	174	One gravity wing	327	Cut granite masonry	Solid granite	160	Variable Radius		Schweizerische Bauzeitung 1925 N.20 Water Works. Sept 1923. p. 530	Arch is built of cut granite with very thin mortar joints.
55	156	None	35000		Solid limestone	32400	Variable Radius	H.E. Gruner	Engineering News-Record 1924 p.128	
9.8							Constant Radius		First World Power Conference, London 1924	Arch is reinforced.
16.4	162	None		1:3:4 and 1:2:2			Variable Radius	B. Hellström	Teknisk Tidskrift 1919 No.3 First World Power Conference, London 1924	Arch is strongly reinforced.

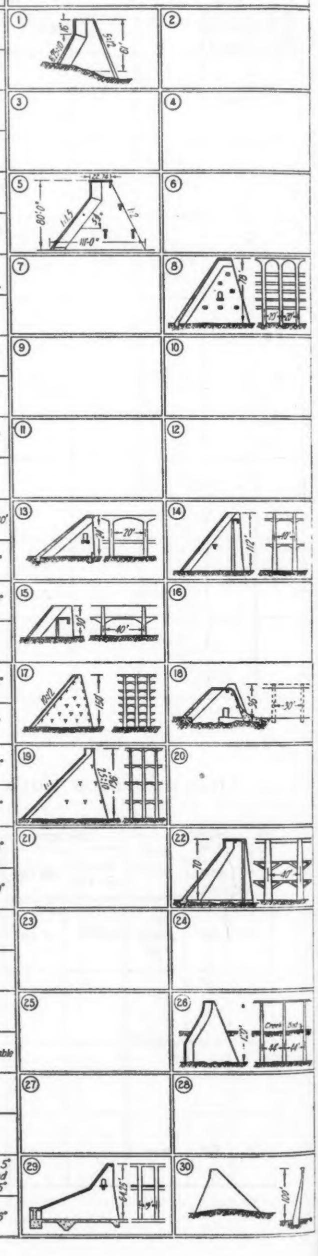
DAMS IN OTHER FOREIGN COUNTRIES.

ARCH THICKNESS AT BASE FEET	MAXIMUM AXIAL CYLINDER STRESS lb./sq. in.	WING WALLS	CONCRETE VOLUME CU. YDS.	CONCRETE MIXTURE	FOUNDATION	RESERVOIR CAPACITY ACRE FT.	TYPE OF DAM	DESIGNER	LIST OF LITERATURE	REMARKS
12.8	326	One gravity wing	2700	1:2:3.85	Andesite	3000	Constant Radius	H. Hawgood	Trans. Am. Soc. C.E. 1915	
							Variable Radius	H.F. Cameron	Engineering Record. Aug. 13, 1913. p.203 E. Wegmann, The Design and Construction of Dams. 7th ed. p.436h.	
							Variable Radius			
73	207	None					Variable Radius		Water Works. Feb 1924. p.340	Arch has a very heavy abutment on one side.

TABLE 13.—PRINCIPAL DATA ON

NUMBER	NAME OF DAM	LOCATION AND STREAM	OWNER	WHEN BUILT	PURPOSE	MAXIMUM HEIGHT FEET	TOTAL LENGTH FEET	SPACING OF BUTTRESSES FEET	NUMBER OF ARCHES	SLOPE OF ARCH BARREL
1	Hume Lake	Ten Mile & Long Meadow Creeks, Sierra Nevada Mountains, Calif.	The Hume-Bennett Lumber Company	1908	Water Supply	61	677	50	12	58°
2	Pecks Lake	Adirondacks New York		1910						
3	Garoga									
4	Sturgis	St. Joseph River Michigan	City Water Works Sturgis, Mich.	1910	Water Supply	30	300	20	15	58°
5	Bear Valley	Bear Valley San Bernardino Mountains California	The Bear Valley Mutual Water Company	1910-1911	Irrigation	92	363	32	10	53°
6	Lost River	Oregon		1911		44		20		62.5°
7	Barton			1912		30	207	20	10	45°
8	Aziscohos	Androscoggin River Maine		1910-1912		78	501	20	25	57°
9	Palmer Falls	Hudson River New York	International Paper Company	1913		38		15		40°
10	Glens Falls	Hudson River New York	International Paper Company	1913		26		9		40°
11	Los Verjels	Oroville California	Los Verjels Land & Water Company	1913-1914		60		20		45°
12	Kennedy	Jackson California		1914		53	486	40	6	
13	Three Miles Falls	Three Miles Falls Oregon		1914		24		20	20	38°-30'
14	Gem Lake	Rush Creek Mono County California	Southern Sierra Power Company	1915-1916	Power	112	690	40	18	50°
15	Agnew Lake	Rush Creek Mono County California	Southern Sierra Power Company	1915-1916	Power	30	280	40	7	50°
16	Argonaut	California	Argonaut Mining Company	1916		50	450	32	14	
17	Mountain Dell	Parley's Canyon, Salt Lake City, Utah	Salt Lake City, Utah	1916 Raised 1924	Water Supply	150	560	35	16	50°
18	Rock Creek	Auburn, California	Pacific Gas & Electric Company, San Fransisco	1916	Power	36	1080	30	36	45°
19	Lake Hodges	San Dieguito River, California	City of San Diego, California	1917	Water Supply	136	550	24	23	45°
20	Murray	San Diego River, California	Cuyamaca Water Company	1917	Water Supply	117	900	30	30	45°
21	San Dieguito	California	San Dieguito Mutual Water Company	1917	Water Supply	52	630	50	14	58°
22	Lake Eleanor	Cherry Creek, Sierra Nevada Mountains, Calif.	Hetch Hetchy Water-Supply of San Fransisco	1917 - 1918	Water Supply	70	800	40	20	50°
23	Eagle's Nest									
24	Superior									
25	Boulder Lake									
26	Cave Creek	Cave Creek Arizona	City of Phoenix Arizona	1922	Water Supply	120	1692	44	38	Variable
27	Gillespie									
28	Green Valley									
29	Sherman Island	Hudson River New York	International Paper Company	1921-1923		80.75	551	19	31	22.5° and 45°
30	Webber Creek	North Fork of Webber Cr. California	El Dorado Water Corporation	1922-1923		120	320	140 93 87	3	76°

TYPICAL SECTIONS



TA ON

Multiple-Arch Dams in the United States.

ARCHES							BUTTRESSES					FOUNDATION	CONCRETE VOLUME CU. YDS.	TOTAL COST OF DAM	RESERVOIR CAPACITY ACRE FT.	DESIGNER	LIST OF LITERATURE	REMARKS
RADIUS	CENTRAL ANGLE 2Φ	THICKNESS TOP	THICKNESS BASE	MIXTURE OF CONCRETE	REINFORCED	SPECIAL WATER-PROOFING	THICKNESS TOP	THICKNESS BASE	STRUTS OR STIFFENERS	MIXTURE OF CONCRETE	REINFORCED							
27'-6"	118°	18"	3'-0"	1:2:4	Yes	Two coats of 1" to 1½" cement plaster	24"	7'-0"	None	1:2:4	Yes	Granite	2267	$46000	10560	J.S. Eastwood	E. Wegmann, The Design and Construction of Dams. Engineering News Dec. 25, 1913	
												Slightly indurated sand and gravel				W. J. Douglas	N. Kelen die Staumauern p. 213 Trans. Am. Soc. C.E. 1917 p. 892	
12'-0"		12"										Clay and Sand		$22000			N. Kelen die Staumauern p. 214 Eng. Record 2. III. 1912 p. 230	The dam is curved in plan with radius of 1200 ft. Arches circular in horiz. planes
17'-0"	140°	12"	2'-2"	1:2:4	Yes	None	18"	4'-6"	4	1:2½:5	No	Rock	4684			J.S. Eastwood	E. Wegmann, The Design and Construction of Dams. W. G. Bligh, Dams & Weirs Journal of Electricity. Oct. 30, 1909	
		12"			No								5536			W. W. Patch E. G. Hopson		
		15"					2'-3"	2'-3"										
Intrados 8'-0" Extrados straight	142°	24"	3'-6"		Yes		4'-0"	4'-0"	7 Struts							S.A. Moulton	Engineering News, March 1911 E. Wegmann, The Design and Construction of Dams. 7th ed. p. 459	
																H. de B. Parsons	Trans. Am. Soc. C.E. 1925 p. 1258	
																H. de B. Parsons	Trans. Am. Soc. C.E. 1925 p. 1258	
		6"			Yes												Western Engineering July 1914	
22'-0"	133°	12"			Yes		12"					Schist		$52025		J.S. Eastwood	N. Kelen die Staumauern p. 215 Engineering News 29. IV. 1915 Eng. Mining Journal 24 I 1917	Arches Three Hinged Raised in 1916 to a height of 50 ft.
18'-0"		12"			Yes		18"	18"	1		Yes						N. Kelen die Staumauern p. 215 Engineering News 27 V 1915	
23'-1"	120°	12"	3'-7"	1:2:4	Yes	Gunite ½" at the crest ¾" at the base	18"	4'-3"	2	1:2½:5	No	Exposed Bed Rock	8537			L.R. Jorgensen	Trans. Am. Soc. C.E. 1917 p. 850 Trans. Am. Soc. C.E. 1928 p. 713	
23'-1"	119°-57'	12"	1'-10¼"	1:2:4	Yes	1:2 plaster coat of cement mortar ½" to ¾"	1'-10½"	2'-9"	1	1:2½:5	No	Exposed Bed Rock				L.R. Jorgensen	Trans. Am. Soc. C.E. 1917 p. 881	
																	E. Wegmann, The Design and Construction of Dams.	
19'-8"	133°	15"	4'-10"	1:2:4	Yes	Lower part waterproofing membrane Gilsonite	18"	8'-0"	22 Struts	Lower part 1:3:6 Upper part 1:2½:5	0.2%	Seamy rock				J.S. Eastwood F.A. Noetzli	N. Kelen die Staumauern p. 221 E. Wegmann, The Design and Construction of Dams	Lower part three hinged arches
16'-10¾"	120°-28'	14"	17"	1:1½:4⅓	Yes	None	19"	2'-8"	1 Strut	1:2½:5	No	Rock	4303	$124000		H. C. Vensano	Journal of Electricity June 1, 1917	Two hinged arches
13'-10"	120°	12"	2'-7"	1:2:4	Yes	1 inch of Gunite	18"	4'-2"	7 Struts	1:2½:5	No	Rock				J.S. Eastwood	Engineering News-Record April 10, 1919 E. Wegmann, The Design and Construction of Dams.	
		12"	2'-6"	1:2:4	Yes		12"	4'-4½"		1:2½:5	No	Porphyritic dyke and old beach hard pan of cemented conglom.				J.S. Eastwood	Journal of Electricity March 15, 1919	
29'-0"	120°								None		No						Journal of Electricity March 15, 1919	
23'-0"	120°	15"	4'-0"	1:2:4	Yes		24"	4'-11"	2 Struts	1:2½:5	Yes	Solid Rock				R.P. McIntosh	Engineering News-Record. Sept. 4, 1919 E. Wegmann, The Design and Construction of Dams.	Arches circular in horizontal planes
24'-3"	130°	12"	4'-0"	1:2:4	Yes	None	12"	8'-10"	None	1:2½:5	Four rows of rails		19000	$600 000		J.S. Eastwood	Engineering News-Record. Jan. 26, 1922 Trans. Am. Soc. C.E. 1924 p. 402 E. Wegmann, The Design and Construction of Dams.	
7'-9"	180°	18"	24"	1:2:4 Lime 6%	Yes		3'-6"	3'-6"	1 Strut	1:3:5	No	Sand and boulders Interlocking steel piles				H. de B. Parsons	Trans. Am. Soc. C.E. 1925 p. 1959 Engineering News-Record. Sept 20, 1923 E. Wegmann, The Design and Construction of Dams.	
77'-2¾" 35'-2"	130° 114°-18' 61°-51'	24"	12'-0"	1:2½:5	No	None	4'-0"		None	1:2½:5	No					J.S. Eastwood	N. Kelen die Staumauern. p. 227 E. Wegmann, The Design and Construction of Dams.	

TABLE 13.—

NUMBER	NAME OF DAM	LOCATION AND STREAM	OWNER	WHEN BUILT	PURPOSE	MAXIMUM HEIGHT FEET	TOTAL LENGTH FEET	SPACING OF BUTTRESSES FEET	NUMBER OF ARCHES	SLOPE OF ARCH BARREL
31	Palmdale	Little Rock Creek California	Little Rock and Palmdale Irrigation Districts	1923	Irrigation	175	648	24	24	45°
32	Florence Lake	San Joaquin River California	Southern California Edison Company	1925-1926	Power	150	3300	50	58	48°
33	Lake Lure	Rocky Broad River North Carolina	Carolina Mountain Power Company	1925	Power	122	574	41	11	45°
34	Alspaugh	Little River North Carolina	Liledoun Manufacturing Company	1917	Power	35	240	35	5	36°-52°
35	Lake Pleasant	Agua Fria River Arizona	Agua Fria Water Conservation Dist. Maricopa Co., Ariz.	1926-1927	Irrigation	256	1975	60	30	47°
36	Sutherland	San Dieguito River California	City of San Diego California	1927-1928	Water Supply	160	1100	60	16	45°
37	Coolidge	Gila River Arizona	United States Government	1926-1928	Irrigation	250	910	180	3	Variable
38	Big Dalton	Big Dalton Creek California	Los Angeles County Flood Control District	1928	Flood Control	165	480	60	8	48°

TYPICAL SECTIONS

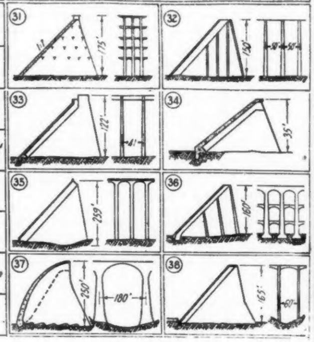
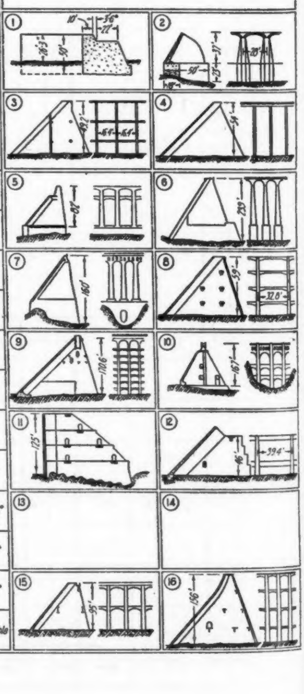


TABLE 14.—PRINCIPAL DATA ON

NUMBER	NAME OF DAM	LOCATION AND STREAM	OWNER	WHEN BUILT	PURPOSE	MAXIMUM HEIGHT FEET	TOTAL LENGTH FEET	SPACING OF BUTTRESSES FEET	NUMBER OF ARCHES	SLOPE OF ARCH BARREL
1	Meer Allum	Hyderabad India	City of Hyderabad India	1806	Water Supply	50	3000	70-147	21	90°
2	Belubula	Belubula River New South Wales	Lyndhurst-Gold Fields Company	1898	Power	60	431	28	7	60°
3	Sélune	Manche France	Société des Forces de la Sélune, France	1916-1917		50	420	16.2	17	45°
4	Belle Isle en Terre	Legner River Cotes du Nord France		1923		54		16	15	45°
5	Scoltenna	Scoltenna Creek Modena, Italy	Società Emiliana Esercizi Elettrici	1918-1920		75	300	31	8	52°
6	Tirso	Tirso River Sardinia Italy	Società Imprese Idrauliche ed Elettriche del Tirso	1917-1923		239	930	49.2	18	57°
7	Gleno	Lombardy Italy	G. Viganò	1918-1923		160	735	26.25	24	53°
8	Piano Sapeio	Calandrino River Genova, Italy	Consorzio Idroelettrico Monte Ajona	1923		65	400	32.8	11	49°
9	Tidone	Tidone Creek Lombardy Italy	Consorzio d'irrigazione di Valle Tidone Piacenza	1924-1925		171	900	32.8	26	45°
10	Pavana	Sambuca Creek Bologna Italy	Government Railroads	1924-1925		177	470	54.1	3	60°
11	Venina	Venina Lake Sondrio, Italy	Società Acciaierie E Ferriere Lombarde	1925-1926		170				90°
12	Suorva Lakes	Lule River Lapland Sweden	Government of Sweden	1919-1923		46 to be increased by 23	Western channel 550 Eastern channel 790	39.4	West 13 East 23	45°
13	Melby	Sweden				36	256	24.6	7	63.5°
14	Great Lake	Tasmania		1923		90	1182	40	27	60°
15	Voehrenbach	Linach River Schwarzwald Baden Germany	City of Voehrenbach			95	500	35.4	12	50°
16	Anyox	Anyox British Columbia Canada	Granby Consolidated Mining, Smelting and Power Co., Ltd. Anyox, B.C., Canada	1923-1924	Power	156	680	24	26	Variable

TYPICAL SECTIONS



Continued).

ARCHES							BUTTRESSES											
RADIUS	CENTRAL ANGLE 2Φ	THICKNESS TOP	THICKNESS BASE	MIXTURE OF CONCRETE	REINFORCED	SPECIAL WATER-PROOFING	THICKNESS TOP	THICKNESS BASE	STRUTS OR STIFFENERS	MIXTURE OF CONCRETE	REINFORCED	FOUNDATION	CONCRETE VOLUME CU. YDS.	TOTAL COST OF DAM	RESERVOIR CAPACITY ACRE FT.	DESIGNER	LIST OF LITERATURE	REMARKS
5'-8"	100°	15"	4'-3"	1:2:4	Yes	Cement wash	15"	5'-0¼"	15 Struts	1:2½:5						J. S. Eastwood	Western Construction News. Nov. 1926 E. Wegmann, The Design and Construction of Dams	
Five centered arch 27'-6"	156°	18"	4'-6"	1:1.9:3.9	Yes	Inertol	2'-3"	7'-10"	3 Stiffeners No struts	1:2.3:4.5	Yes	Granite				So. Calif Edison Co.	H. Kelen die Staumauern p. 230 Western Construction News. July 25, 1927 E. Wegmann, The Design and Construction of Dams.	
3'-1½"	130°	12"	3'-8"	1:2:4	Yes	Gunite ¾" coating	24"	7'-6"	None	1:3:6 plus 12% plums	No					Mees & Mees		
39'-6"	52°	18"	2'-6"		No	None	2'-6"	4'-0"	None		No					Mees & Mees		Designed for 12 ft overflow over crest.
4'-0"	95°-50'	18"	5'-6" at a depth of 180'	1:2:4	Yes		18"	5'-6" at a depth of 200'	None	1:2½:5	Yes		75000		173500	Peckham & James	Western Construction News. III, 25, p. 23 E. Wegmann, The Design and Construction of Dams.	Double-wall buttresses
Three centered arch ...'-0"	130°	18"	6'-4"		Yes		3'-4"	10'-0"	2 Stiffeners		Yes	Granite			21200	A. P. Davis	Western Construction News. Jan. 1927	
Variable	Variable	4'-0"	20'-1"	Compressive strength at 28 days 2200 lb./sq. in.	Yes	Gunite 1½" coating	20'-0"	60'-0"	None	Compressive strength at 28 days 1800 lb./sq. in.	Yes	Quartzite			1300000	C. R. Olberg	E. Wegmann, The Design and Construction of Dams. Western Construction News. Dec. 1926	Multiple dome type of dam
Three centered arch 27'-0"	140°	24"	5'-6"	1:2:4	Yes	None	24"	14'-0"	None	1:2½:5	Yes	Granite			1500	S. M. Fisher	Western Construction News. Oct. 25, 1927	Double wall buttresses.

Multiple-Arch Dams in Foreign Countries.

ARCHES							BUTTRESSES											
RADIUS	CENTRAL ANGLE 2Φ	THICKNESS TOP	THICKNESS BASE	MIXTURE OF CONCRETE	REINFORCED	SPECIAL WATER-PROOFING	THICKNESS TOP	THICKNESS BASE	STRUTS OR STIFFENERS	MIXTURE OF CONCRETE	REINFORCED	FOUNDATION	CONCRETE VOLUME CU. YDS.	TOTAL COST OF DAM	RESERVOIR CAPACITY ACRE FT.	DESIGNER	LIST OF LITERATURE	REMARKS
35' to 77'	180°	8'-6"	6'-6"	Masonry in lime mortar	No		12'-0"	19'-0"	None	Masonry in lime mortar	No						W. G. Bligh, Dams & Weirs	
Arch spandrels filled		19"	4'-0"	Lower part concrete Upper part brickwork	No		5'-0"	12'-0"	None	Lower part concrete Upper part brickwork	No	Rock	5030	$45000		Oscar Schulze	W. G. Bligh, Dams & Weirs Trans. Am. Soc. C. E. 1917 p. 891 H. Kelen, die Staumauern p. 210	
		4½"	7"	1:2½:3	Yes	A layer of reinforced concrete mortar	8"	8"	5 Struts	1:2½:3	Yes					Considère Pelnard Caquot	Genie Civil 12 Mai 1917 H. Kelen, die Staumauern p. 258 E. Wegmann, The Design and Construction of Dams	
		4½"	7½"	Very rich mixture	Yes		8"	8"		Very rich mixture	Yes						Houille Blanche March 1924 E. Wegmann, The Design and Construction of Dams.	The dam is similar in type and design to the Silune Dam.
14'-0"	90°	16"	3'-2"		Yes		4'-6"	8'-4" at a depth of 65'	1 Strut	Masonry in lime mortar	No					G. Ganassini	H. Kelen, die Staumauern p. 232. Elettrotecnica Sept. 25, 1920. E. Wegmann, The Design and Construction of Dams.	
	180°	20"	5'-6"	1:2½:5	Yes	Upper part cement coating Lower part asphaltum	8'-2"	23'-10" at a depth of 200'	6 Struts	0.69 to 0.78 cu. yds. cement mortar to a cu. yd. of masonry	No	Trachyte	214000			Luigi Kambo	Engineering News-Record May 10, 1923 E. Wegmann, The Design and Construction of Dams.	The buttresses are built of hand-laid masonry and are faced with ashlar
9'-10"	180°	16"	2'-7"		Yes		6'-6"	11'-3" at a depth of 90'	None		No	Sandstone				Santangelo	Water Works 1924 p. 375 H. Kelen, die Staumauern p. 235 E. Wegmann, The Design and Construction of Dams.	The center section of the multiple arch was built on a base of gravity section which later failed
16'-2"	140°	10"	15"		Yes		10'	2'-9"	5 Struts		No					Ferrobeton Co.	H. Kelen, die Staumauern p. 251 E. Wegmann, The Design and Construction of Dams	
16'-4"	180°	14"	3'-6"	1:1.8:3.6	Yes		2'-4"	7'-2"	13 Struts	1:2:4	Yes	Shale					Engineering News-Record Oct. 29, 1925 E. Wegmann, The Design and Construction of Dams.	Some of the highest buttresses were based on a slab to reduce pressure on foundation
	180°	24"	5'-7"		Yes	Reinforced Gunite 1½" thick	6'-6"	20'-0"	2 Struts		No		44500			Manfredini	H. Kelen, die Staumauern p. 245 E. Wegmann, The Design and Construction of Dams.	The wings are hollow gravity sections. System Figari.
		20"	4'-1"		Yes											P. Bonetti	H. Kelen, die Staumauern p. 252 E. Wegmann, The Design and Construction of Dams.	The arches are vertical.
23'-2"	115°	2'-7"	5'-7"	1:2½:3	Yes	⅜" coating of Gunite mixture 1:3			2 Struts	1:5:6½	Yes	Solid Rock					Engineering, London. Dec. 14, 1923 Water Works. 1924 p. 381. Water Works 1924 p. 345	On downstream side of arches there is a backfilling of earth about 8 ft. high
				1:3:3	Yes					1:4:6 with 10% plums							H. Kelen, die Staumauern. p. 283 Teknisk Tidskrift. 1923 No. 2	On downstream side of arches there is a backfilling of earth about 8 ft. high.
22'-0"		12"	2'-8"		Yes		22"	5'-4"	Several Struts		Yes						Trans. Am. Soc. C. E. 1924 p. 389 E. Wegmann, The Design and Construction of Dams.	
19'-0"	130°	16"	20" at elev. 18'		Yes	Reinforced Gunite and Inertol	2'-7"	3'-11"	3 Struts	1:3:6	Yes					F. Maier	Beton und Eisen. XXIII 1924 E. Wegmann, The Design and Construction of Dams.	
15'-8"	100°	12"	3'-6"	1:2:4	Yes	None	18"	10'-1"	7 Struts	1:2½:5	No				28000	J. S. Eastwood	Water Works Feb. 1928. p. 65 Southwest Builder & Contractor Jan. 1925 E. Wegmann, The Design and Construction of Dams.	

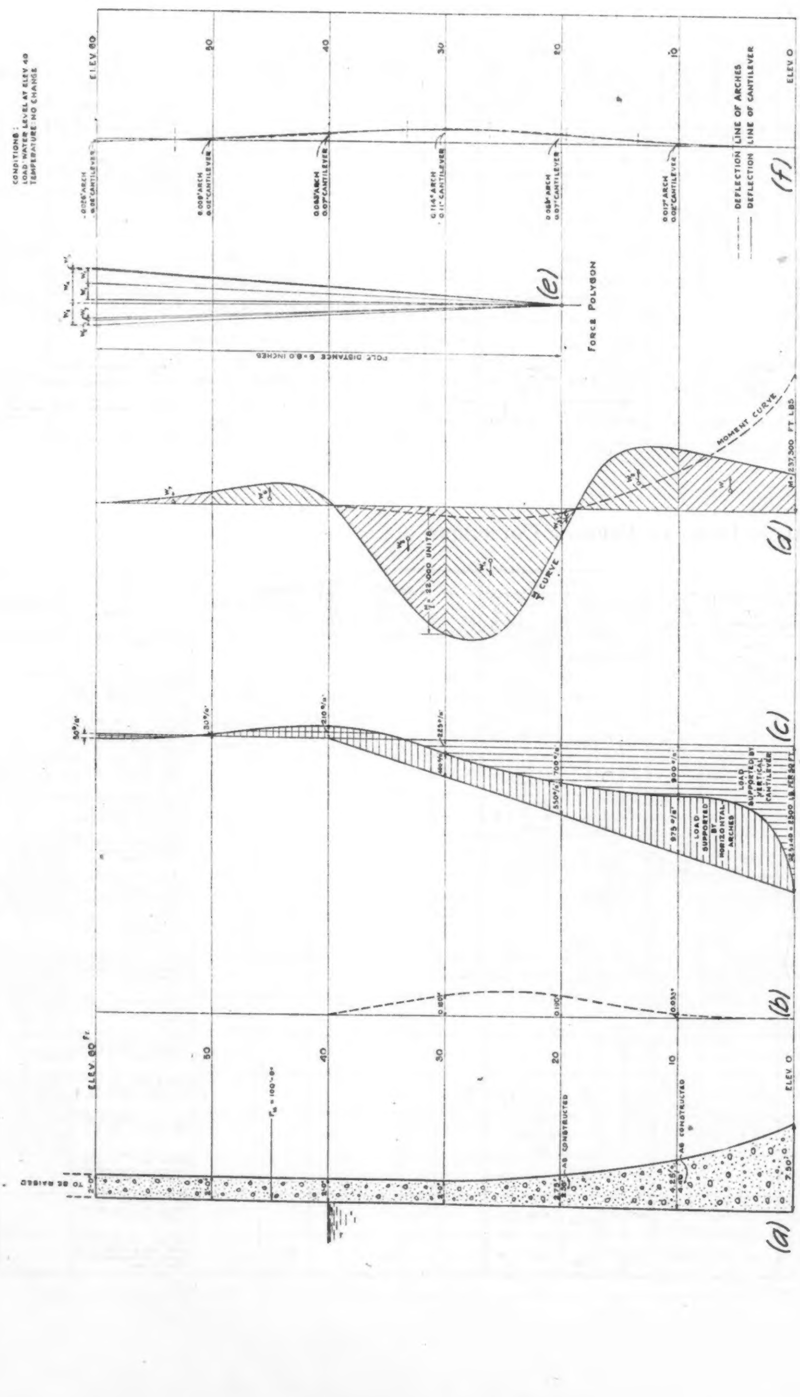


FIG. 5.—DIVISION OF WATER LOAD BETWEEN CANTILEVER AND ARCHES FOR WATER SURFACE AT ELEVATION 40 AND NO CHANGE IN TEMPERATURE AS DETERMINED IN THE DESIGN.

PART III.—FINANCING AND CONSTRUCTING THE EXPERIMENTAL ARCH DAM

By H. W. Dennis,* M. Am. Soc. C. E.

At a meeting of the Committee on Arch Dam Investigation held in San Francisco, Calif., December 1, 1923, W. A. Brackenridge, M. Am. Soc. C. E., Senior Vice-President of the Southern California Edison Company, proposed the construction of a large arch dam wholly for experimental purposes. In behalf of that Company, he offered a large contribution of funds, the use of many facilities for building and testing the dam, and assistance in further financing. This proposal was accepted by the Committee, with the approval of Engineering Foundation. To a sub-committee was delegated the detail of the test of the Stevenson Creek Dam, under the general supervision of the main Committee. Funds, materials, and equipment, contributed by the organizations and individuals (see "Acknowledgments"), for which a cash value was entered in the accounts, totaled, to July 1, 1927, $115 500; and this sum was greatly augmented by many intangibles.

To the writer was assigned general charge of the surveys for this dam and of its construction. In the extremely precipitous canyon of Stevenson Creek, a tributary of the San Joaquin River, in the Sierra Nevada, about 60 miles east of Fresno, Calif., a site was found which had the following advantages (Fig. 6):

1.—Strong granite foundations (Fig. 7);
2.—A symmetrical, triangular profile for the dam;
3.—A very small reservoir, which could be quickly filled and emptied, involving only small consumption of valuable time and water;
4.—Freedom from possibility of damage to persons and property down stream in the event of the breaking of the dam;
5.—Accessibility for men and materials by means of railroad and automobile roads previously built by the Company for its extensive construction work, still in progress;
6.—Close proximity to the tunnel conduit of the Company (3 000 cu. ft. per sec.), from which water could be drawn at will;
7.—Availability of electric power and light, construction equipment, materials, and, above all, trained men of all ranks, from laborers to engineers.

To carry out the experiment, the raising of a fund of $100 000 was undertaken. After a year and a half this fund amounted to $75 000 and as early as August, 1925, work was under way. In the meantime, the design of the dam had been perfected, various special instruments had been devised, methods of construction and tests had been developed, and the preliminary operations at the site had been begun. Excavation continued during the winter months with a comparatively small crew working under a general specification that the completed excavation should give as nearly as practicable the desired symmetrical cross-section with a minimum amount of over-break and with

* Chf. Civ. Engr., Southern California Edison Co., Los Angeles, Calif.

no disturbance of the bed-rock below the excavation lines. To this end the plug and feather method of excavation was used extensively. The final cross-section approximated very closely the theoretical section shown on the plans. A modification in the original plans was made, when the excavation disclosed a natural seam in the rock which, after cleaning out for a width of about 5 ft., became the logical location for the outlet tunnel, or under-sluice. This under-sluice was capped January 25, 1926, by a heavy slab of reinforced concrete, the top of which was made to conform to the longitudinal profile of the design for the dam.

The excavation had been practically completed in February, 1926, when a severe flood occurred on the water-shed of Stevenson Creek and large quantities of débris were deposited in the excavation. This caused a temporary delay, but after the passing of the flood, sluicing operations were organized, all threatening local material, estimated at 20 000 cu. yd., was sluiced from the adjacent walls of the canyon and a large quantity of water, turned out from the Shaver Reservoir of the Southern California Edison Company, washed the excavation clean.

The placing of concrete began April 19, 1926, and continued until June 4, when the last was placed, bringing the dam to a height of 60 ft. At each concreting stage 5 ft. of height for the full length of the dam was built, but the placing was accomplished in four horizontal lifts for each such 5-ft. height. There were no expansion joints and no reinforcing steel. Between the times of pouring one lift and its succeeding lift, the various instruments were placed in position within the forms and numerous readings were taken of instruments already located, not only to secure data while construction was in progress, but also to train the men in taking observations so that when the actual testing should begin the crew of observers would be fully competent to carry out their assignments.

A 24-in. wood-stave pipe was installed to conduct the water of the creek across the top of the dam when not wanted. Two valves and a set of stop-logs were set in the under-sluice. One valve was 24 in. in diameter to permit rapid unwatering of the reservoir, and the other, 6 in., to permit careful regulation and hold the water at a predetermined elevation during test. Scaffolds which did not touch the dam were built up stream and down stream so as to provide ready access to the entire area of each face. The stems of the two valves were carried on the up-stream scaffold. The dam thus stood entirely independent of any contacts, except at its abutments, and yet with means to permit complete and careful inspection (Fig. 6).

The underlying rock is a fine-grained gray granite. This same material was used for the aggregate of the concrete, being taken from the dump of one of the Southern California Edison Company's tunnels, crushed, and screened in an existing construction plant, and brought to the dam in two sizes. The smaller was retained on a $\frac{3}{8}$-in square mesh screen, but passed a screen having holes 1 in. in diameter. The larger was retained on a screen having 1-in. round holes and passed one with 2-in. round holes. For sand it was necessary to use the fine material from the tunnel dump which passed the $\frac{3}{8}$-in. square screen. A washer, however, was installed and all such mate-

Fig. 7.—Foundation Rock, South Abutment, Stevenson Creek Test Dam.

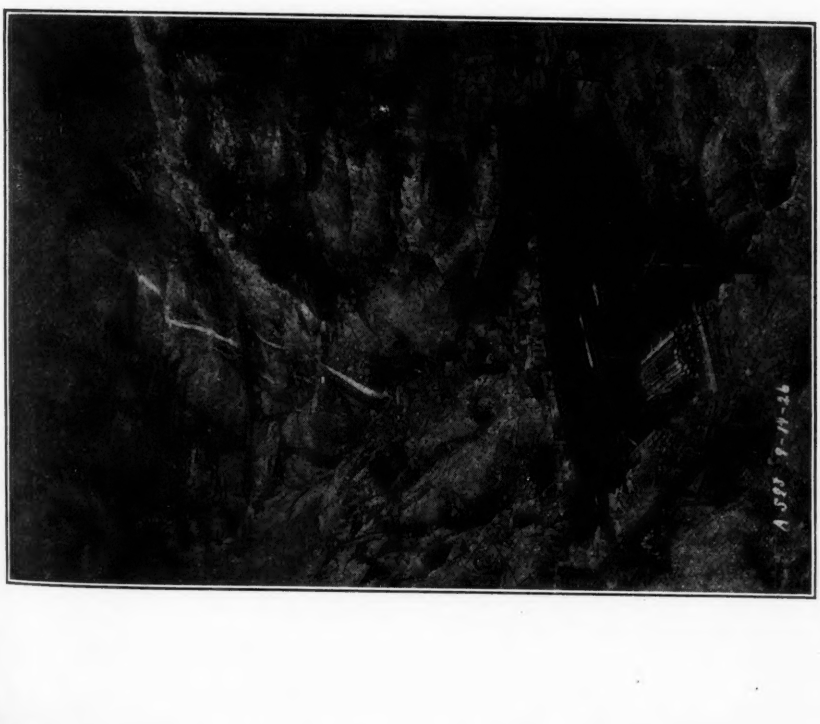


Fig. 6.—Looking Up Stevenson Creek at Test Dam, Showing Tunnel Adit in Upper Left-Hand Corner.

rial
was
large
erall
in m
T
dona
ceme
expe
but
A
the
unde
dam
mad
was
care
limi
the
tern
was
plac
of "
I
by
min
the
I
plen
desc
wer
tech
Eng

or th

rial was washed, with the result that much material finer than the 200 mesh was removed. An average sample of the sand showed that about 20% was larger than No. 4 and that 5% was finer than No. 100. The fact that generally only the aggregate passing the No. 4 sieve is called sand should be kept in mind when considering the amount of material herein classed as sand.

The entire dam was made using "Colton" cement, manufactured and donated by the California Portland Cement Company, of Colton, Calif. The cement was received at the site of the dam in the fall of 1925, when it was expected that the concrete would be poured during the succeeding few months; but the pouring was unavoidably delayed as already recited.

An inundator, a device for measuring sand in water, which was lent by the Blaw-Knox Company, was used for all concrete, and the mixing was done under careful supervision, realizing that uniformity throughout the entire dam was most desirable. With this end in view, special arrangements were made for measuring the constituent elements and the water-cement ratio was very carefully watched. Likewise, the time of mixing was controlled carefully, the minimum time after all the materials were in the mixer being limited to 1½ min. Some further mixing undoubtedly occurred in the chute, the total length of which was about 400 ft. from the outlet of the mixer to the terminus at the 60-ft. level. This chute was lined with thin steel plate and was laid on a grade of about 5 on 12. Below Elevation 20, the concrete was placed in the dam directly from the chute; above this elevation, by the use of "buggies."

Forms were made of ship-lap lumber laid horizontally and held in position by through-bolts, so as to maintain the desired thickness of the dam with minimum variation. The forms were removed two to four weeks after pouring the concrete.

During the construction and tests, a motion picture was prepared to supplement the Committee's report. Realizing that explanation of methods and description of conditions would thus be illuminated, about 2 000 ft. of film were exposed and have been useful on numerous occasions at gatherings of technical men, including several meetings of the American Society of Civil Engineers, where the picture has excited great interest.*

* A copy is available on a rental basis, or copies may be purchased on either the 35 mm. or the 16 mm. size, by application to the Director of Engineering Foundation.

P

1
men
dam
in e
the
fina
co-o
writ
of S
prog
load
and
Cali
J
erec
of t
secu
in s
it n
true
the
con
wou
the
gen
not
yea
tha
equ

tica
inc
lin
fro
abu
pro
eve

Con

PART IV.—REPORT OF TESTS ON STEVENSON CREEK DAM*

By Willis A. Slater,† M. Am. Soc. C. E.

A.—General Description

1.—Introduction.—This report gives the results of load tests on an experimental concrete arch dam to secure information on the behavior of such dams under load, with the expectation that it would be an important step in establishing rational standards for their design. The site had been chosen, the dam had been designed, and its construction and testing had been partly financed when, in 1925, the National Bureau of Standards was invited to co-operate with Engineering Foundation. About December 1, 1925, the writer took charge of the experimental work, as representative of the Bureau of Standards. At that time rock excavation for the foundation was in progress. The dam was constructed during the spring of 1926, and the load tests occupied the latter part of the summer. The reduction of the data and the preparation of this report have been carried out in Los Angeles, Calif., since October 1, 1926.

Just as important as the proper execution of the test was the proper erection of the dam, which was carried out by the Construction Department of the Southern California Edison Company. Special care was taken to secure exactness of dimensions in excavation and construction, and uniformity in strength of concrete and in the curing of the completed dam. Although it may never be possible in practice to build a dam as symmetrical and as true to the designed dimensions, the uncertainties which would have entered the results if the ordinary limitations of construction had been allowed to control, would have been so great that neither practical nor ideal results would have been secured. However, the employment of methods approaching the ideals of design, should not be lightly dismissed as impracticable even in general construction. In the making of good concrete the past few years have noted a marked advance because exact methods have been used which a few years ago would have been rejected as impracticable. It is not improbable that similar efforts extended to other features of construction would produce equally valuable results.

2.—General Description of Test Dam.—In Fig. 1 is shown the plan, vertical section, and developed vertical surface of the dam. The maximum angle included between the radii is 80° 37′, at the 60-ft. elevation. The abutment lines on the developed surface are straight and have an inclination of 45° from the 60-ft. down to the 7.32-ft. elevation. At this lower elevation the abutment line is tangent to a circular arc of 25-ft. radius, which forms the profile of the developed surface below. The radius of the up-stream face is everywhere 100 ft.

---

* Publication approved by the Director of the Bureau of Standards, U. S. Dept. of Commerce.

† Chf. of Masonry Construction Section, National Bureau of Standards, Washington, D. C.

A vertical radial section anywhere in the dam shows for the down-stream face a vertical straight line down to the 30-ft elevation, at which point it is tangent to a circular arc of 84.568-ft. radius, which forms the lower profile. The thickness is 2 ft. from the top down to the 30-ft. elevation, below which it increases by the ordinates to the circular arc, until at the bottom it becomes 7.5 ft. This gives a constant thickness for all parts of any horizontal element (Figs. 1 and 2.)

The under-sluice (Fig. 8) is capped by a slab, having an average thickness of about 4 ft., reinforced heavily with steel rails. Observations during the tests indicated that this slab was as stationary as any other part of the foundation. The throat of the under-sluice (4 by 6.5 ft.) was provided with stop-logs which could be removed when it was desired to let all the water pass the dam. However, even with the stop-logs out, a heavy storm during the winter of 1926-27 caused the under-sluice to clog up and the reservoir to fill to the top of the dam. (See, also, Part III.)

B.—Materials and Features of Construction

3.—General Procedure in Construction.—The excavation for the dam was finished by January 20, 1926, except that for the proposed deflection towers. (See Section 21.) Profiles along the up-stream face of the dam are given in Fig. 1. The dotted line shows the original profile of the rock. The profile of the final excavation is shown as a rough, solid line. The theoretical profile of the dam is shown by the lighter, smooth line. During excavation some rock below the theoretical profile of the foundation was loosened. This loose rock was removed and wherever the variation from the profile appeared to be of importance, the cavities were filled with concrete, several days before construction reached these points. The concreted portions of the over-cut, also the slab over the under-sluice, are shown in Fig. 1.

The slab over the under-sluice (Fig. 8) was concreted on January 25. Storms immediately following caused much damage and delayed the progress for nearly two months before the concreting of the dam began. During the construction of the near-by water tunnel a waste dump of tunnel muck had been deposited on the side of the canyon. The storms washed the fine material from the tunnel muck, thus dislodging larger rocks, which, in turn, did much damage in falling.

Efforts were made to protect the work by means of heavy barricades, but these were ineffectual and, finally, it became necessary to remove from above the dam site almost the entire bank of tunnel muck. Using a 4-in. stream of water under a head of about 100 ft., about 20 000 cu. yd. of rock were washed down. This stream was not sufficient to do more than to wash the rock into the bottom of the canyon. From there it was dislodged by a flood of water produced by opening the gates of the Shaver Lake Reservoir of the Southern California Edison Company on March 17. About 840 acre-ft. of water passed down the canyon in less than 2 hours, effectually cleaning out the bottom of the canyon and clearing the site for construction. The concreting of the dam proper began on April 19. A lift of 5 ft. was added every three or four days until June 4, when the dam was completed to a height of 60 ft.

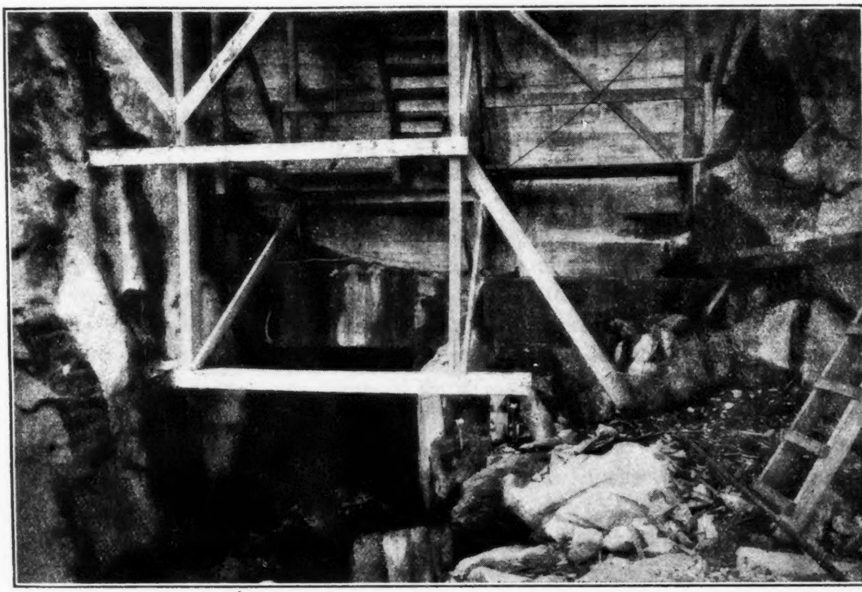


FIG. 8.—DOWN-STREAM SIDE AT BOTTOM OF STEVENSON CREEK TEST DAM, SHOWING UNDER-SLUICE.

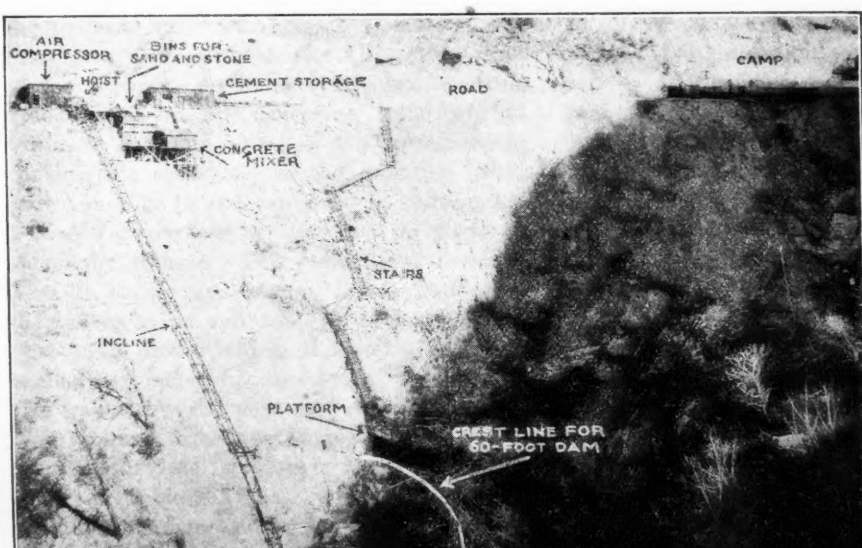


FIG. 9.—VIEW OF PRELIMINARY WORK AT SITE OF DAM.

4.
for t
begi
ceme
in a
dela
cons
side
leng
store
have
T
aggr
2-in.
that
scre
the
thes
T
unw
fine
agg
the
befo
the
that
of t
grad
form
con
sho

tion
1 c
the
pur
exc
siz
loo
res
sho

by
to
of
wh
ho

4.—Materials.—Colton cement was used for the dam and Monolith cement for the tower foundations. When it was expected that construction would begin in the fall of 1925, the supply of cement was shipped. The Colton cement was received at Big Creek on October 12, 1925, and immediately placed in a cement storage house at the site of the test dam (Fig. 9). Owing to the delays mentioned, the cement had been in storage about six months before construction began. Examination of several bags of cement showed a considerable number of lumps. It is probable that they were due entirely to the length of time during which the cement had been in storage. Although it was stored under a tight roof, it is suspected that enough moisture penetrated to have some effect. However, a good concrete of uniform strength was secured.

The aggregates were of three sizes, all of crushed granite. The coarse aggregate was divided into two sizes, that which passed through a screen with 2-in. round holes and was retained on a screen with 1-in. round holes, and that which passed a screen with 1-in. round holes and was retained on a screen with $\frac{3}{8}$-in. square holes. Sizes of 2 in. and 1 in. were not included in the set of Tyler sieves used in making analyses of the aggregates. Hence, these sizes do not appear in Table 15.

The fine aggregate was taken from that which passed the $\frac{3}{8}$-in. screen. The unwashed aggregate passing the $\frac{3}{8}$-in. screen contained about 26% of material finer than the No. 100 sieve. This was believed to be much too fine, so this aggregate was washed to remove some of the fine material. Fig. 10 gives the sieve analyses for all the aggregates, including the fine aggregate, both before and after washing. While the purpose of the washing was to remove the material finer than No. 100 sieve, considerable quantities coarser than that sieve were removed also (Fig. 10). Table 15 gives the sieve analyses of the aggregate on successive days of pouring concrete. In considering the gradation of the fine aggregate, tests were made at the University of California to determine whether the large amount of fine material would cause considerable shrinkage in the concrete. Results to about three weeks are shown in Fig. 10 and explained in Section 10.

5.—Measurement of Materials for Concrete.—The materials were proportioned by volume. The cement was measured in bags assumed to contain 1 cu. ft. each. Medium and coarse aggregates were measured in hoppers and the fine aggregate and water were measured in a Blaw-Knox inundator. For the purpose of proportioning, the weight of the fine aggregate by loose measure, exclusive of moisture, was taken at 91 lb. per cu. ft. The weight of the medium size was nearly the same as that of the coarse and the mixture of the two by loose volume, exclusive of moisture, was taken at 93 lb. per cu. ft. The results of several weighings under different conditions of compactness are shown in Table 16.

Hoppers for measuring the medium and the coarse aggregates were provided by arranging the lower parts of the chutes from the bins so that filling them to a predetermined elevation measured out a known quantity. At the bottom of the chute a hinged gate permitted discharge directly into the batch hopper, which was mounted on a car for hauling to the mixer. The upper part of the hopper was narrowed to a small cross-section to secure accuracy. The deter-

TABLE 15.—SIEVE ANALYSIS OF AGGREGATES ON SUCCESSIVE DAYS OF CONCRETING.

Sieve No.	Percentage Retained on Sieve.													Accum. %.
	4-20-26.	4-27-26.	4-28-26.	5-4-26.	5-8-26.	5-13-26.	5-17-26.	5-21-26.	5-25-26.	5-28-26.	6-2-26.	6-4-26.	Aver.	
Fine.														
4	23.0	20.0	20.3	20.3	19.41	22.20	22.88	18.70	21.98	23.09	20.40	26.23	21.54	21.54
8	19.6	20.4	16.8	20.0	21.42	20.14	20.51	20.90	20.49	18.44	19.65	21.29	19.97	41.51
14	15.5	15.6	14.0	16.7	16.70	16.34	16.28	17.42	16.57	16.48	16.91	9.00	15.63	57.14
28	13.6	14.3	15.3	14.8	15.64	15.30	14.65	14.58	13.61	14.98	14.70	17.51	14.91	72.05
48	13.0	17.8	27.2	19.4	17.22	15.74	12.91	13.32	11.79	13.19	12.02	9.89	15.29	87.34
100	8.1	6.3	4.8	6.2	7.68	6.02	7.00	8.01	8.24	6.90	8.83	9.02	7.26	94.60
200	.....	4.0	1.3	2.0	1.76	2.82	4.27	4.29	4.54	3.17	4.53	4.30	3.08	......
Pan	7.2	1.6	0.8	0.6	0.17	1.44	1.55	2.78	2.78	3.75	2.96	2.76	2.32	......
Total	100.0	100.0	100.0	100.0	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	374.18
F. M.	3.73	3.69	3.67	3.78	3.79	3.83	3.81	3.65	3.73	3.75	3.65	3.81	3.74	3.74
Medium.														
3/4	2.7	Omitted from average.	....	3.45	1.68	1.76	1.08	1.66	0.83	1.50	1.63	4.61	2.02	2.02
3/8	73.2		....	63.00	60.09	73.38	67.00	69.38	70.89	69.84	67.21	72.73	68.18	70.20
4	17.9		....	28.30	33.10	21.00	27.22	24.72	24.00	24.04	26.85	18.98	25.36	95.56
8			....	1.45	1.53	0.78	1.25	0.88	0.29	0.86	0.68	0.60	0.92	96.48
14			....	0.70	0.76	0.49	0.55	0.41	0.31	0.50	0.46	0.36	0.50	96.98
28			....	0.54	0.51	0.48	0.53	0.45	0.60	0.55	0.58	0.47	0.52	97.50
48	6.2		....	0.90	0.78	0.65	0.82	0.80	1.06	0.80	0.82	0.65	0.81	98.31
100			....	1.30	1.07	0.84	0.83	1.16	1.07	0.78	0.80	0.68	0.95	99.26
200			....	0.32	0.48	0.47	0.53	0.39	0.62	0.59	0.57	0.48	0.49	......
Pan			....	0.04	0.00	0.15	0.19	0.15	0.33	0.54	0.40	0.44	0.25	......
Total	100.0		....	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	656.31
F. M.	6.65		....	6.53	6.48	6.63	6.53	6.58	6.55	6.55	6.54	6.68	6.56	6.56
Coarse.														
1½	6.9	Omitted from average.	....	4.24	2.64	2.67	0	3.19	0.90	1.21	3.71	6.88	2.83	2.83
3/4	83.7		....	87.40	83.94	72.52	76.37	74.00	73.19	67.34	75.52	84.60	77.21	80.04
3/8	6.3		....	7.22	11.85	22.37	20.90	18.36	21.33	26.99	18.45	5.06	16.95	96.99
4			....	0.35	0.36	0.68	0.41	2.69	0.62	0.75	0.82	0.74	0.82	97.81
8			....	0.04	0.05	0.10	0.07	0.20	0.12	0.24	0.18	0.14	0.13	97.94
14			....	0.05	0.11	0.14	0.14	0.10	0.21	0.25	0.12	0.13	0.14	98.08
28			....	0.08	0.19	0.22	0.25	0.19	0.53	0.45	0.13	0.31	0.26	98.34
48	3.1		....	0.24	0.19	0.45	0.48	0.39	1.01	0.81	0.23	0.55	0.48	98.82
100			....	0.17	0.39	0.45	0.75	0.40	1.28	0.80	0.36	0.71	0.59	99.41
200			....	0.17	0.22	0.25	0.40	0.32	0.61	0.58	0.26	0.51	0.36	......
Pan			...	0.04	0.06	0.15	0.23	0.16	0.20	0.63	0.22	0.37	0.23	......
Total	100.0		....	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	770.26
F. M.	7.89		....	7.91	7.82	7.68	7.63	7.68	7.53	7.49	7.74	7.82	7.70	7.70

mination of the capacity of the hopper was based on weighing tests made in advance. During construction additional weighing tests were made on the medium and the coarse aggregates from one batch during each day's run.

TABLE 16.—WEIGHTS PER UNIT OF VOLUME OF AGGREGATES FROM BINS AT TEST DAM.

(All Measurements Were Made with Cubical Wooden Measures, Generally of 1.0 Cu. Ft. Capacity, but in a Few Cases of 0.5 Cu. Ft. Capacity.)

Size.	Percentage by volume.	Weight by Loose Measure, in Pounds per Cubic Foot.		
		Moist.	Dry.	Rodded dry.
0 to ⅜	100	85.65	......	113.0
⅜ to ¾	100	......	84.96	93.1
¾ to 1½	100		82.77	93.2
⅜ to ¾	50	......	......	99.3
¾ to 1½	50			
0 to ⅜	46	......	......	121.5
⅜ to ¾	27			
¾ to 1½	27			

The advantage to be expected from use of the inundator is uniformity of quantities of both sand and water. A series of tests carried out in 1922 at the Bureau of Standards* on the reliability of the method indicated that reasonably close control of both water and sand might be secured.

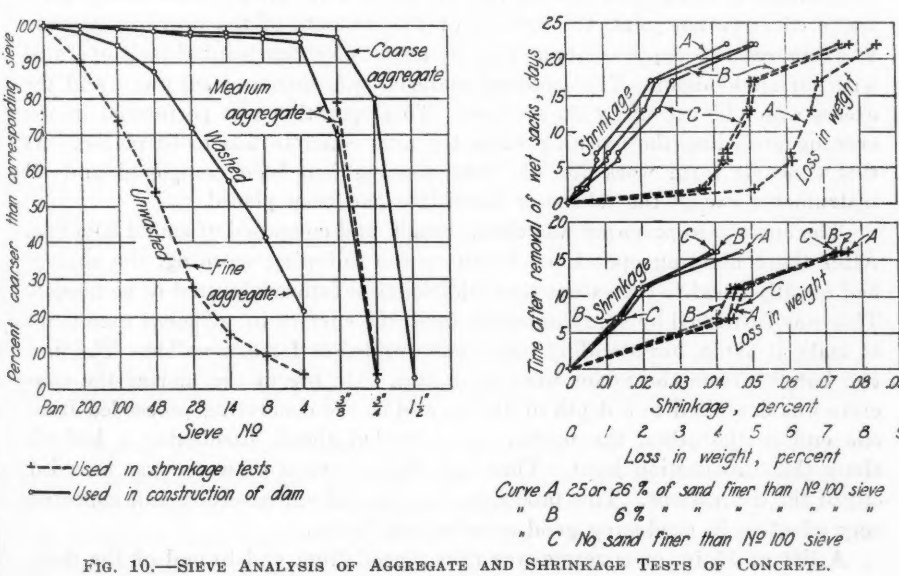


FIG. 10.—SIEVE ANALYSIS OF AGGREGATE AND SHRINKAGE TESTS OF CONCRETE.

6.—*Mixing and Placing Concrete.*—The concrete was mixed 1½ min. in a ½-yd. Ransome mixer about 25 ft. below the Edison Company's road, which

* G. A. Smith and W. A. Slater, M. Am. Soc. C. E., "Inundation Methods for Measurements of Sand in Making Concrete," *Proceedings*, Am. Concrete Inst., Vol. 19, p. 222 (1923).

is about 250 ft. above the base of the dam (Fig. 9). From the mixer it was discharged into the chute 18 in. wide, which was at a slope of approximately 1 vertical to 2.5 horizontal. The stiffness of the mix was such that the concrete would just flow down this slope. This was controlled by the standard slump test, the consistency of the concrete varying between slumps of 2 and 7 in. The concrete was carried in this chute to the approximate elevation of the top of the dam. For the lower 20 ft. of the dam, the concrete was carried to its final position through auxiliary chutes of still greater slope. For the portion of the dam above the 20-ft. level, the concrete was chuted to the elevation where it was to be used and there distributed in buggies.

7.—Construction Joints.—There were no vertical construction joints in the dam. Horizontal joints were necessitated by the fact that it was not practicable to pour the concrete continuously from the bottom to the top of the dam. These horizontal construction joints were placed at vertical intervals of about 5 ft. and were located about 6 in. below the even 5-ft. contours. These locations were determined by the fact that large numbers of strain-gauge, telemeter, and clinometer stations were located at the 5-ft. contours, and it would be undesirable to have them intersected or disturbed by construction joints. These locations also permitted covering the instruments or inserts shortly after the beginning of the day's pouring, and left them free from disturbance while placing the remainder of the concrete.

In general, about four days elapsed between pourings. During this time the surface concrete had usually become quite dry. In preparation for making a construction joint, the surface of the concrete of the previous pouring was removed to a depth of about 1 in. by means of sharp-pointed tools operated with an air-hammer. The loosened material was then washed away and the excess water blown off by an air-hose. This operation was performed in the evening preceding the morning when the next concrete was to be placed. At this time all form work for the next pouring had been completed and all instruments except the telemeter cartridges had been placed.

The surface so prepared was clean, rough, and composed of sound concrete. After the concreting operations began on the following morning, the surface had usually dried to the extent that additional moisture appeared to be needed. This was furnished by spraying water upon the surface in sufficient quantities to make it moist, but standing water was avoided as far as possible. The first two batches from the mixer were of mortar. On top of the mortar the concrete was deposited to a depth of 15 in., and as the concreting proceeded from one end to the other, the mortar was crowded ahead, furnishing a bed all along the construction joint. Thus any chance excess of water was crowded out of the depressions. This procedure was carried out carefully and appeared very effective in producing good construction joints.

A lift of 15 in. of concrete was thus placed from end to end of the dam. The next lift was started at such time that at all points the concrete would have at least an hour for some set to take place between lifts. Ordinarily, one day's pouring consisted of four such lifts, or 5 ft. total. In the second pouring from the 3.2 to 9.5-ft. elevation, an exception was made; less time was allowed between lifts, and there resulted a slight bulging of forms. This

bulging appears in the greater divergence, shown in Table 17, between designed and measured thicknesses of the dam for the second, than for any other, pouring. Even there the maximum divergence was only 0.03 ft.

TABLE 17.—THICKNESS AT 5-FOOT ELEVATIONS AND ELEVATIONS OF CONSTRUCTION JOINTS, STEVENSON CREEK DAM.

Date of Pouring Nominal Elevation Ft. Nominal Thickness Ft.	4-19-26 3.2 6.36	4-23-26 10 4.40	4-27-26 15 3.33	4-30-26 20 2.59	5-4-26 25 2.15	5-8-26 30 2.00	5-13-26 35 2.00	5-17-26 40 2.00	5-21-26 45 2.00	5-25-26 50 2.00	5-28-26 55 2.00	6-2-26 57.5 2.00	6-4-26 60.33 2.00
Station Height of Concrete Thickness at Nominal Elevation												0+03.0 57.40 1.99	0+00 60.35 2.00
Station Height of Concrete Thickness at Nominal Elevation										0+12.0 48.70 2.01	0+10.0 53.70 2.00	0+10.0 57.40 1.99	0+10.0 60.32 2.01
Station Height of Concrete Thickness at Nominal Elevation									0+20.0 43.96 1.99	0+20.0 48.55 1.99	0+20.0 53.70 1.99	0+20.0 57.40 2.00	0+20.0 60.34 2.00
Station Height of Concrete Thickness at Nominal Elevation					0+36.88 24.45 2.16	0+32.0 29.08 2.01	0+30.0 34.30 1.99	0+30.0 39.12 1.99	0+30.0 43.94 2.00	0+30.0 48.65 2.00	0+30.0 53.80 1.99	0+30.0 57.50 2.00	0+30.0 60.36 2.01
Station Height of Concrete Thickness at Nominal Elevation				0+40.0 19.20 2.58	0+40.0 24.58 2.17	0+40.0 29.01 2.00	0+40.0 34.33 2.00	0+40.0 39.17 1.99	0+40.0 43.98 2.00	0+40.0 48.79 2.00	0+40.0 53.80 2.00	0+40.0 57.40 2.00	0+40.0 60.36 2.01
Station Height of Concrete Thickness at Nominal Elevation	0+58.0 * 6.25	0+51.91 9.40 4.41	0+50.0 14.20 3.32	0+50.0 19.13 2.58	0+50.0 24.40 2.16	0+50.0 29.05 1.99	0+50.0 34.32 1.99	0+50.0 39.12 2.00	0+50.0 43.96 2.00	0+50.0 48.85 1.99	0+50.0 53.85 2.00	0+50.0 57.60 1.99	0+50.0 60.37 2.00
Station Height of Concrete Thickness at Nominal Elevation	0+64.0 * 6.31	0+60.0 9.26 4.42	0+60.0 14.40 3.32	0+60.0 19.19 2.59	0+60.0 24.40 2.17	0+60.0 29.02 1.99	0+60.0 34.31 1.99	0+60.0 39.18 2.00	0+60.0 44.00 1.99	0+60.0 48.85 2.00	0+60.0 53.80 2.00	0+60.0 57.80 2.00	0+60.0 60.37 2.00
Station Height of Concrete Thickness at Nominal Elevation	0+69.4 3.20 * 6.33	0+70.36 9.44 4.42	0+70.36 14.45 3.32	0+70.36 19.19 2.58	0+70.36 24.30 2.16	0+70.36 29.02 1.99	0+70.36 34.31 2.00	0+70.36 39.43 1.99	0+70.36 44.00 1.99	0+70.36 48.86 1.99	0+70.36 53.65 2.00	0+70.36 57.90 2.00	0+70.36 60.37 2.01
Station Height of Concrete Thickness at Nominal Elevation	0+76.9 * 6.35	0+80.0 9.56 4.41	0+80.0 14.20 3.32	0+80.0 19.20 2.58	0+80.0 24.35 2.15	0+80.0 29.04 1.99	0+80.0 34.28 2.00	0+80.0 39.45 2.01	0+80.0 43.99 1.99	0+80.0 48.87 1.99	0+80.0 53.65 1.99	0+80.0 57.80 2.01	0+80.0 60.36 2.01
Station Height of Concrete Thickness at Nominal Elevation	0+80.6 * 6.36	0+88.8 9.65 4.43	0+90.0 14.30 3.32	0+90.0 19.23 2.59	0+90.0 24.40 2.16	0+90.0 29.11 2.01	0+90.0 34.35 2.01	0+90.0 39.23 2.00	0+90.0 44.00 1.99	0+90.0 48.82 2.00	0+90.0 53.65 2.00	0+90.0 57.80 2.00	0+90.0 60.36 2.01
Station Height of Concrete Thickness at Nominal Elevation				1+00.0 18.90 2.60	1+00.0 24.45 2.16	1+00.0 29.20 2.00	1+00.0 34.38 2.00	1+00.0 39.19 2.00	1+00.0 43.85 2.00	1+00.0 48.90 2.00	1+00.0 53.70 2.00	1+00.0 57.75 2.00	1+00.0 60.34 2.00
Station Height of Concrete Thickness at Nominal Elevation					1+03.84 24.45 2.16	1+09.5 29.20 2.00	1+10.0 34.39 2.00	1+10.0 39.13 2.00	1+10.0 43.83 2.00	1+10.0 48.93 2.00	1+10.0 53.75 1.99	1+10.0 57.60 1.99	1+10.0 60.36 2.00
Station Height of Concrete Thickness at Nominal Elevation								1+20.0 39.15 2.00	1+20.0 43.80 2.00	1+20.0 48.92 2.00	1+20.0 53.80 2.00	1+20.0 57.65 2.00	1+20.0 60.34 2.00
Station Height of Concrete Thickness at Nominal Elevation										1+26.0 48.92 1.99	1+30.0 53.85 1.99	1+30.0 57.65 1.99	1+30.0 60.36 2.01
Station Height of Concrete Thickness at Nominal Elevation												1+38.0 57.70 2.00	1+40.71 60.34 2.00

* Widths as shown are measured at actual height of concrete. Top of concrete at Station 0 + 58.0 is approximately 0.15 higher, and at Station 0 + 86.0 approximately 0.15 lower than at Station 0 + 69.4. All dimensions are in feet. Stationing is on 100 ft. radius.

8.—Forms and Form Removal.—The form ribs, 2 by 12 in., were horizontal and 5 ft. apart. Studs, 2 by 6 in., spaced 18 in. apart, spanned vertically from rib to rib, and 1-in. ship-lagging spanned the studs horizontally. On the up-stream side the studs were 16 ft. long, hence were continuous across supporting ribs. On the down-stream face the forms were built up as the construction progressed, that is, 5 ft. at a time, hence the studs were simple beams from rib to rib. During construction, that is, up to June 4, the forms were left in place four weeks after the pouring of the concrete. At that time a change in the program was made in order to hasten the work, and all forms which had been in place two weeks or more were removed. From then on,

forms were removed whenever they had been in place two weeks, thus completing the removal on June 18.

9.—Control Specimens.—The control specimens for the compressive strength of concrete were made regularly during the concreting operations. The results of the tests are given in Table 18. These specimens were 6 by 12-in. cylinders and were selected as follows: On each day of placing concrete twenty-four cylinders were made. Eight cylinders were taken from each of the first three 15-in. lifts of concrete. From the fourth (and last) lift, no specimens were taken. Of the eight specimens taken from any lift, the first four were from one batch and the other four were from another batch. These eight specimens were numbered consecutively, beginning with the number following a multiple of eight. For example (see Table 18), for the third lift placed on April 30, Specimens 89 to 96 were taken. Specimens 89 to 92 were made from one batch and Specimens 93 to 96 were made from another batch.

The specimens having odd numbers were stored in damp sand at a temperature of about 21° cent. (70° Fahr.) in a curing room in the adit to Water Tunnel No. 3 of the Southern California Edison Company. The temperature in this adit was very nearly constant, but artificial heat was required to make it high enough for proper curing of the specimens. The temperature at all times was recorded with an autographic thermometer. The specimens having even numbers were stored at the dam under conditions as nearly like those of the curing of the dam as appeared feasible. Of each eight specimens taken on any one day, the first two were tested at seven days, the second two at twenty-eight days, the third two after three months, and the last two are being held for later tests.

Some exception to this procedure has been made due to the fact that certain specimens were sent to the University of California, at Berkeley, for the determination of the modulus of elasticity, as will be detailed hereinafter.

During the construction of the dam a beam 12 in. square and 8 ft. long was made for determining the modulus of elasticity. A known load was applied to the beam. The resulting deflection was measured and from the relation between the load and the deflection, the modulus of elasticity was computed. Eight control cylinders were made with the beam, four of which were cured at the site of the dam along with the other control cylinders taken from the dam. These cylinders were Nos. 281 to 288, inclusive, Table 18. A hose carried to the test beam was so connected that when water was turned upon the dam it was turned upon the beam also.

On June 16, twelve days after the completion of the dam, two sacks of cement were mixed together and from the mixture seventeen 2-qt. pails were filled and sealed thoroughly with solder. At that time the cement had been in the construction warehouse since about October 12, 1925, or approximately eight months. It was already about four months old at the time of placing in the warehouse. These samples were sealed for the purpose of future study if occasion should arise and are stored at the Materials Testing Laboratory of the University of California. Eight sacks of the cement provided for

the dam were shipped to the University for use in the tests of concrete made with similar materials and of the same proportions as those that went into the dam. These tests were planned in May, 1926, but owing to lack of funds, were postponed. This cement was kept in a dry place all summer and shipped to the University about September 17.

10.—Proportions and Strength of Concrete.—Proportions and water-cement ratio for the concrete on the different days of pouring are shown in Tables 18 and 19. Table 18 gives also the strength and slump of the concrete as determined from the test specimens. The proportions given in this table are those determined by weighings made on one batch during each day of pouring. The results of these weighings are shown in Table 19. Except for the first and the last two days, the proportions were approximately 1:3:2. Within the same period the water-cement ratio varied from 0.87 to 1.03. At the beginning of construction the proportions were aimed to give a 28-day strength of 1 800 lb. per sq. in. Before the time came for the second pouring it was found from tests of 6 by 12-in. cylinder specimens that the proportions used on the first day gave a strength of only about 1 200 lb. per sq. in. Accordingly, the proportions were changed and the mix decided upon was used until at the 54-ft. elevation the supply of cement became exhausted, and a new supply of Colton cement was used for the remainder of the construction.

From the data of Table 18, Fig. 11 was prepared to show the strength of the curing-room specimens distinct from those cured at the dam site for the three different ages, and for the various heights from which they were taken. It shows also the average of the strengths for all ages. The strength of the specimens from the first pouring was markedly lower than from any other pouring for each age. This is evidently due to the fact that a leaner mix was used there than in all the portions above it. For the most part, the strengths were quite uniform with the exception of the lowest specimens and those from heights of 52, 56, and 59 ft.

The lack of uniformity in the specimens near the top of the dam is probably due in some manner to the change in cement at about the 54-ft. elevation. The strengths for the 52-ft. elevation represent the last of the cement in the stock pile, and it is not unlikely that it came largely from the outer portions of the pile where the greatest deterioration should be expected, with a correspondingly low strength. The strengths for the 56 and 59-ft. elevations represent the new cement, and although an effort was made to change the mix in such a way as to hold the strength constant, it seems evident that this was not successful. A marked similarity of form is to be noted in all the graphs of Fig. 11, indicating that the same kind of variation ran through all the sets of specimens and that they represent actual variations in the strength of the concrete in the dam, and not merely in the manipulation of the laboratory specimens.

The 28-day tests show an average strength of about 2 000 lb. per sq. in., or a little more than 10% greater than the designed strength of 1 800 lb. per sq. in. With the exception of the extreme top and the extreme bottom,

TABLE 18.—Summary of Concrete Mix, Strength, and Slump, Stevenson Creek Dam.

DATE OF POURING 1926	CONCRETE POURED TO HEIGHT OF FEET	MIX OF CONCRETE *	WATER RATIO	7 DAY TESTS						28 DAY TESTS						3 MONTH TESTS						NOT YET TESTED					
				CURED IN DAMP SAND			CURED AT DAM			CURED IN DAMP SAND			CURED AT DAM			CURED IN DAMP SAND			CURED AT DAM			CURED IN DAMP SAND			CURED AT DAM		
				Nº	SLUMP	STRENGTH	Nº	SLUMP	STRENGTH	Nº	SLUMP	STRENGTH	Nº	SLUMP	STRENGTH	Nº	SLUMP	STRENGTH	Nº	SLUMP	STRENGTH	Nº	SLUMP	STRENGTH	Nº	SLUMP	STRENGTH
4-19	3.5	1:3.67:2.92	1.15	1	7.0	905	2	7.2	800	3	3.5	1120	4	7.0	1100	5	4.5	1460	6	5.5	1160	7	7.0		8	2.8	
				9	2.7	990	10	1.0	920	11	5.0	1740	12	0.9	1420	13	4.0	1940	14	2.2	1410	15	5.5		16	0.5	
AVERAGE					4.9	947		4.1	860		4.2	1430		4.0	1260		4.2	1700		3.8	1285		6.3			1.8	
4-23	9.5	1:3.0:2.0	1.00	25	5.5	1410	26	0.7	1280	27	1.3	1990	28	1.5	1550	29	1.7	2850	30	1.2	2520	31	1.0		32	0.8	
				33	4.7	1400	34	1.8	1310	35	2.0	1960	36	2.5	1880	37	3.0	2480	38	5.0	2570	39	1.8		40	5.2	
				41	4.3	1370	42	6.0	1220	43	5.0	1690	44	5.5	1460	45	5.5	2610	46	2.8	2380	47	3.2		48	3.3	
AVERAGE					4.7	1393		2.7	1270		2.7	1880		3.2	1563		3.5	2647		3.0	2490		2.0			3.1	
4-27	14.5	1:3.0:2.0	1.00	49	3.0	1170	50	3.2	1170	51	3.0	1860	52	2.7	1730	53	4.5	2890	54	4.0	2660	55	3.0		56	2.5	
				57	2.7	1280	58	3.0	1260	59	3.0	1970	60	2.5	2060	61	3.0	2850	62	2.7	3180	63	3.0		64	2.0	
				65	1.5	1590	66	1.5	1470	67	1.5	2290	68	1.3	2140	69	2.2	3180	70	2.0	3140	71	1.3		72	1.3	
AVERAGE					2.5	[illegible]		2.5	1300		2.5	2040		2.2	1977		3.2	2973		3.0	2993		2.4			1.9	
4-30	19.5	1:3.0:2.0	1.00	73	1.5	1440	74	2.0	1310	75	1.7	2050	76	1.2	2010	77	2.0		78	2.2		79	1.8		80	2.3	
				81	2.2	1360	82	2.0	1230	83	1.8	1840	84	1.8	1500	85	3.5	2800	86	3.5	2690	87	3.5		88	3.0	
				89	2.0	1370	90	2.2	1220	91	1.7	1810	92	2.0	1960	93	3.0	2650	94	2.3	2610	95	2.7		96	2.5	
AVERAGE					2.0	1390		2.0	1253		1.7	1900		1.7	1823		2.8	2725		2.7	2650		2.7			2.6	
5-4	24.5	1:3.14:2.06	0.98	97	4.0	1320	98	3.7	1150	99	3.7	1920	100	4.0	1820	101	7.0	2810	102	7.0	2720	103	6.0		104	4.8	
				105	2.0	1320	106	2.0	1100	107	2.0	1890	108	2.0	1740	109	1.5	2790	110	2.7	2980	111	2.5		112	2.0	
				113	2.2	1480	114	2.0	1280	115	2.2	2050	116	1.7	2220	117	3.7	2960	118	3.0	3040	119	3.0		120	2.5	
AVERAGE					2.7	1373		2.5	1177		2.7	1953		2.5	1927		4.0	2853		4.2	2913		3.8			3.1	
5-8	29.5	1:3.08:1.97	1.02	121	6.5	1660	122	6.2	1380	123	4.7	2200	124	4.0	2010	125	7.0		126	7.0		127	5.5		128	8.0	
				129	3.2	1500	130	2.8	1320	131	3.5	2210	132	2.7	1840	133	7.0	2640	134	7.0	2920	135	7.3		136	5.8	
				137	3.8	1510	138	5.2	1180	139	3.8	2040	140	3.5	1990	141	4.5	2810	142	4.7	2620	143	4.7		144	4.7	
AVERAGE					4.5	1560		4.7	1290		4.0	2150		3.5	1947		6.2	2825		6.2	2770		5.8			5.5	
5-13	34.5	1:2.87:2.03	0.87	145	4.0	1220	146	4.7	1170	147	4.5	2090	148	4.5	1960	149	7.0	2790	150	6.0	2760	151	5.3		152	6.0	
				153	4.0	1160	154	6.3	1150	155	5.5	1990	156	4.0	1840	157	3.2	2860	158	3.2	2750	159	2.5		160	2.3	
				161	4.3	1290	162	4.5	1230	163	4.2	2090	164	3.7	1970	165	4.0	2910	166	4.0	3000	167	4.5		168	4.3	
AVERAGE					4.1	1220		3.2	1180		4.7	2057		4.0	1923		4.7	2853		4.3	2837		4.1			4.2	
5-17	39.0	1:3.12:2.02	0.89	169	3.5	1100	170	3.5	1130	171	3.0	1780	172	2.8	1910	173	3.0		174	3.0		175	2.5		176	2.3	
				177	3.0	1190	178	3.0	1130	179	3.0	1860	180	2.7	1900	181	4.0	2540	182	3.8	2860	183	3.3		184	2.7	
				185	3.5	1230	186	4.0	1180	187	4.0	1905	188	3.0	2070	189	2.5	2920	190	2.5	3280	191	2.7		192	2.7	
AVERAGE					3.3	1173		3.5	1147		3.3	1848		2.8	1960		3.2	2730		3.1	3070		2.8			2.6	
5-21	44.0	1:3.09:2.21	1.00	193	3.3	1510	194	3.5	1430	195	3.0	2270	196	3.0	2380	197	3.5	2860	198	3.3	2980	199	3.3		200	2.8	
				201	2.8	1380	202	2.5	1250	203	2.3	2150	204	1.3	2180	205	3.3	2920	206	2.8	3190	207	3.3		208	3.0	
				209	3.0	1470	210	3.5	1290	211	3.0	2100	212	3.3	2220	213	4.0	2820	214	3.8	2900	215.	4.0		216	3.0	
AVERAGE					3.0	1453		3.2	1323		2.8	2173		2.5	2260		3.6	2867		3.3	3023		3.5			2.9	
5-25	49.0	1:3.18:2.05	0.934	217	3.3		218	3.0		219	2.8		220	3.0		221	3.0	2660	222	3.3	2560	223	2.8		224	2.5	
				225	2.8	1480	226	2.8	1500	227	2.8	2380	228	2.5	2420	229	3.3	3110	230	3.5	3280	231	3.8		232	3.0	
				233	2.8	1620	234	3.0	1495	235	3.3	2300	236	4.0	2280	237	4.5	3120	238	5.0	3310	239	5.3		240	4.3	
AVERAGE					2.9	1550		2.9	1497		3.0	2340		3.2	2340		3.6	2963		3.9	3050		3.9			3.3	
5-28	54.0	1:3.10:2.04	1.03	241	2.5	1470	242	2.5	1330	243	2.5	2040	244	1.5	2040	245	3.8	2560	246	3.8	2740	247	3.3		248	2.5	
				249	3.0	1470	250	3.3	1490	251	3.8	1610	252	3.0	2190	253	4.0	2960	254	3.8	2720	255	3.5		256	3.5	
				257	4.8	1410	258	4.5	980	259	4.5	1540	260	3.0	1460	261	3.5	2740	262	3.5	2800	263	3.8		264	3.0	
AVERAGE					3.4	1450		3.4	1267		3.6	1730		2.5	1897		3.8	2753		3.7	2753		3.5			3.0	
6-2	58.0	1:3.57:1.99	1.193	265	5.0	1740	266	8.3	1530	267	6.0	2660	268	8.0	2630	269	7.0	2750	270	7.3	2760	271	6.8		272	6.8	
				273	3.3	1760	274	3.0	1750	275	2.8	2760	276	2.8	2680	277	3.5	2650	278	3.5	3210	279	2.2		280	3.2	
				281	2.5	1920	282	2.2	1600	283	2.5		284	2.2		285	3.0		286	3.2		287	3.0		288	3.0	
AVERAGE					3.6	1807		4.5	1627		3.8	2710		4.3	2655		4.5	2700		4.7	2985		4.0			4.3	
6-4	60.33	1:3.66:2.29	1.06	289	2.5	1630	290	2.0	1280	291	2.3	2320	292	2.0	2510	283	2.5	2970	294	2.8	2930	295	2.5		296	4.0	
				297	6.0	1530	298	7.0	1320	299	7.0	2050	300	5.3	1975	301	4.0	3170	302	4.0	3090	303	4.0		304	3.3	
				305	3.5	1540	306	4.5	1460	307	4.5	2290	308	3.5	2510	309	6.5	3010	310	4.0	2660	311	2.5		312	2.5	
AVERAGE					4.0	1567		4.5	1347		4.6	2220		3.6	2332		4.3	3050		3.6	2893		3.3			3.3	

* Mix by loose volume

TABLE 19.—Data of Concrete Mix.

(In computing proportion by volume 91 lb. of sand and 93 lb. of crushed stone, exclusive of moisture, were each assumed to give 1 cu. ft. loose measure.)

			4-9-26	4-13-26	4-14-26	4-22-26	4-28-26	4-29-26	4-30-26	5-4-26	5-8-26	5-13-26	5-17-26	5-21-26	5-25-26	5-28-26	6-2-26	6-4-26
1	DATE		4-9-26	4-13-26	4-14-26	4-22-26	4-28-26	4-29-26	4-30-26	5-4-26	5-8-26	5-13-26	5-17-26	5-21-26	5-25-26	5-28-26	6-2-26	6-4-26
2	MOISTURE IN SAND	CU. FT.	0.467		0.99	1.05	1.093	1.181	1.408	1.367	1.361	* 1.006	1.063	1.240	0.996	1.443	1.264	1.193
3	MOISTURE IN 3/16" TO 1 1/2" AGGREGATE	"								0.208	0.175	* 0.136	0.138	0.153	0.142	0.168	0.152	0.200
4	TOTAL WATER PER BATCH	" "								3.426	3.565	* 3.042	3.125	3.508	3.268	3.605	3.583	3.182
5	CEMENT PER BATCH	BAGS				3.5	3.5	3.5	3.5	3.5	3.5	3.5	3.5	3.5	3.5	3.5	3.0	3.0
6	WATER-CEMENT RATIO								1.00	0.98	1.02	* 0.870	0.894	1.000	0.934	1.030	1.193	1.06
7	AGGREGATES EXCLUSIVE OF MOISTURE	0" TO 3/16" LBS.	636.0		880.0	878.0	959.0	978.0	968.0	996.0	974.0	* 985.0	994.7	982.0	1004.5	986.96	979.2	1003.0
8		3/16" TO 3/4" "								371.0	376.5	398.0	377.6	381.0	396.0	373.8	340.0	368.8
9		3/4" TO 1 1/2" "								302.0	266.0	265.0	280.8	340.1	272.0	292.2	246.9	273.0
10	FINENESS MODULUS OF AGGREGATE	0" TO 3/16"				3.73	3.67			3.78	3.79	3.83	3.81	3.65	3.73	3.75	3.65	3.81
11		3/16" TO 3/4"				6.65				6.53	6.48	6.63	6.53	6.58	6.55	6.55	6.54	6.68
12		3/4" TO 1 1/2"				7.89				7.91	7.82	7.68	7.65	7.68	7.53	7.49	7.74	7.82
13		0" TO 1 1/2" (Mixed)								5.14	5.08	* 5.18	5.15	5.11	5.01	5.06	4.89	5.12
14	MIX (Parts)	BY WEIGHT								1:3.03:2.04	1:2.96:1.95	* 1:2.81:2.01	1:3.02:2.00	1:2.99:2.19	1:3.08:2.05	1:3.00:2.02	1:3.47:1.97	1:3.56:2.27
15		BY VOLUME (Loose)							1:3.0:2.0	1:3.13:2.06	1:3.08:1.97	* 1:2.90:2.03	1:3.12:2.02	1:3.09:2.21	1:3.16:2.05	1:3.10:2.04	1:3.57:1.99	1:3.66:2.29

* Appears to be in error.

at which places the cause for variations in strength is fairly well understood, the uniformity seems to be very good. Excluding the first pouring, the average strength at the age of 3 months was about 2 800 lb. per sq. in. By averaging the strengths for all ages, as shown in Fig. 11, the effect of accidental variations is reduced to a minimum. Therefore, this average strength curve indicates best the degree of uniformity attained.

FIG. 11.—STRENGTH OF CONCRETE AT VARIOUS ELEVATIONS.

11.—Thickness and Alignment of Dam.—The measured thicknesses of the dam at various heights and the elevations of the construction joints are shown in Table 17. The day after each pouring, except the first, the elevation of the top of the concrete was determined accurately at each even 10-ft. station along the dam, and the distance between up-stream and down-stream forms was measured at these stations. These measurements were taken at the 10-ft. elevation and thereafter at even 5-ft. contours to the top of the dam. These distances, it is believed, may fairly be taken as the thickness at these contours, although the concrete was usually from 5 to 10 in. below the contour on which the thickness was measured.

For the first pouring, the thickness was determined at the surface of the concrete. The exact elevation at the points at which these thicknesses were measured was not determined, and, due to the taper, slight variations in thickness would accompany the variation in level of the top of the concrete. This is sufficient to account for the differences in thickness at the average elevation of 3.2 ft., shown in Table 17.

The variation from the designed thickness (Table 17) was generally not more than $\frac{1}{8}$ in. In one instance (at the 10-ft. elevation) the thickness was $\frac{3}{8}$ in. too great. This was the greatest variation found anywhere. In that

pouring the height of the concrete was increased approximately 6 ft. in 1 day, and this may account for the greater variation in thickness at the 10-ft. elevation than elsewhere, since less time than usual elapsed between the pouring of the successive 15-in. lifts. These thicknesses were measured at approximately the elevations of the form ribs, which were 5 ft. apart. Measurements taken at the 37.5 ft. elevation showed a bulging of the lagging between the form ribs of about ¼ in.

It has not been possible to determine accurately the movements of the dam at each elevation, beginning with the time immediately after the pouring of the concrete. Theodolite readings were taken on the morning after each pouring, for the purpose of comparing the location of the up-stream face of the dam with the position required by the design. Of course the theodolite readings gave no information on the amount of deflection after the concrete was placed, because they did not distinguish between deflection and error of placing. Generally, the up-stream form was placed so that at the top of a new lift it was about ⅛ in. down stream from the position called for by the design, while the down-stream form was placed as specified by the design. If the theodolite readings showed the up-stream face to be in the position called for by the design, it indicated a spreading of the forms.

The first clinometer readings were taken when the dam was 15 ft. high. Thereafter the first reading on any 5-ft. lift of concrete was taken on the morning after the lift in question had been poured. However, since the highest clinometer bracket was generally about 4 ft. below the construction joint (temporarily the top of the dam), the highest readings at this elevation represent, principally, deflection of the portion of the dam below the top lift.

It was, therefore, generally five days or more after a given lift of concrete was poured before the first reading was available which would give information on the deflection within the upper 4 ft. of that lift. Before this period had elapsed, the chemical heat of the concrete had been developed and disappeared, and the clinometer readings would be of no value in interpreting the effect of the chemical heat.

A study has been made to determine as nearly as possible from the clinometer readings and the measured distances from the concrete to the hole in the bracket and from the hole to the ball on the bracket the actual position of successive points on the down-stream face of the dam during the construction period. In Fig. 12, giving the results of this study, the movements are shown for successive days at various elevations. No reason appears for thinking that there was an appreciable deflection of the dam, either up stream or down stream, during the construction period, except as changes in temperature may have caused it to move either way temporarily.

12.—Curing of Concrete.—To assist in retaining the moisture in the concrete, the forms were left in place for a minimum of 14 to a maximum of 32 days. (See Section 8.) For the lower 30 ft., forms were wet with a hose three times a day. In spite of these precautions, a separation of the dam from the foundation at an elevation of about 12 ft. was observed. In the hope that more water would prevent shrinkage, which seemed to be responsible for this separation, a pipe perforated at intervals of about 18 in.,

was carried along the up-stream, and another along the down-stream, face for the full length of the dam at approximately the top of the concrete. As the height of the concrete was raised, the pipes were raised. The water was then kept flowing on the forms all night and at such times during the day as did not interfere with construction operations and instrument readings. These pipes were first used at the 35-ft. elevation. After completion of the concreting, one pipe was placed on the top of the dam and the water was kept flowing whenever it did not interfere with the work, except that in the latter part of June the water was discontinued for about 7 days in order to determine the effect on the dam of a certain amount of drying.

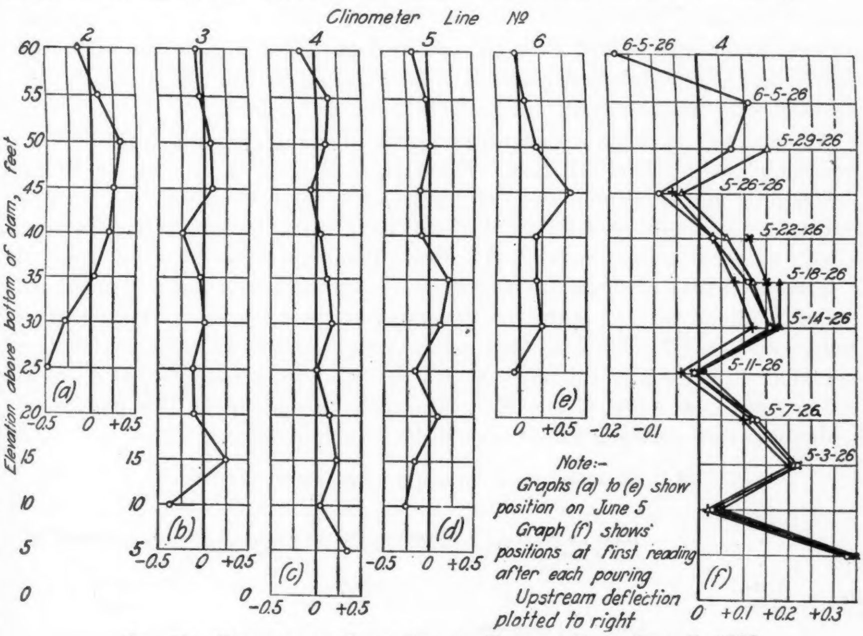


FIG. 12.—POSITION OF DOWN-STREAM FACE OF DAM, JUNE 5, 1926.

C.—GENERAL METHOD OF TAKING MEASUREMENTS

13.—*Purpose in Testing.*—The purpose of the test was fundamentally to determine the distribution of the load. All measurements of strains and deflections were aimed as directly as possible at giving information on this subject. In addition, some information on shrinkage and temperature effects was secured. All the measurements taken may be grouped under four headings: Deflections, strains,* change in width of cracks, and change in temperature.

* Wherever in Part IV the word, "strain", is used, it denotes a change of length per unit of length. The expression, "temperature strain", where used refers to expansion or contraction due to temperature change. The expression, "total strain", denotes the total change of length within the entire gauge length or whatever length is under consideration. Wherever the word, "stress", is used, it designates intensity per unit of area of the internal force. This usage is consistent with the recommendations of Committee E-1 of the American Society for Testing Materials, see, *Proceedings*, Am. Soc. for Testing Materials, Vol. 23, Pt. 1, p. 937 (1923); also, Vol. 25, Pt. 1, p. 879 (1925). By measured stress is meant the stress computed from the measured strain.

14.—General Procedure.—During the construction, accurate observations were made to determine changes of temperature which accompanied the setting of the cement and the corresponding strains in the concrete. Deflections of the dam also were observed. During the subsequent curing period, measurements were taken to determine the effect of changes of temperature and moisture on the deflections, and the strains. These measurements were taken at various times during the day. Soon it became evident that temperature effects during the day were so great that the agreement of readings taken even a few minutes apart was affected by the condition of shadow or sunshine at the station under consideration. This made it evident that for subsequent tests it would be necessary to take readings at such a time that a uniform temperature would prevail over the entire dam and that comparatively little change would take place during the time required for making the test. An approximation of such conditions prevailed only at night.

The wiring and other arrangements for the lighting system around the dam were completed on June 22, 1926. From June 24 until the conclusion of the testing program, in September, 1926, practically all observations on the dam were taken at night or early in the morning before the sun had risen high enough to shine on any part of the dam. Readings were taken on June 24 to serve as initial readings for the study of temperature effects. Additional sets of readings without water in the reservoir were taken on the nights of June 30 and July 3. The load testing began on July 12 and was completed on September 23. Usually the observations extended from about 12:00 P. M. to 7:00 or 8:00 A. M. (See Table 20.) The temperature during this portion of the day was very nearly constant, as will be seen from Fig. 47.

TABLE 20.—GENERAL SUMMARY OF TEST SCHEDULE.

Test No.	Date, 1926.	Head of Water, in Feet, for:		Time of Taking Readings (Morning, Except as Starred):			
				No load.		Load.	
		Key readings.	Complete readings.	From	To	From	To
1	July 12	10	20	12:50	3:25	5:15	7:10
2	" 15	10	20	12:50	3:15	4:30	6:15
3	" 19	20	30	12:30	2:45	6:30	8:10
4	" 23	20	30	1:15	3:30	5:00	6:45
5	August 9	20	30	12:40	3:10	4:35	6:30
6	" 27, " 28	30	40	11:35*	2:00	4:50	7:00
7	September 1	30	40	12:15	2:55	4:00	6:00
8	" 4	None.	40	12:00	2:30	6:00	8:00
9	" 8	None.	50	12:00	2:20	5:00	7:15
10	" 11	None.	50	12:00	2:25	4.00	6:30
11	" 18, " 19	None.	60	2:20	4:15	10.45*	12:45
12	" 21, " 22	50	60	2:00	4:00	9 50*	11:55*
13	" 22, " 23	30, 40, 50, 60.	None.	10:05	12:40	7:55*	11:55*

D.—INSTRUMENTS

15.—Strain-Gauge.—The strain-gauge used is a variation from the Berry instrument. Before deciding on the type and the length, a study of strain-

gauges was made. It might be expected that the 20-in. gauge length would give greater accuracy than the 10-in. length. The total strains to be measured in 20 in. would generally be greater than in 10 in., and if the total errors are the same, the percentage of error would be correspondingly smaller. It was found, however, that the increase of error due to the greater difficulty of handling a longer instrument was sufficient to over-balance the advantage. For this reason a 10-in. gauge length was selected. Fig. 13 is a diagram of this instrument. The principle upon which it operates is that of a parallelogram with one leg of the instrument attached to one corner of the parallelogram and the other to the diagonally opposite corner. The two sides of the parallelogram are connected by short-leaf springs, flexible enough to permit the necessary movement and to serve as hinges. The idea originated with Mr. H. L. Whittemore, of the Bureau of Standards, who, at the request of the Committee, designed and constructed one for its use. The final design of the instrument, shown in Fig. 13, was worked out by G. S. Binckley, M. Am. Soc. C. E., a member of the Sub-Committee on Test Dam. The strain-gauges were made from invar steel having a coefficient of expansion of about 0.0000021 per degree centigrade and were so constructed as to compensate to a large extent for the effects of temperature changes.

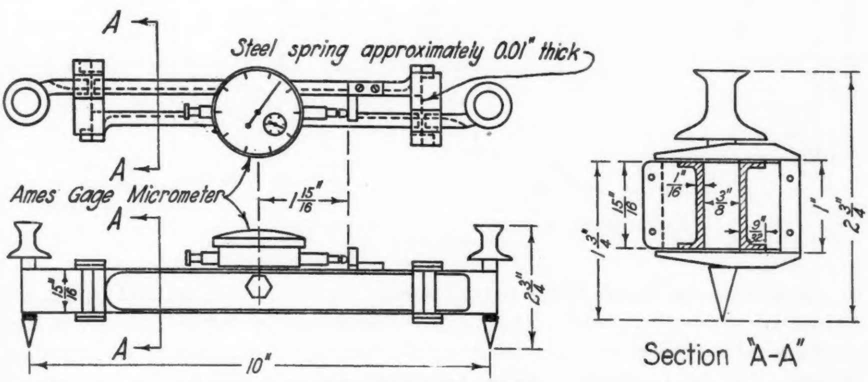


FIG. 13.—STRAIN-GAUGE.

The error of a reading with the strain-gauge may be said to be made up of the error inherent in the micrometer gauge mounted upon it and the error of observation with the strain-gauge as a whole. Owing to the difficulty of manipulating the strain-gauge so as to duplicate readings on any gauge line, the latter error is more than merely that of reading the dial on the gauge.

In order to determine the error inherent in the micrometer, an Ames gauge which formed the micrometer of one of the strain-gauges used in the test was calibrated at the Bureau of Standards. In this calibration it was found that there were errors at certain points in the travel of the instrument which were larger than those at other points. The calibration curves for selected portions of the range are shown in Figs. 13 and 14. The curves showing the total error are not drawn through the observed points, but have been smoothed out by connecting averages for pairs of consecutive points. This is justified

on the ground that in calibrating the instrument, the readings were taken to the nearest 0.00001 in., whereas the gauge is graduated only to 0.0001 in. Part of the error must be in estimating the reading and this is allowed for by drawing the curve through the averages for all pairs of consecutive values. The method used for correcting the error of reading the Ames gauge is arbitrary, but it seems to give reasonable results. The use of individual values instead of the modified curve would give a maximum percentage of error for Fig. 14 almost four times as great, and for Fig. 15 about twice as great, as the maximum errors they show.

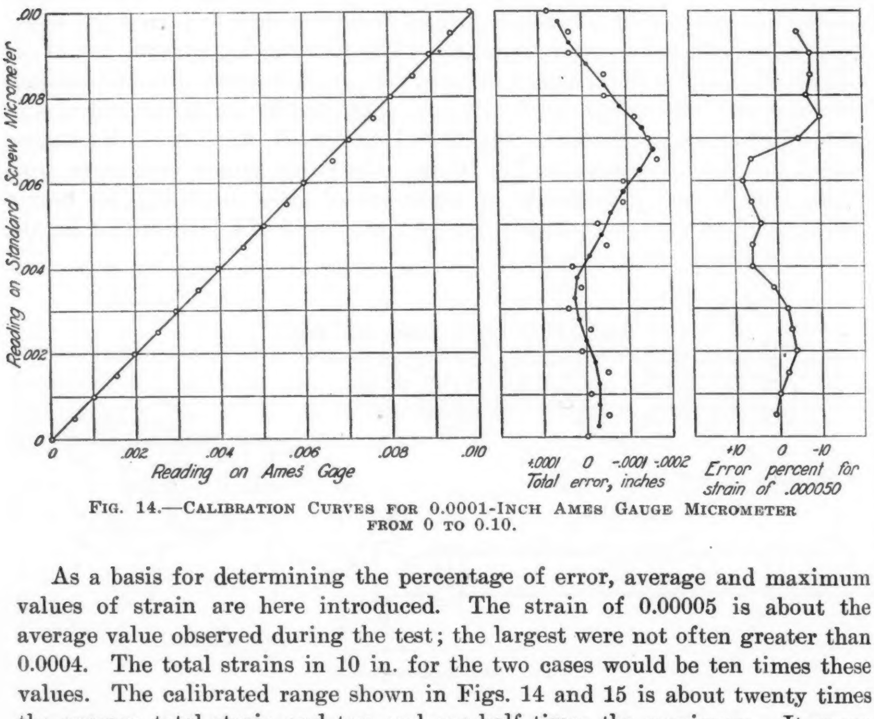


Fig. 14.—Calibration Curves for 0.0001-Inch Ames Gauge Micrometer from 0 to 0.10.

As a basis for determining the percentage of error, average and maximum values of strain are here introduced. The strain of 0.00005 is about the average value observed during the test; the largest were not often greater than 0.0004. The total strains in 10 in. for the two cases would be ten times these values. The calibrated range shown in Figs. 14 and 15 is about twenty times the average total strain and two and one-half times the maximum. It seems fair, then, to assume that the calibrated range covers the largest errors likely to be found in the Ames gauge.

The percentage of error for a strain of 0.00005 at various positions in the range of the gauge is shown in Figs. 14 and 15. For strains larger than 0.00005, the percentage of error would, in all cases, be smaller than that shown. For strains smaller than 0.000050, the percentage of error may be larger than that shown, but it is not likely to be much larger. As shown in Figs. 14 and 15, the maximum error for a strain of 0.000050 was about 10 per cent. For the strain of 0.00040, the maximum error from Figs. 14 and 15 is found to be about 5 per cent.

In order to determine the portion of the error due to inability to repeat readings on a given gauge line, twenty gauge lines were selected for a study.

The readings used were taken in the course of carrying out Load Tests 11 and 12. Each of three observers took two readings on each gauge line for each test and the probable error of the mean of each of the two readings was computed. The probable error of the mean of two observations is 0.954 v, in which, v is the variation of either reading from the average. This is so close to the value, v, itself that the constant, 0.954, has been dropped and the residual, v, has been used as the probable error.

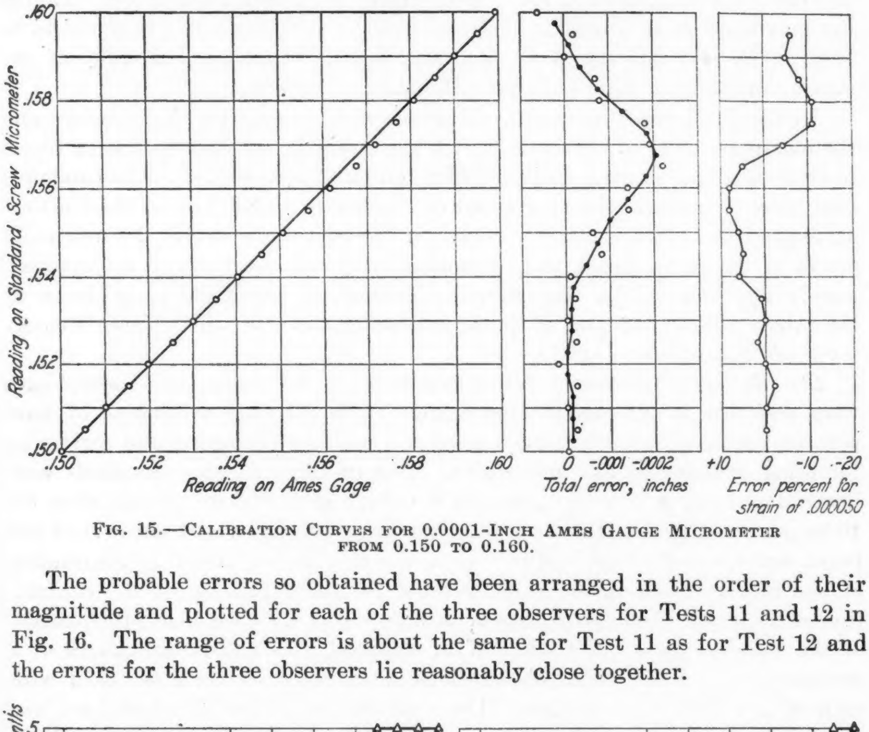


Fig. 15.—Calibration Curves for 0.0001-Inch Ames Gauge Micrometer from 0.150 to 0.160.

The probable errors so obtained have been arranged in the order of their magnitude and plotted for each of the three observers for Tests 11 and 12 in Fig. 16. The range of errors is about the same for Test 11 as for Test 12 and the errors for the three observers lie reasonably close together.

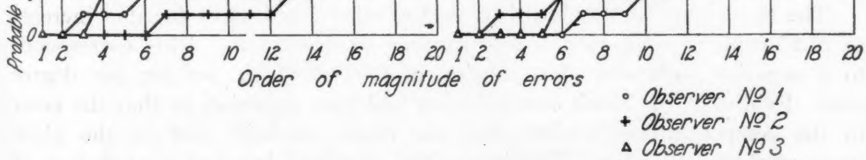


Fig. 16.—Probable Error of Average Reading with Strain-Gauge.

In the testing procedure the maximum permissible variation between any two readings on a single line was fixed at 0.0001 in. If any two readings dif-

fered from each other by more than this amount, additional readings were taken until two consecutive readings agreed within 0.0001 in. The variation of either of the two readings from the average could not then be greater than 0.00005 in. Dividing by 10 (the gauge length) to reduce to terms of strain, it will be seen that no error in strain could be found in Fig. 16 greater than 0.000005. Evidently, the number of gauge lines selected for this study (twenty) is just about sufficient to include the maximum error of 0.0001, since this error is not repeated in these figures more often than other errors. Taking the maximum error in strain as five millionths (0.000005) this is found to be 10% of the average strain of fifty millionths (0.000050), and 1¼% of the maximum strain of four hundred millionths (0.000400).

In the foregoing paragraphs, the maximum error of the Ames gauge and the maximum error of observation with the strain gauge have each been found to be 10% of the average strain of fifty millionths (0.000050). The combined error may be taken as the square root of the sum of the squares of these errors, or 14% of the average strain. Based on the maximum strain, the maximum errors of the Ames gauge and of reading with the strain-gauge are approximately 5% and 1.25%, respectively. Taking the combined error again as the square root of the sum of the squares of these errors, gives approximately 5.1% of the maximum strain.

16.—Reference Standard for Strain-Gauge.—Although the strain-gauges were designed to compensate for change of length due to change of temperature, reference standards were used as a means of detecting and correcting for other changes in the instrument. Two of the reference standards were invar steel bars, 1 in. square, having a total length slightly greater than the 10-in. gauge length. A third standard bar was constructed with invar steel and brass, and the parts combined in such a way that the expansion or contraction of one metal was designed to compensate for the expansion or the contraction of the other. An approximate determination of the coefficient of expansion of the standard bars was made. With standard bars and strain-gauges at a temperature of 21.7° cent., observations were taken on each standard with each of the four strain-gauges. The temperature of the standard bars was then changed to 48.9° cent. and observations were again taken on all standards with each of the four strain-gauges, the latter still having the former temperature of 21.7 degrees. The difference of temperature of the standard was, therefore, 27.2° cent.

The total expansion measured for Standard Bars Nos. 1 and 2 was 0.00051 in. in a total length of 10 in., or 0.00000187 in. per in. per degree cent. for each.

The compensating standard bar, on the other hand, gave for this increase of 27.2° cent. in temperature, a shortening of 0.00022 in. This corresponds to a negative coefficient of expansion of 0.00000081 in. per in. per degree cent. Evidently, too much compensation had been provided, so that the error in the compensating standard bar was about one-half that in the plain invar steel standard bar. The invar steel standard bar had a coefficient of expansion of 0.0000019 in. per in. per degree cent., or about one-fifth that of ordinary steel. For most cases, the expansion would be small enough to be negligible in its effect on strain-gauge observations.

FIG. 17.—VIEW SHOWING USE OF STRAIN-GAUGE.

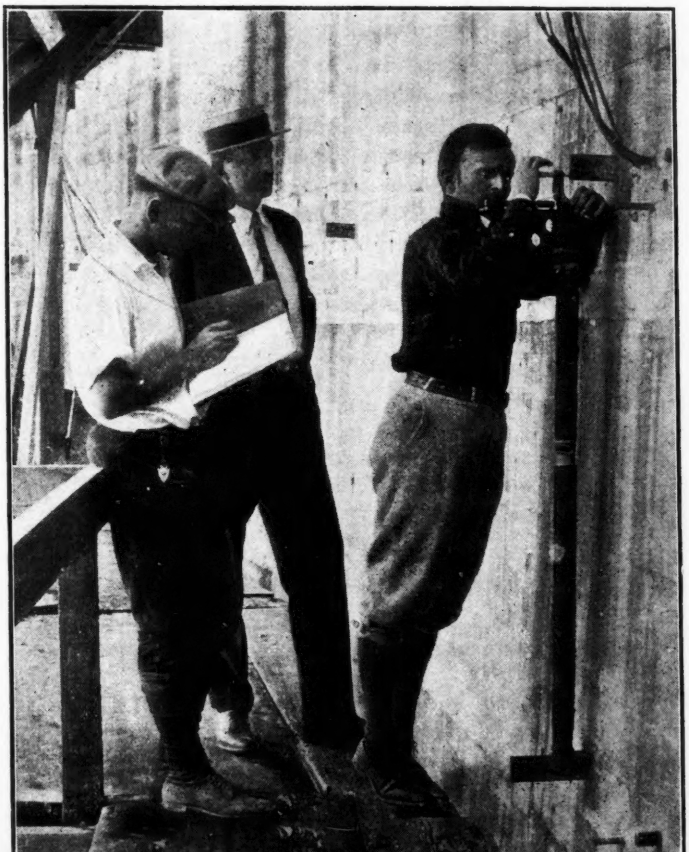


FIG. 18.—VIEW SHOWING CLINOMETER IN USE.

In taking strain-gauge observations (Fig. 17), duplicate readings on twelve gauge lines generally intervened between two successive readings on the reference standard. The entire twelve readings were first taken and then in the same order duplicate readings were taken. This gave sufficient time between duplicate readings to prevent the observer from being prejudiced in any reading by the reading immediately preceding it.

17.—Electric Telemeter.—The electric telemeter is an instrument for measuring microscopic changes of length and temperature by means of varying electrical resistances. By installing lead wires the observations may be taken at any convenient distance from the telemeter cartridge. The electric telemeter used was designed by Messrs. Burton McCullom and O. S. Peters, of the Bureau of Standards. The principle by which the changes of length are measured has been described in detail.* As stated by the inventors, the electric telemeter "depends upon the well-known fact that if a stack of carbon plates is held under pressure, change of pressure will be accompanied by a change of electrical resistance and also a change of length of the stack, both of which are reversible, the stack of plates under change of pressure performing like an elastic body."

In this test the portion of the instrument embedded in the concrete has been termed the cartridge. The cartridge consisted of a stack of carbon plates enclosed in a cylindrical steel casing 6 in. long, and having end-bearing plates of steel in contact with the stack so that the strain in 6 in. of concrete was transmitted to the carbon plates. Fig. 19 shows the details of this cartridge. Fig. 20 gives typical resistance-strain diagrams for several cartridges. There was no means of checking the telemeters against a reference standard during the test. Therefore, dependence had to be placed on the calibration curves after the cartridges had once been embedded in the concrete.

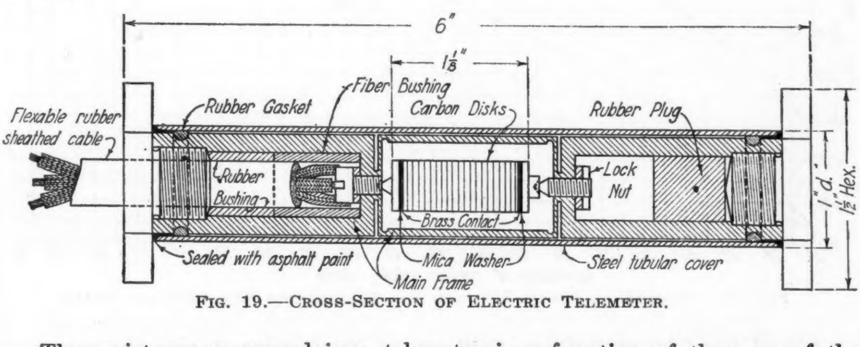


FIG. 19.—CROSS-SECTION OF ELECTRIC TELEMETER.

The resistance measured in a telemeter is a function of the sum of the distances between the carbon disks in a stack. These distances will be affected, not only by strains in the concrete, but also by expansion or contraction of the carbon disks themselves, or of the metal frame and capsule which carries the stack of disks. It is essential for other reasons that all moisture be kept out of the instrument, and temperature changes will be the principal,

* *Technologic Paper 247*, National Bureau of Standards.

if not the only, cause of expansion and contraction. Therefore, in order to make possible the interpretation of the strains observed in the telemeter, it is necessary to observe the temperatures within the cartridges, and, knowing the coefficient of expansion of the carbon disks and the metal frames, to compute the corrections to be applied to the observed strains. The temperatures in each cartridge were determined from measured resistances of a coil of No. 40 enameled copper wire wound on a vulcanized fiber spool. The resistance of this wire at 20° cent. was specified to be 512 ohms. A change of 1° cent. in the temperature of the wire, caused a change in resistance of 2 ohms. Such a device, known as a resistance thermometer, was contained within the cartridge of each telemeter. It was found that a rise or fall in temperature of 1° cent. corresponded to a change in length of the instrument of 0.000016 in. per in., and this value was used as a coefficient for making temperature corrections to the telemeter readings.

In all, 140 telemeter cartridges were embedded in the dam. Each cartridge had three lead-wires, one from the stack of carbon disks, one from the temperature coil, and one common to both stack and coil. All the leads were carried to a switchboard where there was a terminal for each lead.

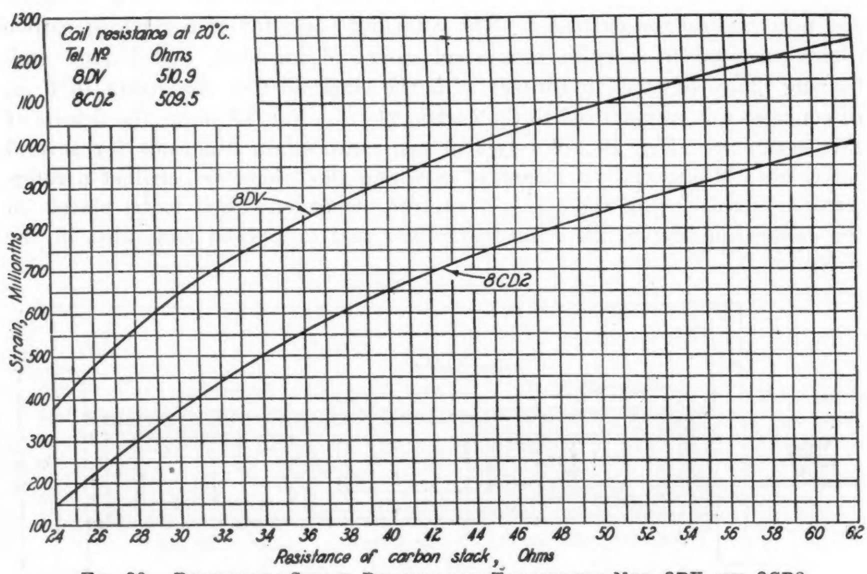


FIG. 20.—RESISTANCE STRAIN DIAGRAM FOR TELEMETERS NOS. 8DV AND 8CD2.

Six of the telemeter stacks went out of commission during the construction, curing, and testing periods without any definitely known cause. The penetration of small amounts of moisture into cartridges has been suspected as the cause. Recent improvements in the manufacture of the cartridges have aimed at making such leakage impossible. Generally, the failure of the telemeter stack was accompanied by failure of the temperature coil, but, in addition, thirty-two of the temperature coils failed while the tele-

meter stacks continued to function. After any temperature coil failed, the temperature from another coil in the same group was used for determining the temperature corrections in subsequent tests. In only two groups did all the temperature coils go out of commission. These were at the center line of the dam at Station 7 and on the down-stream face at Station 13. For Station 13 the corrections for the up-stream face were used for the telemeters in both the up-stream and the down-stream faces after the down-stream temperature coils failed. One of the center-line telemeters at Station 7 did not go out of commission until nearly the end of the investigation; hence it was available for making corrections until the test was nearly completed.

18.—Radius Meter.—The instrument here termed a radius meter measures the change in mid-ordinate of a 40-in. arc of the flexure curve of a horizontal rib of the dam. Its usefulness depends on the fact that the bending moment in a member is proportional to its change of curvature. Expressed algebraically, $M = E\,I\left(\frac{1}{r} - \frac{1}{r_0}\right)$, in which, r and r_0 are the radii of initial and final curvature, E is the modulus of elasticity, I is the moment of inertia, and M is the bending moment. If E, I, and r vary throughout the gauge length, their average values will give the moment at the center of the gauge length with only slight error.

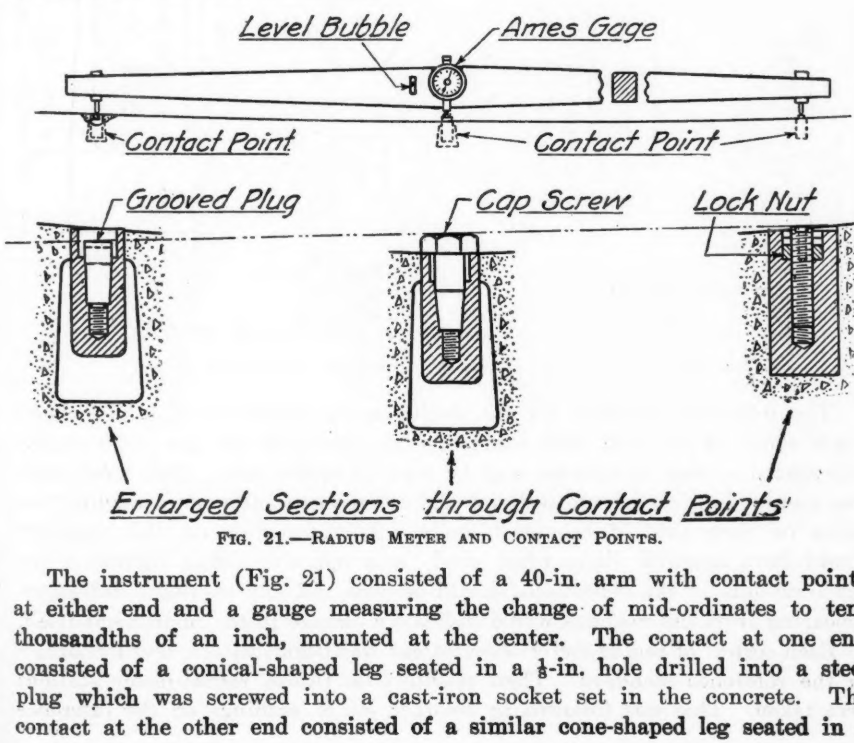


FIG. 21.—RADIUS METER AND CONTACT POINTS.

The instrument (Fig. 21) consisted of a 40-in. arm with contact points at either end and a gauge measuring the change of mid-ordinates to ten-thousandths of an inch, mounted at the center. The contact at one end consisted of a conical-shaped leg seated in a ⅛-in. hole drilled into a steel plug which was screwed into a cast-iron socket set in the concrete. The contact at the other end consisted of a similar cone-shaped leg seated in a

$\frac{1}{8}$-in. groove in a similar steel plug. The groove permitted the extension of the gauge length relative to the instrument without affecting the measurement of the change of mid-ordinate. The contact point at the center consisted of a bearing of the plunger of the Ames gauge on a flat metal surface. The plug used for one end point of each station was adjustable as to depth, so that it could be set to give a reading within the range of the instrument. Fig. 21 shows these many features of the radius meter. Fig. 22 shows the details of the inserts used with the radius meter.

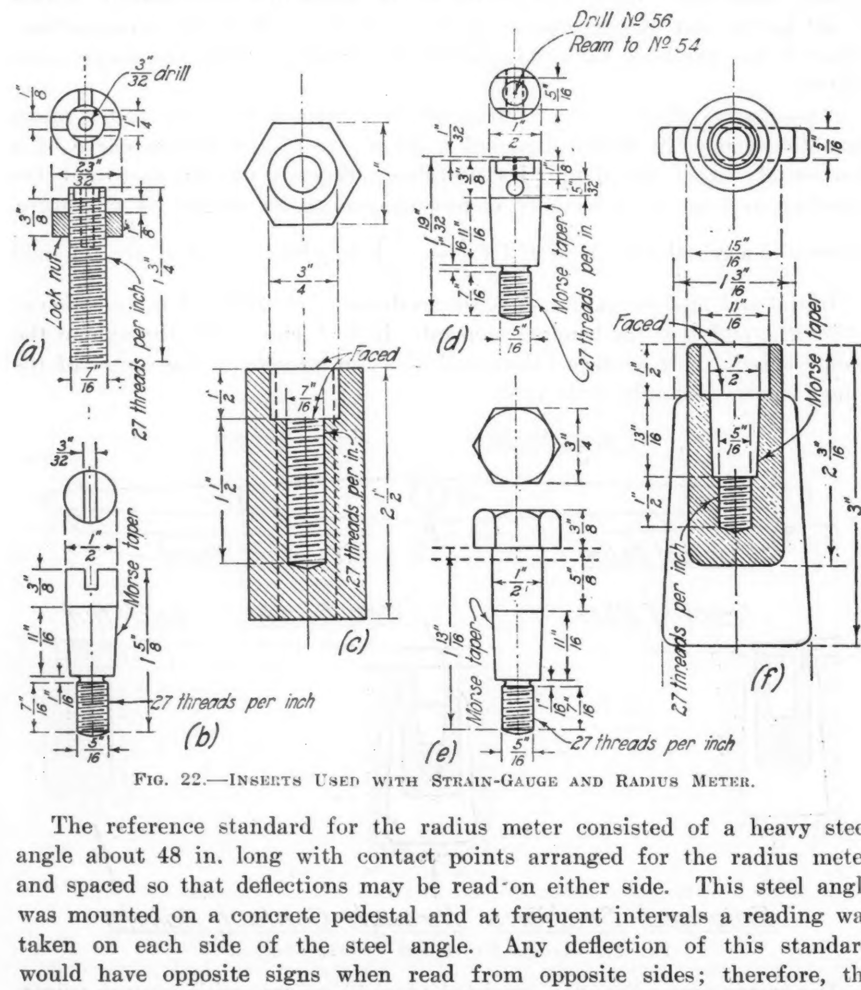


FIG. 22.—INSERTS USED WITH STRAIN-GAUGE AND RADIUS METER.

The reference standard for the radius meter consisted of a heavy steel angle about 48 in. long with contact points arranged for the radius meter and spaced so that deflections may be read on either side. This steel angle was mounted on a concrete pedestal and at frequent intervals a reading was taken on each side of the steel angle. Any deflection of this standard would have opposite signs when read from opposite sides; therefore, the algebraic sum of the deflections should be zero and any resultant deflections appearing from the readings would indicate a change in the instrument itself.

Each series of radius-meter observations was begun with a set of readings on the reference standard. Then readings on twelve radius-meter stations were taken. This was followed by another set of readings on the reference

standard, then by readings on the other eleven radius-meter stations, and, finally, by a third set of readings on the reference standard.

19.—Clinometer.—The clinometer (Figs. 18 and 23) was used to measure the deflections of the dam. It consists of a vertical staff provided with, first, a level bubble used to bring the axis of the staff into the vertical position; second, a means of bringing the lower end of the staff to the same position relative to the dam time after time; and, third, a micrometer at the top for measuring variation in distance from a fixed point on the dam to the axis of the vertical staff. The clinometer is 4 ft. 10 in. in length from the point of bearing on the ball to the tip of the cone at the bottom and is used as shown in Fig. 18. The conical point at the lower end of the clinometer seats in a $\frac{1}{8}$-in. hole in the lower bracket (Fig. 23). The plunger of the Starrett gauge bears against a $\frac{3}{8}$-in. steel ball which is inserted in the end of the upper bracket. The vertical distance from the tip of the cone at the bottom of the clinometer to the center of the $\frac{3}{8}$-in. steel ball below, is 2 in. This gives a vertical distance of 5 ft. between stations.

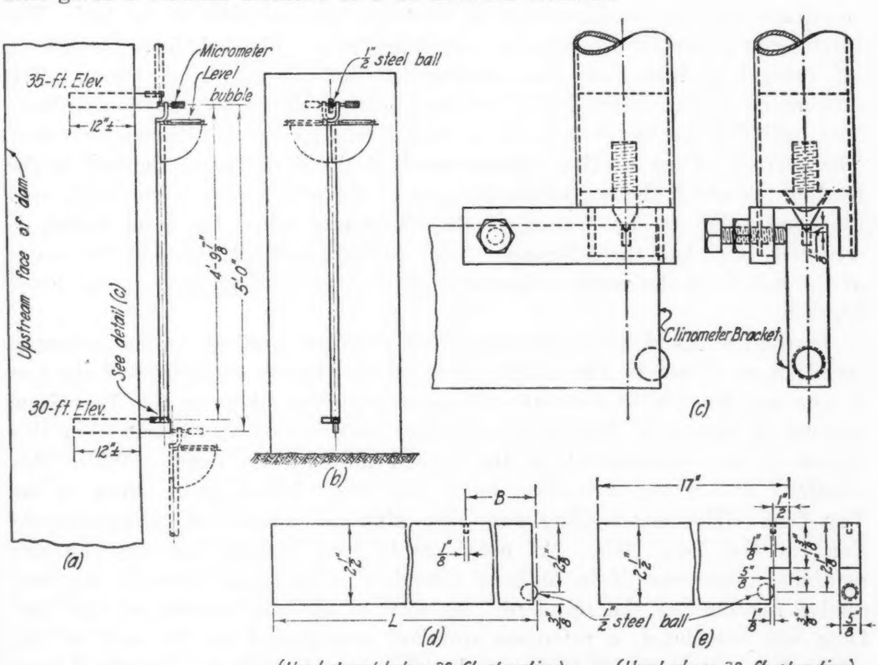


FIG. 23.—SECTIONS OF CLINOMETER: (*a*) AS USED FOR MEASURING DEFLECTIONS OF DAM; AND (*b*) AS USED ON REFERENCE STANDARD.

By means of the micrometer head the upper end of the clinometer is moved in a horizontal direction until the bubble is centered in the level vial and the micrometer is read to an estimated ten-thousandth of an inch. The instrument is then moved to the next span below or above, as shown in Fig. 23 (*a*) in dotted lines, and a similar reading is obtained. In this way readings were taken from the top to the bottom of the dam. Such a set

of readings taken at any time may be referred to a similar set taken at a previous time and the differences at corresponding positions give the deflection in the corresponding 5 ft. By adding the deflections cumulatively from the bottom to the top, the total deflection at any point in the height of the dam is obtained.

If the instrument were always kept in the same adjustment, it would not matter whether the reading for the exact vertical position of its axis were known or not, since the differences would be the same regardless of the verticality of the axis. However, since the adjustment may not remain the same, it is necessary to provide a means of determining the reading of the micrometer at which the end of the plunger will be vertically over the cone point at the bottom of the staff when the bubble is in its central position.

The manner in which this is done is illustrated in Fig. 23 (*b*). Two brackets the same distance apart vertically as the clinometer brackets on the Test Dam are provided. The steel ball is inserted in the upper clinometer bracket in such a way that readings may be taken on opposite sides of the ball. The instrument is placed as shown by the solid lines. The bubble is brought to its central position and the reading of the clinometer is taken. The clinometer is then rotated 180° to the position shown by the dotted lines. The bubble is again leveled and a second reading of the clinometer taken. The average of the readings minus one-half the diameter of the ball is the reading for which the end of the plunger of the micrometer is vertically over the cone point at the bottom of the clinometer when the level bubble is centered. One-half the difference of the readings is the distance of the center of the ball from the vertical line through the center of the hole in the lower bracket.

In the process of using the clinometer, frequent readings on the reference standard, as shown in Fig. 23 (*b*), were taken. In the early part of the test it was not feasible to have the reference standard close to the Test Dam because of danger of damage to it during construction operations. For this reason it was constructed in the tunnel adit at the camp. While this standard was in use, a reading on it was taken before going down to the Test Dam. The next reading was taken after the completion of the observations at the Test Dam. As considerable time elapsed between the two readings, there was likely to be a considerable variation between the successive readings on the standard. As soon as the construction of the Test Dam was completed, a reference standard was placed on the rock of the south abutment at the 30-ft. level. Readings on the reference standard were thereafter taken at intervals during each series of observations frequently enough to eliminate all appreciable error due to variation in the adjustment of the clinometer.

In order to obtain a measure of the accuracy of the readings taken with the clinometer, a study of probable errors for deflections on all stations was computed. These are shown in Fig. 24.

20.—Level Bar.—This instrument was designed to determine the change in inclination of the foundation rock and of the lower portions of the dam.

It consists of a flat bar a little more than 10 in. long which forms the distance piece and on which is mounted, with a hinge at one end and a coil spring bearing at the other end, an auxiliary bar carrying a level vial. The vial is graduated to indicate 4.5″ change of inclination for each change of one division in the position of the bubble. To measure the change of inclination, a bracket attached to the distance piece carried a Starrett micrometer, the plunger of which had its bearing on the end of the auxiliary bar directly over the spring. To the flat distance piece a conical pointed leg was attached at each end. In using the level bar the conical point at one end had its bearing on the circumference of a hole drilled with a No. 54 drill (0.055 in. in diameter). The point of the leg at the other end of the instrument bore upon the edges of a groove of rectangular cross-section cut longitudinally with the gauge length of the instrument.

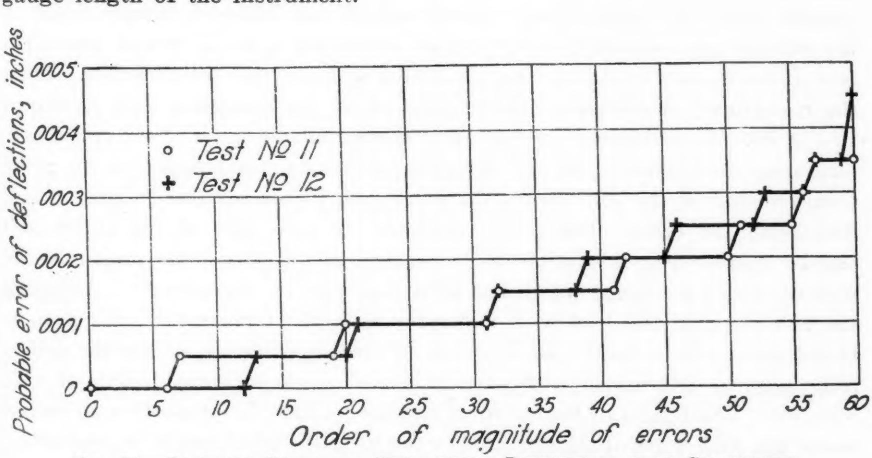


FIG. 24.—PROBABLE ERROR FOR DEFLECTIONS DETERMINED WITH CLINOMETER.

To take observations with the level bar, the instrument is placed on the contact points of the station and the level bubble is brought to the center of the vial by turning the micrometer screw. The coil spring always keeps the auxiliary bar in position against the lower end of the plunger of the gauge. The difference of readings on the Starrett gauge for different observations indicates the change of inclination which has taken place during the interval between the observations. Since the pointed leg at either end of the instrument may be used either in the drill hole or in the groove, the instrument may be reversed in direction for successive observations. This compensates for changes in the instrument, obviating the necessity of a reference bar.

21.—Resistance Micrometer.—In addition to the determination of deflections by means of the clinometer, provisions were made for measuring the deflections relative to steel towers erected a few feet from the dam on the down-stream side. The tower bases were placed just previous to the construction of the dam. The towers were designed to be 5 ft. square, protected by a wooden sheath with 1 ft. clearance on all sides. For convenience, the

erection of the towers was postponed until after the completion of the dam, but at that time the finances of the Committee did not permit the construction of the towers, and they were not erected. When the test is continued it may be desirable to build the towers to facilitate observation at times when it may not be safe to take readings with the clinometers.

An instrument for measuring the deflection relative to the steel towers was developed by Mr. Binckley. Although this instrument, the resistance micrometer, has not been used for measuring the deflections of the dam, it has been adapted to the under-water measurement of the change in width of a crack.

The resistance micrometer consists essentially of a variable resistance with a sliding contact so arranged that the distance through which this contact moves is equal to the deflection and approximately proportional to the change in resistance. The coil of resistance wire is wound around a core and both ends of the wire are connected with an instrument for measuring the resistance. A circular contact slides along the resistance coil, as shown in Fig. 25. A third wire is carried from this circular contact to the resistance-measuring instrument. As the deflection increases, the contact moves away from one end of the coil toward the other end, increasing one resistance and decreasing the other. Since the resistance of each part of the entire coil can be measured it is possible to determine, at any time, the proportionate part of the entire range which has been traversed by the contact. Although the resistance in the lead wire may vary with the temperature, the proportional effect will be nearly the same on all three resistances so that the deflections may be determined without correcting for temperature, length of lead wire, etc. The length of lead wire will thereby affect the proportionate resistances, but with wires of the size used in the test, this effect would be negligible.

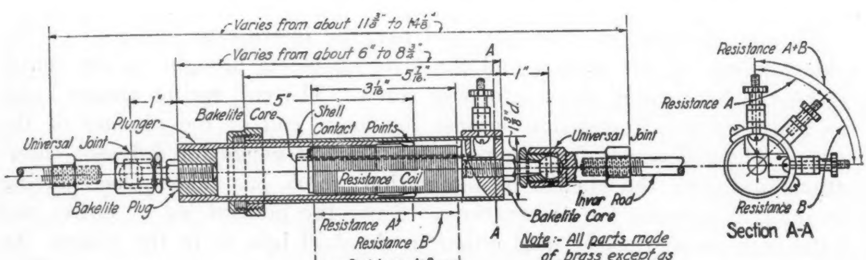


FIG. 25.—RESISTANCE MICROMETER.

A calibration curve for the instrument is shown in Fig. 26. This calibration was made just before attaching the instrument to the dam. As with the telemeters, no reference standard could be used during the test and dependence had to be placed on the calibration curve. In adapting the instrument for measuring the change in the width of a crack, the essential feature was its protection against leakage of water into the resistance coil and into the leads to the Wheatstone bridge. The instrument operated satisfactorily (Fig. 76).

After the completion of the load tests, the resistance micrometer was removed from the place of attachment at the base of the dam, and it was found that water had penetrated the envelope in considerable quantity. It is possible that water had been inside the cover during the taking of the measurements, but it is not likely that it had been in contact with the resistance coil.

FIG. 26.—CALIBRATION CURVE OF RESISTANCE MICROMETER USED IN MEASUREMENT OF WIDTH OF FOUNDATION CRACK.

E.—LOCATION OF STATIONS FOR OBSERVATIONS

22.—Basis for Arrangement of Stations.—The stations for observing deflections and strains were laid out in such a way that curves of deflection and strain would be obtained for various vertical and horizontal sections. These curves did not necessarily give the maximum strains, but they furnished a convenient system for studying the data, and the strains and moments on sections selected may be compared directly with the analytical results for the same sections. To adopt a system which involves only the sections giving the maximum strains would be next to impossible, but with the data secured it is feasible (theoretically at least) to determine the amount and direction of the maximum strains. This is true, both for the strains determined from the deflections and for those measured directly with the strain-gauge.

23.—Arrangement of Strain-Gauge Stations.—The number and arrangement of the strain-gauge stations are shown in Fig. 27. The first one or two digits in any strain-gauge station number indicates the elevation of the station. Where a station involves more than one gauge line, the letter, A, is appended as the designation for the vertical gauge line and the letters, B, C, and D for the gauge lines making clockwise angles of 45°, 90°, and 135°, respectively, with the vertical.

The stations are arranged in vertical and horizontal lines all over the down-stream face, and above the 30-ft. elevation on the up-stream face, of the dam. The horizontal rows of stations are generally 5 ft. apart. On the down-stream face, alternate stations at even 10-ft. elevations in all vertical rows have four gauge-lines each, 45° apart. These four-gauge-line stations were designed to be used in determining the magnitude and direction of the maximum strains. They also were of use in determining the amount of the torsional moment. In addition to these stations 5 ft. apart, vertically, a continuous row of stations 10 in. apart, was placed on the vertical center line and on a horizontal row at the 30-ft. elevation.

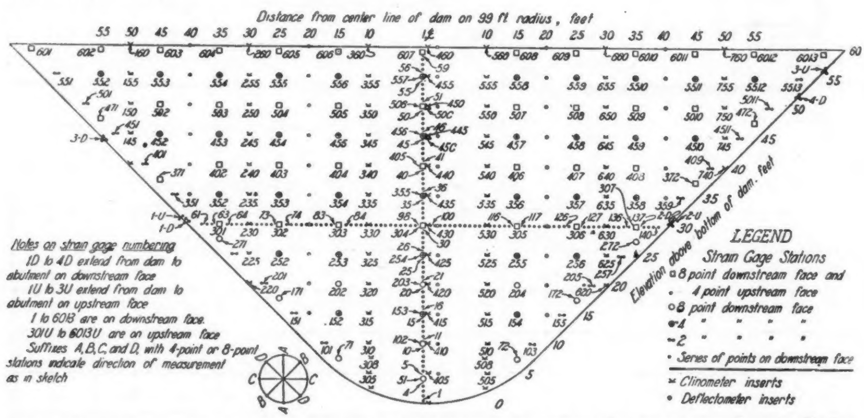


FIG. 27.—LOCATIONS OF STATIONS FOR OBSERVATIONS WITH CLINOMETER, STRAIN-GAUGE, AND DEFLECTOMETERS.

A few special strain-gauge stations not included in this system were used for showing the movement of the dam relative to the abutments and the variation in the width of the crack at the center line in the upper portion of the dam. These are Stations 1*D* to 4*D* on the down-stream face; Stations 1*U* to 3*U* on the up-stream face; Stations *T*1, *T*2, and *E*1 to *E*4 on the top of the dam at the ends; and Stations 45*C* and 50*C* on the down-stream face of the dam at elevations of 45 to 50 ft. across the crack at the center line. Most of these special stations have three gauge lines each, which form the three sides of an equilateral triangle. One corner of the triangle is in the rock and the other two are in the concrete. The three gauge lines are designated as *A, B*, and *C*, of which, Gauge Line *C* is entirely in the concrete and the others span from the concrete to the rock. All the stations are shown in Fig. 27.

24.—Arrangement of Telemeter Stations.—The numbers and locations of telemeter stations are shown in Fig. 28. Arabic numbers from 1 to 30, followed by initials showing the position and direction of measurement, identify the various telemeter cartridges. Of the initials following the Arabic numeral, the first indicates whether the location is up stream, down stream, or central in the dam, by the letters *U, D*, and *C*, respectively. The second initial indicates the direction of measurement, whether it is longitudinal (extending in the direction of a horizontal tangent to the dam at the point under consideration), vertical, or diagonal (using *L, V*, and *D*, respectively).

There are no diagonal telemeter cartridges at the down-stream face. At the up-stream face there were, in some cases, diagonal cartridges bisecting an angle made by vertical and longitudinal cartridges. Midway between the up-stream and the down-stream faces there were vertical and longitudinal cartridges, also diagonal cartridges in vertical radial planes; but there were no diagonal cartridges in a vertical longitudinal plane except at the up-stream face. A diagonal cartridge with its upper end closer to the up-stream, than the down-stream, face, is indicated by the initial, *D*1; and a diagonal with its upper end closer to the down-stream face is indicated by *D*2.

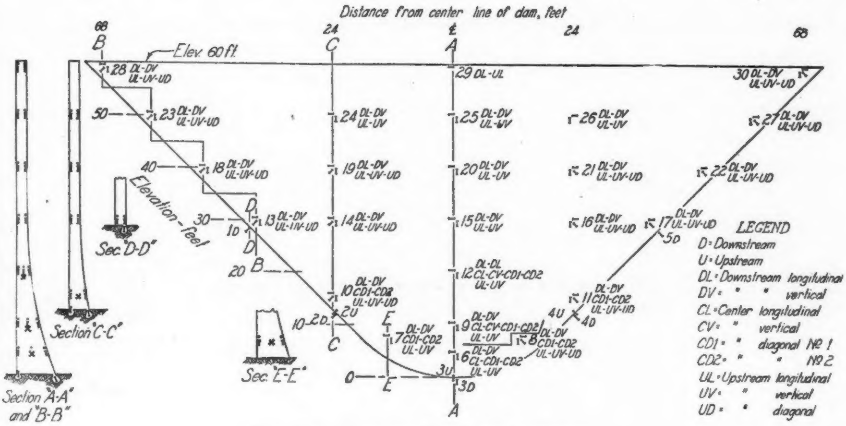


FIG. 28.—LOCATION OF TELEMETERS.

Usually a telemeter cartridge was placed in the up-stream face opposite each point where one was used in the down-stream face. It is now believed that better use would have been made of the cartridges available if, for the most part, they had been placed in the dam close to the connection with the foundation, in order to determine the boundary conditions.

Eight telemeters were placed at five stations in the foundation rock to measure temperatures and strains. Three of these were close to the up-stream face of the dam and were embedded partly in the rock and partly in the dam. The other five were near the down-stream face and were embedded entirely in the foundation rock. Fig. 28 shows the location and the manner of embedding these telemeters.

25.—Arrangement of Radius Meter Stations.—The radius meter stations, 40 in. in length, cover the entire extent of the dam at the 30-ft. elevation. It was originally planned to have thirty additional stations located as shown in Fig. 29, and the inserts for these stations were placed, but they were not used in the test. The reasons for their disuse were: First, the difficulty in operating the radius meter; and, second, the effectiveness of other measurements which made that instrument less necessary than otherwise. Although it was difficult to use, it gave information that has been useful in the interpretation of the test data.

26.—Arrangement of Clinometer Stations.—The clinometer stations are arranged on vertical lines at the center, and 10, 30, and 50 ft. from the vertical center line on either side of it, as shown in Fig. 27. Lines at 20 and 40 ft. from the center line were omitted, as it was expected that deflections on those lines would be measured with the resistance micrometer from the deflection towers. The stations were placed at elevations of 0, 5, 10, 15 ft., etc., except on the lines 10 ft. either side of the vertical center line. Since the bottom of the dam is about 2.5 ft. higher at this point than at the vertical center line, the lowest observed deflection on these two lines was at an elevation of 7.5 ft. instead of 5 ft. This gives an overlapping of the two lower clinometer spans. All the other stations were placed at elevations which are a multiple of 5 ft., in order to obtain deflection curves for horizontal sections.

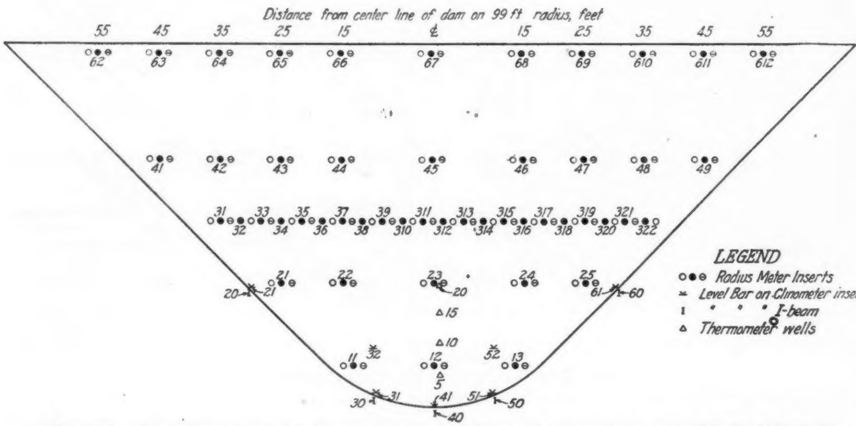


FIG. 29.—LOCATIONS OF STATIONS: RADIUS METER, LEVEL BAR, AND THERMOMETER.

27.—Arrangement of Level Bar Stations.—There were twelve stations for measuring the change of inclination with the level bar, of which five were on the foundation rock near the bottom of the dam, and seven were in the concrete of the dam near the bottom. There were no stations above the 20-ft. elevation. The numbers and the locations of these stations are shown in Fig. 29. Inclination of the foundation rock was observed upon I-beams embedded in the rock (shown in Fig. 29 by the letter, *I*). Inclinations of the dam were measured on clinometer brackets.

28.—Arrangement of Stations for Resistance Micrometer.—The locations designed for the measurement of deflections by means of the resistance micrometer are shown in Fig. 27. As already noted, this micrometer has not been used, except for the measurement of the width of a crack (Section 40).

29.—Placing of Instruments and Inserts.—The appearance of the interior of the forms during the construction of the dam is illustrated in Figs. 30 to 33, which show how the instruments and inserts were held in place. Steel plates about $\frac{1}{16}$ in. thick were used as templates to fasten, temporarily, the cast-iron inserts into which the strain-gauge plugs were inserted. The templates used for the 4- and 8-point stations were 14 in. square. Those used

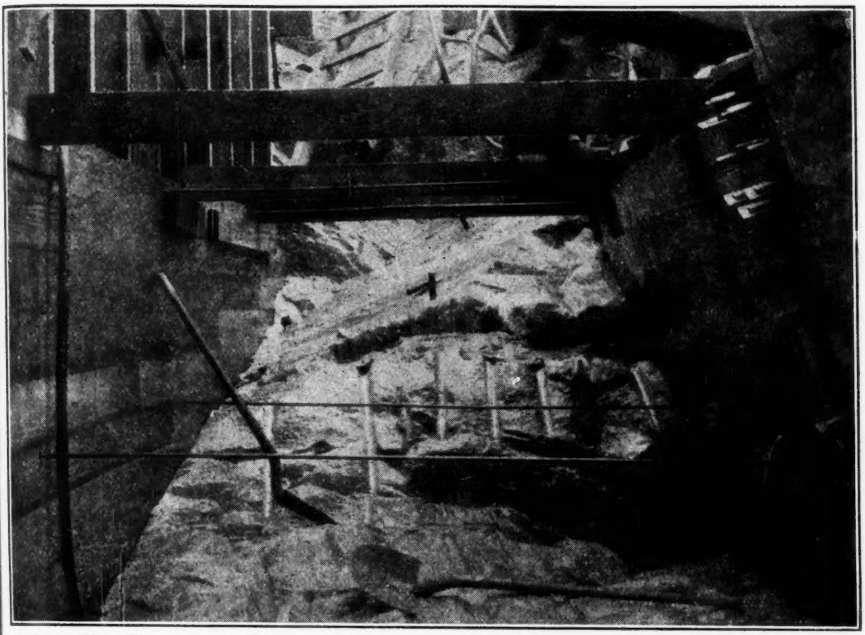


FIG. 30.—FOUNDATION ROCK AND INTERIOR OF FORMS BEFORE CONCRETING, STEVENSON CREEK TEST DAM.

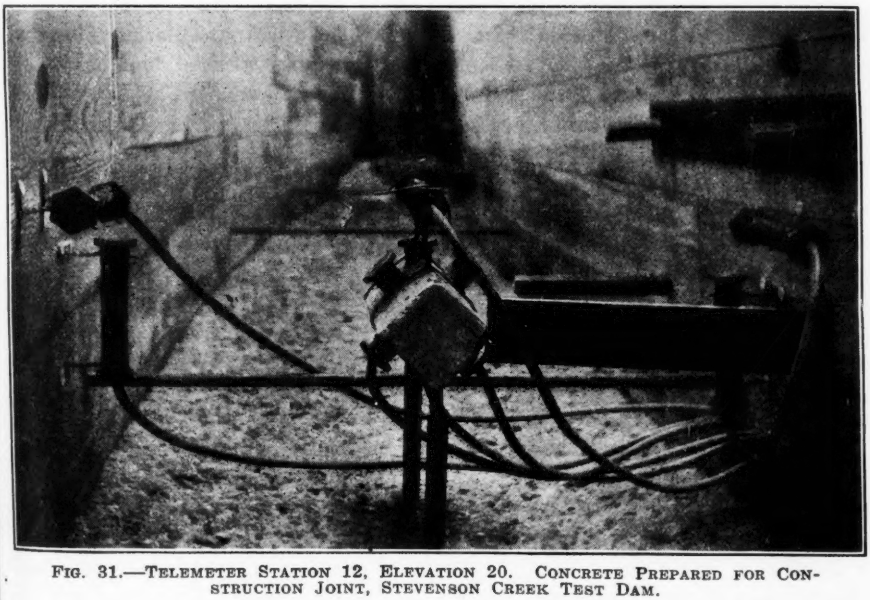


FIG. 31.—TELEMETER STATION 12, ELEVATION 20. CONCRETE PREPARED FOR CONSTRUCTION JOINT, STEVENSON CREEK TEST DAM.

for con
mitting

The
and st
and on
The ca
for pla
insert
harden
plugs
on the

The
socket
over th
gauge
hole w
magni
ing the
to stat

In
to rece
used t
the con

Th
positio
were p
tridge
suppor
and 3
cartri
block,
little
the co
free t
of the
end pl

Tw
of the
enter
placed
to fill
vision
was n
there
had b

Fo
end a

for continuous rows of gauge lines were 3 in. wide and 44 in. long, thus permitting four complete 10-in. gauge lengths per template.

The inserts were drilled and tapped (Fig. 22 (*f*)) to receive the cap-screws and strain-gauge plugs (Fig. 22 (*d*)). The taper shank on the cap-screw and on the plug fitted into the corresponding taper in the recess of the inserts. The cap-screws were used to assemble the templates and inserts into units for placing in the concrete, as shown in Figs. 30 and 33. The taper in the insert assisted in drawing it to an accurate center. After the concrete had hardened, the cap-screws and templates were removed and the gauge-hole plugs were inserted for taking strain-gauge observations. The taper shank on the plug helped to center the plug in the insert and to draw it to a bearing.

The large recess in the upper end of the insert furnished access with socket wrench for inserting the plugs and permitted placing a protecting cap over the end of the plug when tests were not in progress. After drilling the gauge hole in the plug (using a No. 55 drill), the plug was hardened and the hole was lapped inside to smooth the surface and remove all burr. Under a magnifying glass the edges of the hole appeared sharp. This method of placing the plugs resulted in accurate gauge lengths. The variation from station to station was generally within 0.03 in.

In Fig. 22 are shown the cast-iron inserts that were set into the concrete to receive the steel strain-gauge plugs; also, the plug and the brass cap-screw used to hold the inserts in place during the pouring and the hardening of the concrete.

The manner of placing and retaining the telemeter cartridges in correct positions before concreting is shown in Figs. 31 and 32. Most of the cartridges were placed near the surface so that as nearly as possible the axis of the cartridge lay parallel to, and 2 in. from, the inside face of the form. They were supported and held by twisted wire brackets tacked to the form (Figs. 31 and 32). Fig. 31 shows quite clearly the method of mounting the telemeter cartridges in the mortar blocks for use in the interior of the dam. From the block, slightly less than 4 in. square, each end of the telemeter projected a little more than 1 in. All movement of the cartridge is transmitted from the concrete through the end plates, and this arrangement leaves the ends free to receive the pressure. Only slight force is required to operate the parts of the telemeter, and the projections shown give sufficient bond between the end plate and the concrete to develop either tensile or compressive movements.

Two short pieces of iron pipe (Fig. 31) project upward from the surface of the concrete and two ½-in. bars passing entirely through the mortar blocks enter these pipes and hold the block in position. After the block had been placed, it was held from moving horizontally by wedges placed in the pipe to fill up the space around the ½-in. bars. During concreting very close supervision was maintained to avoid displacement of the telemeters. This feature was more important with the telemeters than with other instruments, because there was no means of determining their actual position after the concrete had been poured.

For each radius-meter gauge line, three plugs were required, one at each end and one at the center. Those at the ends were of the same form as the

strain-gauge plugs, and used the same kind of inserts. On account of the small range of the gauge used with the radius meter, it was necessary that one of the three plugs (an end plug) be made adjustable as to depth so that the plugs could be set to give a reading within the range of the instrument. This required room for a lock-nut on the thread of the plug and necessitated a special insert to accompany this end plug (Fig. 22).

The templates to hold the inserts for the radius-meter plugs during concreting were 3 by 44 in. by about $\frac{1}{16}$ in. (the same as for the strain-gauge inserts). Cap-screws were used to attach the inserts to the templates, which were tacked to the forms in position before the concreting (Figs. 31 and 32). After the concrete had been placed and had hardened, the cap-screws and templates were removed and the plugs were inserted and adjusted.

The clinometer brackets were steel bars, 0.5 by 2.5 in. in cross-section, extending about 12 in. into the concrete. The length of the projecting portion, illustrated in Fig. 23, depended on the elevation in the dam. It was just long enough to bring the extreme point of the ball at the end of one bar, as nearly as possible, vertically above the bearing hole of the bar next below it in which the conical point of the clinometer staff rested. The ball on which the micrometer plunger bore, of hardened steel, $\frac{3}{8}$ in. in diameter, was inserted in a recess in the end of the bar and was retained by peening around the circumference of the hole. The hole in the lower bar was $\frac{1}{8}$ in. in diameter by $\frac{1}{4}$ in. deep. The distance, B, in Fig. 23 from the bearing point on the ball to the center of the $\frac{1}{8}$-in. hole in which the cone-shaped point of the clinometer rested, was just equal to the batter in a 5-ft. height of the down-stream face of the dam. Above the 30-ft. elevation the face of the dam was vertical, that is, the batter was zero, and in order to bring the center of the bearing hole directly over the bearing point on the ball, an offset in the end of the bar, such as that shown in Fig. 23 (*e*), was provided.

The clinometer brackets were held in place in the dam during construction by rigid frames of small structural steel channels, placed back to back. The lower end of the frame was attached to a clinometer bracket which had been placed during the previous pouring. Into the upper end of the frame (5 ft. above its lower end), the bracket to be placed was clamped into the position it was to occupy in the dam. This frame held the upper bracket at the proper elevation during the succeeding pouring of the concrete. It was necessary to maintain continuous inspection during concreting in order to prevent its displacement laterally. The frame was left in position until the concrete had thoroughly hardened, and then was removed and raised 5 ft., engaging the new clinometer bracket at the bottom of the frame and in a like manner holding the one above in position for the next pouring.

Changes in inclination of the foundation rock were measured on the upper surface of 6-in. I-beams embedded in the foundation. For placing the I-beams, trenches in the rock about 6 in. wide and 8 in. deep were extended across the foundation along a radius of the dam. Within the dam about 1 ft. from either face, a small area in the bottom of the trench was left slightly higher than the remainder to afford definite bearing places for the I-beam.

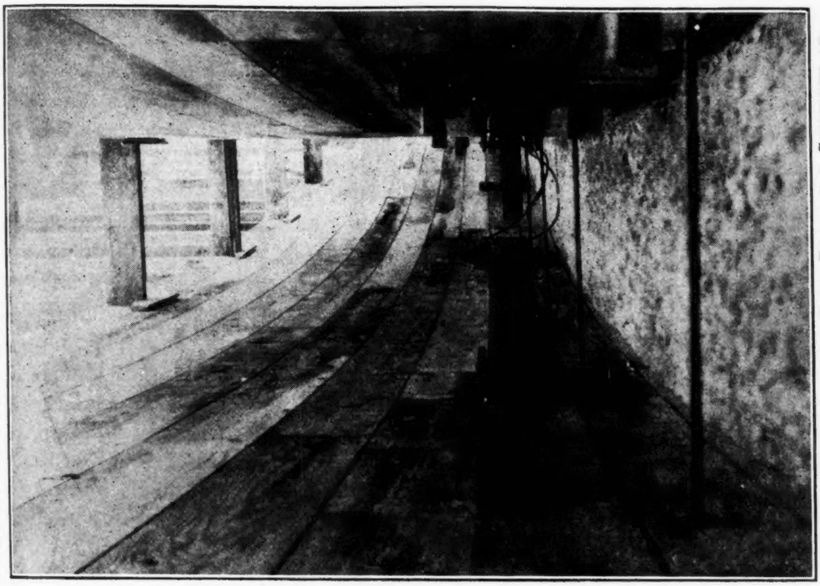


FIG. 33.—INTERIOR OF FORM READY FOR CONCRETE AT ELEVATION 40, STEVENSON CREEK TEST DAM.

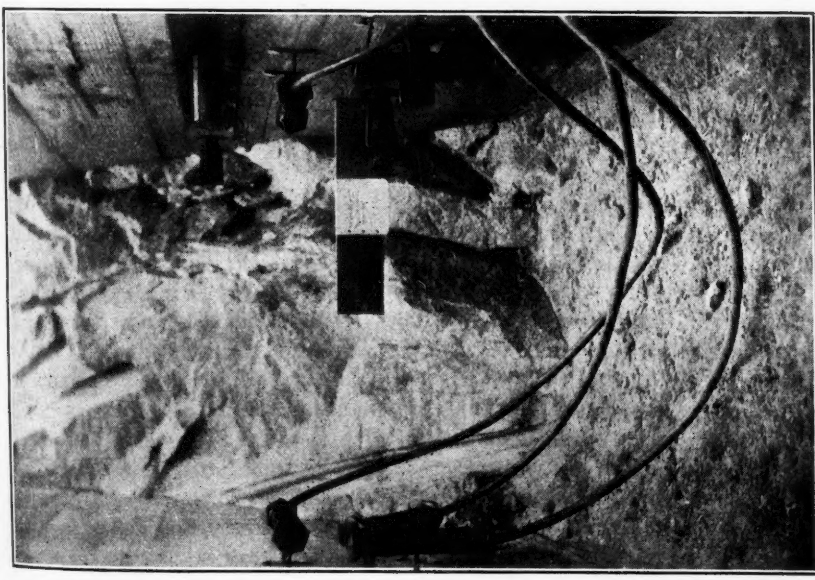


FIG. 32.—TELEMETERS AT LEFT-HAND ABUTMENT, STEVENSON CREEK TEST DAM.

The
rock o
at each
on the
and w
One e
of the
face, e
about
drilled
groove

Th
genera
section
I-bea
the ch
bar, t
planne
determ
is mo

30.
data v
the sh
radius
was t
readin
37 to
nomet
was a
and 3
the as
averag
The c
the se
which
short
elong
correc
strain

F
differ
in th
same
to rig

The I-beam was placed in the trench and anchor-bolts were set in the rock on each side opposite the bearing points. The strap across the I-beam at each of the two bearings was drawn down as firmly as possible by nuts on the anchor-bolts. Rather thin cement mortar was then placed in the trench and worked around the I-beam in the effort to secure a uniform bearing. One end of the beam extended to within a few inches of the up-stream face of the dam, and the other projected about 12 in. beyond the down-stream face, entirely free from contact with the rock. The level-bar station occupied about 10 in. of the 12-in. projection. The station consisted of a small hole drilled for the bearing of the pointed leg at one end of the instrument and a groove for the bearing of the similar leg at the other end.

The hole was drilled on the center line of the bar with a No. 54 drill, generally near the face of the dam. The groove was rectangular in cross-section, about $\frac{1}{16}$ in. wide and $\frac{1}{2}$ in. long and extended lengthwise of the I-beam. The groove and the hole were both about $\frac{1}{8}$ in. deep. For measuring the change of inclination of the dam above the foundation with the level bar, the projecting ends of the clinometer brackets were used. It had been planned to use the level-bar data in connection with the clinometer data to determine strains at the bottom of the dam, but it was not feasible. This is more fully discussed in Section 44.

F.—Records of Test Data

30.—Specimens of Original and Reduced Data.—The form in which the data were taken is indicated in Figs. 34, 35, and 36. To a considerable extent, the sheets are self-explanatory. For strain-gauge, clinometer, level bar, and radius meter, all readings were duplicated and the average of the two readings was taken as one observation. From the original data sheet, the average reading was copied to a reduction sheet, samples of which are shown in Figs. 37 to 39 for strain-gauge, clinometer, and level bar. For strain-gauge, clinometer, and radius meter, an arbitrary zero reading on the reference standard was assumed, and the corrections shown in the reduction sheets of Figs. 37 and 38 are the differences between the actual readings on the reference bar and the assumed zero reading. The corrected reading is the algebraic sum of the average readings transferred from the original data and the correction. The corrected difference is the difference between the corrected readings for the series under consideration and the corrected reading for the series with which the comparison is being made. For the strain-gauge the elongation or shortening is in 0.00001 in. in 10 in. To obtain the strain (that is, the elongation or shortening in inches per inch), it is necessary to divide the corrected difference by 10, since the gauge length was 10 in. Therefore, the strain is given in millionths.

For the clinometer the deflection at any point is obtained by adding to the difference for that point the sum of the differences for all the points below it in the same vertical radial plane. On the reduction sheets, the stations in the same vertical radial plane are so arranged that the cumulative sum from left to right gives the total deflection for the corresponding stations.

Records of Tests made in co-operation with U S Bureau of Standards FORM 1

ENGINEERING FOUNDATION
COMMITTEE ON
ARCH DAM INVESTIGATION

STEVENSON CREEK DAM
ORIGINAL DATA OF TEST
Strain Gage Test № 12

Sheet 1 Date 9-21-26
Series 1 Inst. No. A-2
Observer Stout
Recorder Barry

Time		Std.												
	Gage Line / Group No.		51				71				72			
			A	B	C	D	A	B	C	D	A	B	C	D
~~A. M.~~ 9:45 P. M.	Readings	16285	17478	16403	18362	17970	17400	16278	17214	18542	17156	16660	18081	18070
		16285	17468	16405	18360	17970	17404	16286	17208	18550	17156	16668	18080	18070
	Av. Reading	16285	17473	16404	18361	17970	17402	16282	17211	18546	17156	16664	18080	18070
A. M.	Gage Line / Group No.	Std.	102				171				172			
			A	B	C	D	A	B	C	D	A	B	C	D
P. M. 9:55	Readings	16285	17670	16670	17875	18468	17370	16330	17322	17730	16860	18640	17412	15672
		16285	17680	16678	17870	18458	17370	16335	17325	17740	16870	18640	17410	15670
	Av. Reading	16285	17675	16674	17872	18463	17370	16333	17324	17735	16865	18640	17411	15671
A. M.	Gage Line / Group No.	Std.	202				203				204			
			A	B	C	D	A	B	C	D	A	B	C	D
P. M. 10:05	Readings	16285	17422	16468	17520	18370	15208	17782	17872	15735	17035	16910	18060	18302
		16280	17422	16472	17520	18370	15212	17790	17870	15732	17030	16912	18062	18305
	Av. Reading	16283	17422	16470	17520	18370	15210	17786	17871	15733	17032	16911	18061	18303
A. M.	Gage Line / Group No.	Std.	152			153		154						
			A	C		A	C	A	C					
P. M. 10:15	Readings	16280	16895	17570		15958	16970	17655	17738					
		16300	16900	17575		15960	16970	17658	17740					
	Av. Reading	16290	16897	17573		15959	16970	17657	17739					
A. M.	Gage Line / Group No.	Std.	252		253		254		255		256			
			A	C	A	C	A	C	A	C	A	C		
P. M. 10:20	Readings	16300	19845	17284	18438	16720	17320	16978	17870	17970	17550	17922		
		16300	19850	17290	18440	16720	17330	16980	17872	17978	17542	17920		
	Av. Reading	16300	19847	17287	18439	16720	17325	16979	17871	17974	17546	17921		
A. M.	Gage Line / Group No.	Std	101	103	151	155	201	205	251	257			R1	
			C	C	C	C	C	C	C	C			V	
P. M. 10:30	Readings	16300	17010	16850	17038	16910	16690	16720	16960	16370			19460	
		16300	17010	16858	17032	16910	16690	16728	16960	16375			19468	
	Av. Reading	16300	17010	16854	17035	16910	16690	16724	16960	16372			19464	
A. M.	Gage Line / Group No.	Std												
P. M.	Readings													
	Av. Reading													
A. M.	Gage Line / Group No.	Std												
P. M.	Readings													
	Av. Reading													

Notes and Sketches: Test № 12 60 ft. head of water

Inst. № A-2 and the compensating standard bar used for these readings

FIG. 34.—COPY OF PORTION OF ORIGINAL STRAIN-GAUGE DATA FOR TEST 12.

Records of Tests made in co-operation with U S Bureau of Standards FORM

ENGINEERING FOUNDATION
COMMITTEE ON
ARCH DAM INVESTIGATION

STEVENSON CREEK DAM
ORIGINAL DATA OF TEST
Clinometer Test № 12

Sheet 1 Date 9-21-26
Series 1 Inst. No. 1
Observer Marchand
Recorder Fenwick

Time		Std.												
	Gage Line / Group No.		(1)											
			R	L										
~~A. M.~~ 9:50 P. M.	Readings	Before	6994	8069	$\frac{R+L}{2}-2500=5030$									
			6991	8065										
	Av. Reading		6992	8067										
A. M.	Gage Line / Group No.	Std.	145	150	155	160	225	230	235	240	245	250	255	260
			(55)	(56)	(57)	(58)	(46)	(45)	(44)	(43)	(42)	(41)	(40)	(39)
P. M.	Readings		6382	5710	5038	4530	6253	6494	6804	6075	5688	5150	5160	4109
			6379	5706	5031	4532	6256	6495	6803	6079	5688	5146	5159	4115
	Av. Reading		6380	5708	5035	4531	6255	6495	6803	6077	5688	5148	5160	4112
A. M.	Gage Line / Group No.	Std.	308	310	315	320	325	330	335	340	345	350	355	360
			(33)	(32)	(29)	(28)	(23)	(22)	(17)	(16)	(11)	(10)	(5)	(4)
P. M.	Readings		5710	5970	5959	6349	6704	6502	6735	5908	5871	5703	4914	4172
			5711	5964	5961	6347	6706	6502	6735	5901	5878	5701	4920	4170
	Av. Reading		5711	5967	5960	6348	6705	6502	6735	5905	5875	5702	4917	4171
A. M.	Gage Line / Group No.	Std.	405	410	415	420	425	430	435	440	445	450	455	460
			(37)	(34)	(30)	(27)	(24)	(21)	(18)	(15)	(12)	(9)	(6)	(3)
P. M.	Readings		5326	6436	6486	6661	6555	6035	6332	6060	7021	5771	5451	5103
			5325	6439	6492	6659	6557	6038	6333	6058	7016	5773	5450	5107
	Av. Reading		5325	6438	6489	6660	6556	6037	6333	6059	7018	5772	5450	5105
A. M.	Gage Line / Group No.	Std.	508	510	515	520	525	530	535	540	545	550	555	560
			(36)	(35)	(31)	(26)	(25)	(20)	(19)	(14)	(13)	(8)	(7)	(2)
P. M.	Readings		5291	6930	6580	6172	6748	5846	6051	6146	6753	5272	4623	4304
			5293	6930	6578	6167	6747	5847	6058	6142	6753	5269	4620	4304
	Av. Reading		5292	6930	6579	6170	6747	5847	6054	6144	6753	5270	4621	4304
A. M.	Gage Line / Group No.	Std.	745	750	755	760	625	630	635	640	645	650	655	660
			(62)	(61)	(60)	(59)	(47)	(48)	(49)	(50)	(51)	(52)	(53)	(54)
P. M.	Readings		6043	5298	4119	4380	5692	5939	6288	5972	6185	4944	5144	4296
			6047	5297	4121	4376	5690	5935	6284	5970	6187	4938	5144	4297
	Av. Reading		6045	5298	4120	4378	5691	5937	6286	5971	6186	4941	5144	4297
A. M.	Gage Line / Group No.	Std	(38)											
			R	L										
P. M. 11:00	Readings		6995	8071	$\frac{R+L}{2}-2500=5031$									
			6991	8068										
	Av. Reading		6993	8069										
A. M.	Gage Line / Group No.	Std	(63)											
			R	L										
P. M. 11:55	Readings	After	6992	8070	$\frac{R+L}{2}-2500=5032$									
			6992	8074										
	Av. Reading		6992	8072										

Correction = 0031

Notes and Sketches: Test № 12 60 ft. head of water

Numbers in parenthesis show order in which readings were taken

FIG. 35.—COPY OF PORTION OF ORIGINAL CLINOMETER DATA FOR TEST 12.

Records of Tests made in co-operation with U. S. Bureau of Standards — FORM 1

ENGINEERING FOUNDATION
COMMITTEE ON
ARCH DAM INVESTIGATION

STEVENSON CREEK DAM
ORIGINAL DATA OF TEST
Level Bar Test № 12

Sheet — Date 9-21-26
Series 1-2&3 — Inst. No.
Observer Lyse
Recorder DeYoe

Time	Gage Line / Group No.	61 W	61 E	60 W	60 E	21 W	21 E	20 W	20 E	32 W	32 E	31 W	31 E
~~A. M.~~ 9:55 P. M.	Readings	0480	0532	-0204	1260	1404	-0368	0700	0328	-0025	1045	2569	-1561
		0488	0540	-0209	1264	1402	-0370	0702	0324	-0026	1050	2570	-1562
	Av. Reading	0484	0536	-0207	1262	1403	-0369	0701	0326	-0026	1048	2570	-1562

Time	Gage Line / Group No.	30 W	30 E	52 W	52 E	51 W	51 E	50 W	50 E	41 W	41 E	40 W	40 E
~~A. M.~~ P. M. 11:05	Readings	0285	0732	2019	-1011	2390	-1360	1056	-0038	1764	-0764	0914	0078
		0289	0729	2015	-1014	2390	-1361	1056	-0030	1766	-0759	0922	0079
	Av. Reading	0287	0730	2017	-1012	2390	-1360	1056	-0034	1765	-0762	0918	0079

A. M. Gage Line | Group No. Std.
P. M. Readings
Av. Reading

SERIES NO. 2
9-22-26

Time	Gage Line / Group No.	61 W	61 E	60 W	60 E	21 W	21 E	20 W	20 E	32 W	32 E	31 W	31 E
A. M. 12:35 ~~P. M.~~	Readings	0436	0566	-0230	1278	1295	-0300	0699	0316	-0115	1143	2548	-1508
		0442	0571	-0239	1270	1300	-0306	0700	0326	-0109	1135	2552	-1508
	Av. Reading	0439	0569	-0235	1274	1298	-0303	0700	0321	-0112	1139	2550	-1508

Time	Gage Line / Group No.	30 W	30 E	52 W	52 E	51 W	51 E	50 W	50 E	41 W	41 E	40 W	40 E
A. M. 1:45 ~~P. M.~~	Readings	0282	0745	1915	-0931	2356	-1340	1058	-0052	1735	-0708	0934	0085
		0281	0748	1916	-0928	2358	-1341	1060	-0056	1726	-0706	0935	0081
	Av. Reading	0282	0747	1915	-0930	2357	-1341	1059	-0054	1730	-0707	0935	0083

A. M. Gage Line | Group No. Std.
P. M. Readings
Av. Reading

SERIES NO. 3

Time	Gage Line / Group No.	61 W	61 E	60 W	60 E	21 W	21 E	20 W	20 E	32 W	32 E	31 W	31 E
A. M. 2:05 ~~P. M.~~	Readings	0165	0869	-0276	1310	1134	-0091	0700	0340	-0400	1448	2406	-1368
		0168	0868	-0272	1316	1135	-0090	0698	0341	-0400	1444	2405	-1368
	Av. Reading	0167	0869	-0274	1313	1135	-0090	0699	0341	-0400	1446	2405	-1368

Time	Gage Line / Group No.	30 W	30 E	52 W	52 E	50		50 W	50 E	41 W	41 E	40 W	40 E
A. M. 3:10 ~~P. M.~~	Readings	0221	0809	1636	-0610	2240	-1234	1061	-0059	1598	-0580	0872	0110
		0222	0811	1640	-0612	2240	-1236	1068	-0058	1600	-0580	0878	0112
	Av. Reading	0222	0810	1638	-0611	2240	-1235	1065	-0059	1599	-0580	0875	0111

Notes and Sketches:

Test № 12

Series № 1 - 60 ft. head of water

Series № 2 - 50 ft. head of water

Series № 3 - 0 head of water

FIG. 36.—COPY OF ORIGINAL LEVEL-BAR DATA FOR TEST 12.

Records of Tests made in co-operation with U. S. Bureau of Standards — FORM 2

ENGINEERING FOUNDATION
COMMITTEE ON
ARCH DAM INVESTIGATION

TEST OF
STEVENSON CREEK DAM
Strain Gage

Instrument No. AE Comp. Std. — Sheet No. 5

Test № 12

Test № / Series / Date / Time / Head of Water / Observer / Recorder	Station	Std. Bar.	51 A	51 B	51 C	51 D	71 A	71 B	71 C	71 D	72 A	72 B	72 C	72 D	Std. Bar.
12 / 1 / 9-21-26 / 9:45 P.M. / 60 ft / Stout / Barry	Corrected Zero Reading	16499													
	Av. Reading	16285	17473	16404	18361	17970	17402	16282	17211	18546	17156	16664	18080	18070	
	Correction	+00214	+214	+214	+214	+214	+214	+214	+214	+214	+214	+214	+214	+214	
	Corrected Reading	16499	17687	16618	18575	18184	17616	16496	17425	18760	17370	16878	18294	18284	
	Corrected Difference		0	0	0	0	0	0	0	0	0	0	0	0	
12 / 2 / 9-22-26 / 1:50 A.M. / 0 / Stout / Barry	Av. Reading	16298	17558	16445	18349	17970	17466	16365	17261	18524	17203	16579	18062	18220	
	Correction	+00201	+201	+201	+201	+201	+201	+201	+201	+201	+201	+201	+201	+201	
	Corrected Reading	16499	17759	16646	18550	18171	17667	16566	17462	18725	17404	16780	18263	18421	
	Corrected Difference		+00072	+00028	-00025	-00013	+00051	+00070	+00037	-00035	+00034	-00098	-00031	+00137	
	Strain - Millionths		+72	+28	-25	-13	+51	+70	+37	-35	+34	-98	-31	+137	
	Av. Reading														
	Correction														
	Corrected Reading														
	Corrected Difference														

+ = Tension
− = Compression

FIG. 37.—COPY OF PORTION OF REDUCED STRAIN-GAUGE DATA FOR TEST 12.

Records of Tests made in co-operation with U. S. Bureau of Standards — Form 2

ENGINEERING FOUNDATION — COMMITTEE ON ARCH DAM INVESTIGATION | **TEST OF STEVENSON CREEK DAM** — Clinometer Line No. 4 ℄ | Instrument No. | Sheet No. | Test No. 12

Test No. / Series / Date / Time / Head of Water / Observer / Recorder	Station	Std. Bar.	405	410	415	420	425	430	435	440	445	450	455	460
12 / 3 / 9-22-26 / 2:00-3:35 A.M. / 0 / Marchand / Fenwick	Corrected Zero Reading													
	Av. Reading	#1 -0035	4866	5905	5840	5951	5898	5530	6029	5968	7049	5827	5505	5180
	Correction		-0033	-0033	-0033	-0033	-0033	-0033	-0033	-0033	-0033	-0033	-0033	-0033
	Corrected Reading	#62 -0032	4833	5872	5807	5918	5865	5497	5996	5935	7016	5794	5472	6147
	Corrected Difference													
12 / 2 / 9-22-26 / 12:25-1:30 A.M. / 50 ft / Marchand / Fenwick	Av. Reading	#1 -0031	5247	6312	6347	6486	6352	5847	6153	5940	6978	5759	5421	5070
	Correction	#38 -0031	-0031	-0031	-0031	-0031	-0031	-0031	-0031	-0031	-0031	-0031	-0031	-0031
	Corrected Reading		5216	6281	6316	6455	6321	5816	6122	5909	6947	5728	5390	5039
	Corrected Difference = Deflection		-0383	-0409	-0509	-0537	-0456	-0319	-0126	+0026	+0069	+0066	+0082	+0108
	Σ Deflections		-0383	-0792	-1301	-1838	-2294	-2613	-2739	-2713	-2644	-2578	-2496	-2388
12 / 1 / 9-21-26 / 9:50-11:55 P.M. / 60 ft / Marchand / Fenwick	Av. Reading	#1 -0030	5325	6438	6489	6660	6556	6037	6333	6059	7018	5772	5450	5105
	Correction	#38 -0031	-0031	-0031	-0031	-0031	-0031	-0031	-0031	-0031	-0031	-0031	-0031	-0031
	Corrected Reading	#63 -0032	5294	6407	6458	6629	6525	6006	6302	6028	6987	5741	5419	5074
	Corrected Difference = Deflection		-0461	-0535	-0651	-0711	-0660	-0509	-0306	-0093	+0029	+0053	+0053	+0073
	Σ Deflections		-0461	-0996	-1647	-2358	-3018	-3527	-3833	-3926	-3897	-3844	-3791	-3718

FIG. 38.—COPY OF PORTION OF REDUCED CLINOMETER DATA FOR TEST 12.

Records of Tests made in co-operation with U. S. Bureau of Standards — Form 3

ENGINEERING FOUNDATION — Committee on Arch Dam Investigation — **STEVENSON CREEK DAM**

GENERAL DATA SHEET
Subject Level Bar
Test No. 12
Observer-Lyse Recorder-DeVoe
Date 9-21-26 Sheet 1 of 2 Sheets

Head of water feet	Series	Station 20 W	20 E	21 W	21 E	30 W	30 E	31 W	31 E	32 W	32 E
0	3	+0699	+0341	+1135	-0090	+0222	+0810	+2405	-1368	-0400	+1446
50	2	+0700	+0321	+1298	-0303	+0282	+0747	+2550	-1508	-0112	+1139
	Diff.	-1	+20	-163	+213	-60	+63	-145	+140	-288	+307
	Mean	-10 =	-5"	-188 =	-94"	-62 =	-31"	-142 =	71"	-298 =	149"
Change in 60 in.		-0015		-0274		-0090		-0207		-0434	
60	1	+0701	+0326	+1403	-0369	+0287	+0730	+2570	-1562	-0025	+1048
	Diff.	-2	+15	-268	+279	-65	+80	-165	+194	-375	+398
	Mean	-8 =	-4"	-274 =	-137"	-72 =	-36"	-180 =	-90"	-386 =	-193"
Change in 60 in.		-0012		-0398		-0105		-0262		-0561	
		40 W	40 E	41 W	41 E	50 W	50 E	51 W	51 E	52 W	52 E
0	3	+0875	+0111	+1599	-0580	+1065	-0059	+2240	-1235	+1638	-0611
50	2	+0935	+0083	+1730	-0707	+1059	-0054	+2357	-1341	+1915	-0930
	Diff.	-60	+28	-131	+127	+6	-5	-117	+106	-277	+319
	Mean	-44 =	-22"	-129 =	64".6	+6 =	+3"	-112 =	-56"	-298 =	-149"
Change in 60 in.		-0064		-0188		+0009		-0163		-0434	
60	1	+0918	+0079	+1765	-0762	+1056	-0034	+2390	-1360	+2017	-1012
	Diff.	-43	+32	-166	+182	+9	-25	-150	+125	-379	+400
	Mean	-38 =	-19"	-174 =	-87"	+17 =	+8".5	-138 =	69"	-390	-195"
Change in 60 in.		-0055		-0253		+0025		-0201		-0567	

FIG. 39.—COPY OF PORTION OF REDUCED LEVEL-BAR DATA FOR TEST 12.

For the radius meter, a method of reduction similar to that of the strain gauges was used. The change in mid-ordinate for each station is proportional to the strain, and to obtain deflections from them a double instead of a single integration (cumulative addition) would be necessary.

Telemeter data were recorded and reduced in a different manner. Since temperature corrections for parts of the instrument, the resistance of the stack, and the resistance of the temperature coils are factors in the reduction of the data, a different kind of form was necessary. Fig. 40 gives a sample of the original and reduced data for Test 12.

Records of Tests made in co-operation with U.S. Bureau of Standards — Form 4

ENGINEERING FOUNDATION
COMMITTEE ON ARCH DAM INVESTIGATION

TEST OF
STEVENSON CREEK DAM
TELEMETERS & RESISTANCE THERMOMETERS

WHEATSTONE BRIDGE NO. 117061
OBSERVER Carlson
RECORDER Barworth
DATE 9-21-26
SHEET 1

Test 12
Series 1
Time 9:55 to 10:45 PM.
Head of Water 60

Telemeter	1D	1U	2D	2U	3D	3U	4D	4U	5D	5U	6DL	6DV	6CL	6CD1	6CD2	6UL	6UV	7DL
Stack, Ohms	32.7		29.9		50.2		22.3		47.6			31.9	45.4	57.5	48.4	35.4	46.8	41.5
Coil, Ohms	501		500		503		495		502			503	503	498.5	500.5	498		504
Indicated Strain	300		140		782		242		458			297	746	890	730	467	258	686
Temp. Correction	-96		-88		-80		-136		-96			-88	-80	-92	-92	-104	-104	-64
Corrected Strain	204		52		702		106		362			209	666	798	638	363	154	622
Temperature	14.0		14.5		15.0		11.5		14.0			14.5	15.0	14.25	14.25	13.5		16.0

Telemeter	7DV	7CD1	7CD2	7UL	7UV	8DL	8DV	8CD1	8CD2	8UL	8UV	8UD	9DL	9DV	9CL	9CV	9CD1	9CD2
Stack, Ohms	45.3		44.1	42.9	49.6	45.1	45.0	47.4	48.4	45.1	50.3	47.9		36.6	40.3	56.2	50.8	50.8
Coil, Ohms	501			498	498	502.5	500	500		498		497		502	500		500	
Indicated Strain	738		627	630	702	1125	1020	970	812	652	558	482		402	530	874	670	655
Temp. Correction	-64		-64	-96	-104	-84	-88	-96	-96	-104	-104	-112		-88	-88	-88	-88	-88
Corrected Strain	674		563	534	598	1041	932	874	716	548	454	370		314	442	786	582	567
Temperature	16.0			14.0	13.5	14.75	14.5	14.0		13.5		13.0		14.5	14.5		14.5	

Telemeter	9UL	9UV	10DL	10DV	10CD1	10CD2	10UL	10UV	10UD	11DL	11DV	11CD1	11CD2	11UL	11UV	11UD	12DL	12DV
Stack, Ohms	41.7	37.4	41.6	49.8	46.9	44.7	43.8	77.5	50.0	43.2	40.9	42.4	50.5	46.9	49.5	45.8	40.8	39.4
Coil, Ohms	495	497	502		500	499.5	501	498	501.5	499	498		498		501	500	502.5	503
Indicated Strain	625	425	628	748	710	582	738	240	832	615	407	537	602	690	620	535	570	520
Temp. Correction	-104	-104	-72	-72	-96	-92	-96	-104	-100	-96	-96	-96	-96	-96	-96	-96	-60	-64
Corrected Strain	521	321	556	676	614	490	642	136	732	519	311	441	506	594	524	439	510	456
Temperature		13.5	15.5		14.0	14.25	14.0	13.5	13.75	14.0	14.0		14.0		14.0	14.0	16.25	16.0

Telemeter	12CL	12CV	12CD1	12CD2	12UL	12UV	13DL	13DV	13UL	13UV	13UD	14DL	14DV	14UL	14UV	14UD	15DL	15DV
Stack, Ohms	45.8	39.3	48.4	53.0	45.2	39.4	33.5		45.8	54.4	56.1	36.7	35.4	48.9	46.9	48.3	43.4	31.2
Coil, Ohms	502			501	499					502			501.5		501	502	497.5	
Indicated Strain	808	580	708	725	712	500	595		793	842	810	650	472	967	780	690	665	212
Temp. Correction	-96	-96	-96	-96	-120	-120	-96		-96	-96	-96	-100	-100	-96	-96	-88	-100	-100
Corrected Strain	712	484	612	629	592	380	499		697	746	714	550	372	871	684	602	555	112
Temperature	14.0			14.0	12.5					14.0			13.75		14.0	14.5	13.75	

FIG. 40.—COPY OF PORTION OF ORIGINAL AND REDUCED TELEMETER DATA FOR TEST 12.

For each telemeter cartridge a calibration curve (Fig. 20) was prepared before the instruments were embedded in the concrete. On the form (Fig. 40) there is a column for each telemeter row, for each series of observations. The resistance of the carbon stack is recorded to the nearest 0.1 ohm as "stack, ohms." The resistance of the temperature coil is recorded to the nearest 0.5 ohm in the line marked "coil, ohms," and is used to determine the temperature by means of the constant furnished with the calibration curves for the telemeters.

The indicated strain is the value of the strain, which in the calibration curve corresponds to the resistance "stack, ohms." The temperature correction is that portion of the change of length recorded by the stack, which is due to change of temperature of the carbon and the mountings. This correction has been found to be proportional to the temperature change and the same constant was applied to all the telemeters. The value found to be most consistent for all the data was 0.000016 per degree centigrade in the 6-in. cartridges used.

The corrected strain is found by adding the temperature correction (a strain) to the indicated strain with the proper sign. The temperature corrections were referred to 20° cent., as a zero correction. The "corrected strain" shown on Fig. 40 represents an arbitrary value for the study of strain and is without meaning until compared with similar "corrected strain" for the same telemeter under other conditions; that is, the net strain is found by subtracting the "corrected strain" for one series of readings from the corrected strain for the series with which it is to be compared.

31.—Filing of Data.—The original and the reduced data have been filed with Engineering Foundation and will be available for reference to those who wish to examine them for purposes of reference. These data, whether in the form of tabulations or of graphs, are bound in suitable form and are indexed according to the numbers given in Appendix A.

G.—Behavior of Dam During Construction and Curing Period

32.—Temperature During Hardening and Curing of Concrete.—On April 18, 1926, before any concrete had been placed in the dam, readings were taken on Telemeters 3 *U* and 3 *D* just before they were grouted into the foundation rock. The next readings were taken just before concreting began on April 19. From then on frequent readings were taken. All the temperatures for Telemeters 3 *U* and 3 *D* are shown in Fig. 41. The difference of about 4° cent. between the first two readings in Fig. 41 probably represents partly the daily variation in temperature and partly the difference between air temperature (before grouting) and the temperature of the rock and grout.

After the concrete had been placed, the temperature increased rapidly for about two days and then decreased slowly. Air temperatures for the same time (Fig. 42) were taken from daily temperature graphs recorded by an autographic thermometer stationed at the dam site. Previous to the completion of the dam, the autographic thermometer was kept in a partly enclosed hoist house about 100 ft. down stream from the dam and approximately at its top elevation. On June 27, after the removal of the forms, the thermometer was moved to a position about 5 ft. down stream from the dam at an elevation of about 35 ft.

The two upper graphs of Fig. 42 give the maximum and the minimum daily temperatures. The difference was usually about 15° during the construction and the curing period and decreased to about 10° toward the close of the testing period. The lower curve shows the air temperatures for the same times of day that the telemeter observations (Figs. 41, 43 to 46) were taken.

The temperature rise at Station 6, on the center line of the dam at an elevation of 4 ft. is shown in Fig. 43. The thickness of the dam at this station is about 6 ft. 2 in. The temperature rose to a maximum of 47° cent. in the center of the dam in about 2 days. From then on it gradually decreased, becoming constant at about 17° after 3 weeks. No other station had so great a rise as this, although others were nearly as great. Near the surface the temperature rose to a maximum of about 35° cent. within 1 day and from then on decreased at about the same rate as at the center, reaching a steady temperature of about 11° after 2 weeks.

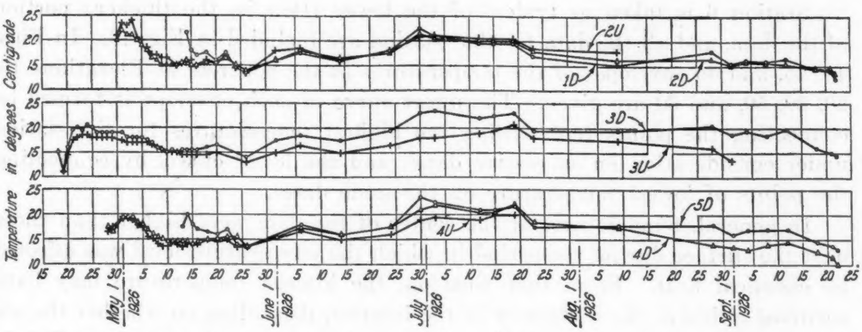


FIG. 41.—TEMPERATURE OF FOUNDATION ROCK AT TELEMETER STATIONS 1, 2, 3, AND 5 DURING SETTING AND CURING OF CONCRETE.

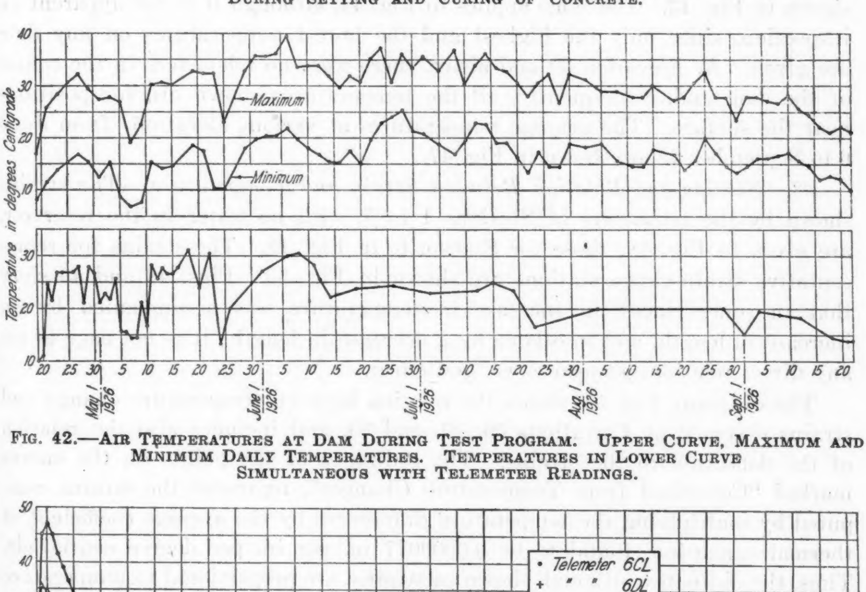


FIG. 42.—AIR TEMPERATURES AT DAM DURING TEST PROGRAM. UPPER CURVE, MAXIMUM AND MINIMUM DAILY TEMPERATURES. TEMPERATURES IN LOWER CURVE SIMULTANEOUS WITH TELEMETER READINGS.

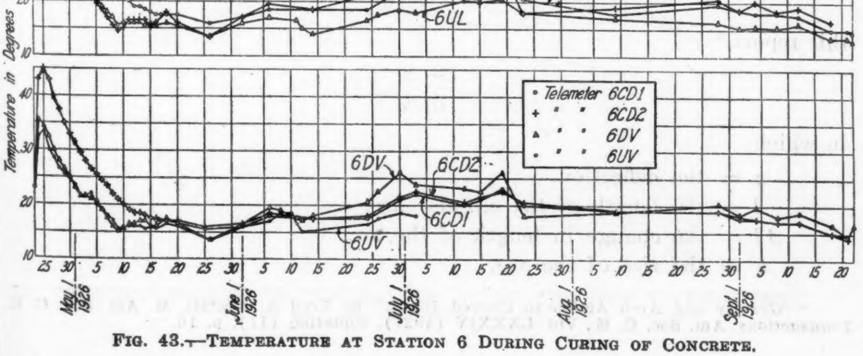


FIG. 43.—TEMPERATURE AT STATION 6 DURING CURING OF CONCRETE.

Station 6 is taken as typical of the lower (that is, the thicker) portion of the dam, and all the data for that station are included in Fig. 43. In Figs. 44, 45, and 46 envelopes of the temperatures in the concrete at Elevations 10, 20, 40, 50, and 60 are given. The upper curve of each envelope is formed by connecting the points representing the highest temperatures for the station under consideration on successive dates, and the lower curve, by connecting the points of lowest temperature on the same dates.

In general, temperatures at the center of the dam were higher than those near the surface during the period in which the temperature level was affected by chemical heat. From that time on, the highest temperature may have occurred either at the surface or in the interior, depending on whether the air temperature at the time was higher or lower than that in the interior. This is shown in Fig. 43. The same applies to Fig. 44, although it is not apparent on inspection, since only the highest and the lowest temperatures on any date are given. At Elevation 30 and above, there were no telemeters in the center of the dam and, consequently, all the temperatures shown are for positions near the surface. The average temperatures at various elevations from June 6 to September 22 are shown in Fig. 47.

33.—Strains and Relation Between Strain and Temperature.—The strains shown by the telemeters of Stations 1 to 5, with no water in the reservoir, are given in Fig. 48; those for Station 6, in Fig. 49. The strains for representative strain-gauge stations are shown in Fig. 50. Figs. 48 and 49 show that in some places an increase in temperature was accompanied by an increase in length, and in others, by a decrease in length; it is not easy to see any direct relation between cause and effect.

The diagram, Fig. 51, shows the relation between temperature change and strains observed at Elevations 30, 40, and 50, and includes also the relation of the deflection to the strains. The ordinates of the points on the curves marked "Computed from Temperature Changes", represent the strains computed by multiplying the temperature differences by the average coefficient of thermal expansion, found to be 0.0000077 in. per in. per degree centigrade. Thus, the ordinates, although shown as strains, are proportional to temperature changes. The strain, ϵ, computed from the deflections was obtained by solving for the average strain $\frac{\Delta l}{l}$, the approximate formula. Reduced to the notation of this report,*

$$z = \frac{3}{16} \frac{l^2}{h} \Delta l$$

in which,

z = the deflection.
l = the length of the circular arc.
Δl = the change in length of the arc.
h = the rise of the arc.

* "Gravity and Arch Action in Curved Dams," by Fred A. Noetzli, M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. LXXXIV (1921), Equation (11), p. 10.

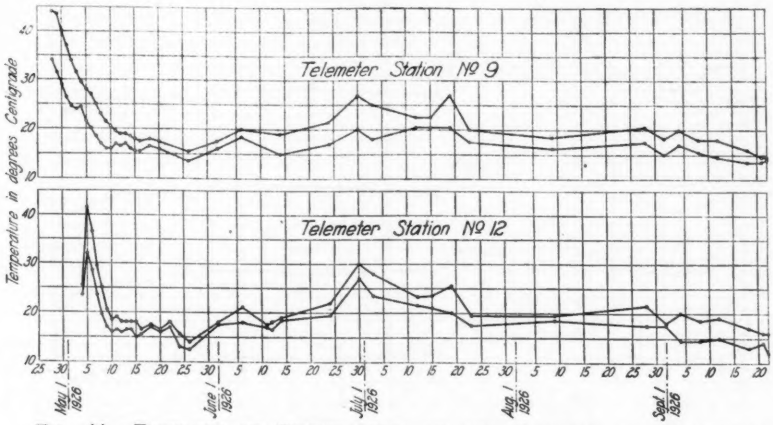


FIG. 44.—ENVELOPE OF TEMPERATURES IN THE CONCRETE AT ELEVATIONS 10 AND 20 (STATIONS 9 AND 12).

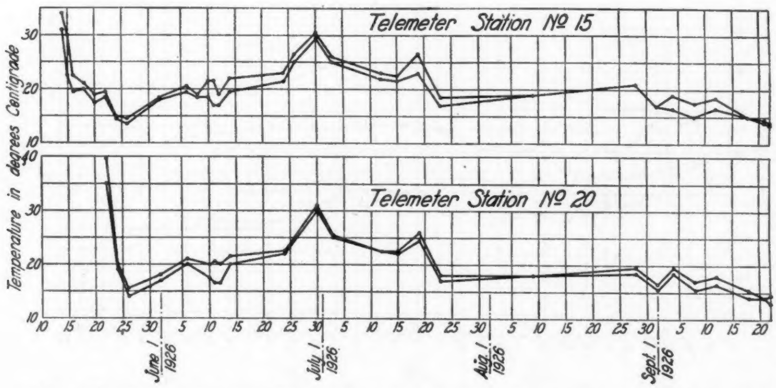


FIG. 45.—ENVELOPE OF TEMPERATURES IN THE CONCRETE AT ELEVATIONS 30 AND 40.

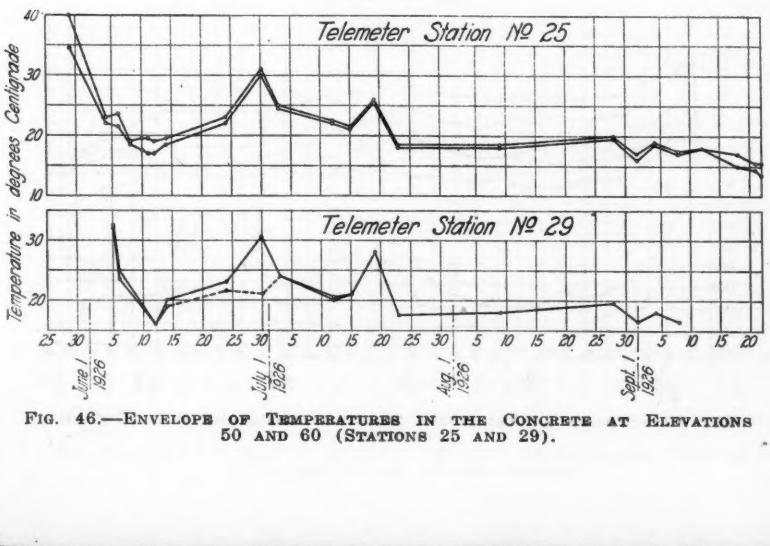


FIG. 46.—ENVELOPE OF TEMPERATURES IN THE CONCRETE AT ELEVATIONS 50 AND 60 (STATIONS 25 AND 29).

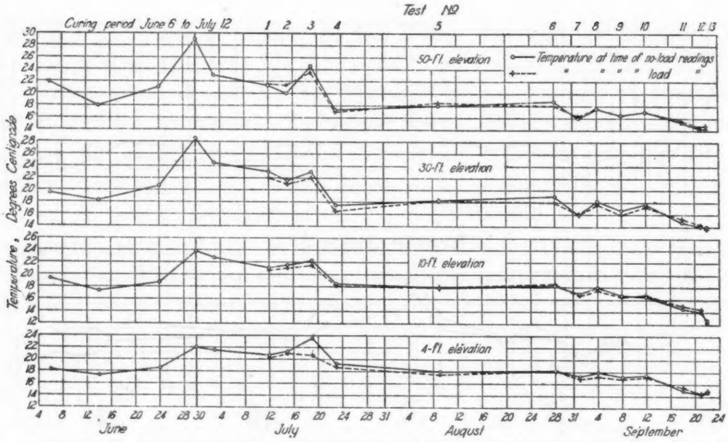


FIG. 47.—AVERAGE INTERNAL TEMPERATURES AT VARIOUS ELEVATIONS FROM JUNE 6 TO SEPTEMBER 22.

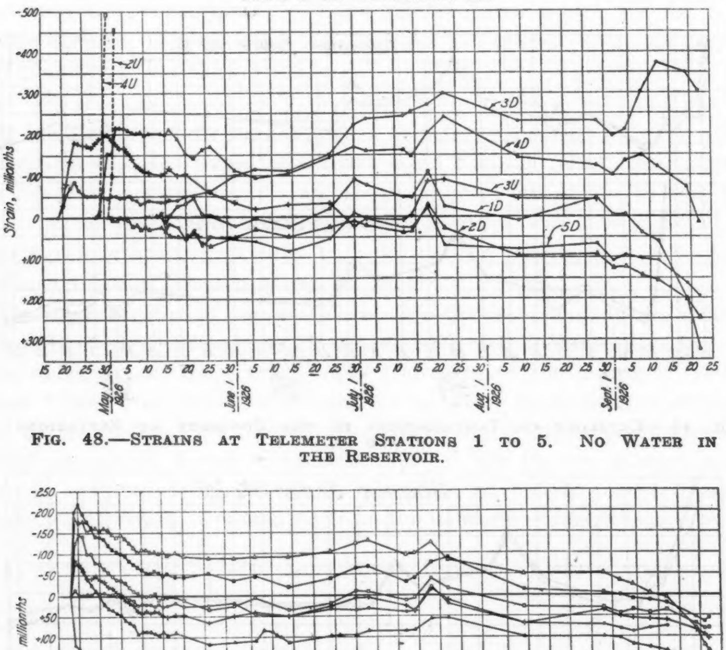


FIG. 48.—STRAINS AT TELEMETER STATIONS 1 TO 5. NO WATER IN THE RESERVOIR.

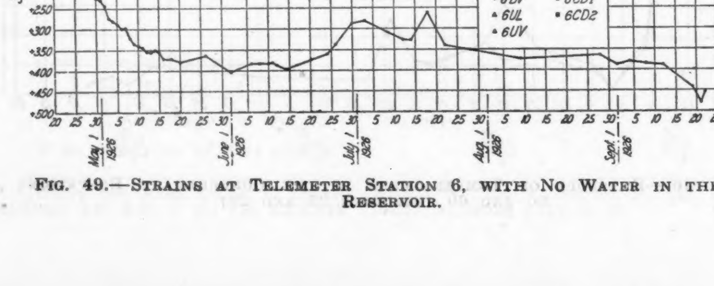


FIG. 49.—STRAINS AT TELEMETER STATION 6, WITH NO WATER IN THE RESERVOIR.

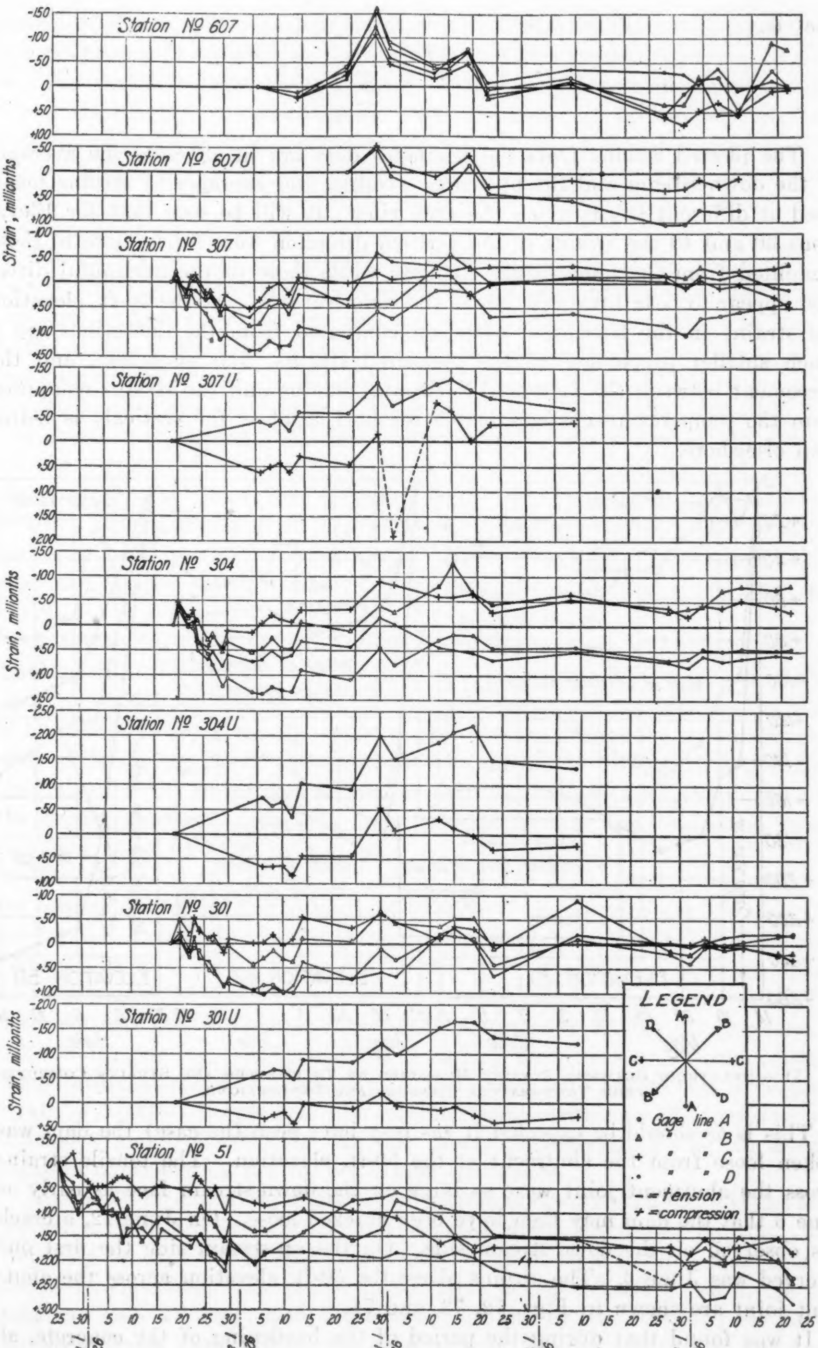


FIG. 50.—NO-LOAD STRAINS AT REPRESENTATIVE STRAIN-GAUGE STATIONS TO THE END OF THE TESTING PROGRAM.

that is,

$$\varepsilon = \frac{\Delta l}{l} = \frac{z}{\frac{3}{16}\frac{l^2}{h}}$$

The plotted strains from the telemeter data are in all cases the averages of the down-stream and the up-stream strains, and averages of strains measured at different points along the arch ring. It will be seen that for Elevations 30 and 40 the strains in the vertical direction were fairly close to those computed from the temperature changes, while those in the horizontal direction appear to bear little relation to the temperatures. At the 50-ft. elevation the strains in the horizontal direction computed from the deflections are a much smaller proportion of the measured strains than elsewhere, and the agreement between the measured horizontal strains and the strains computed from the temperature changes (same for horizontal as for vertical) is better than elsewhere.

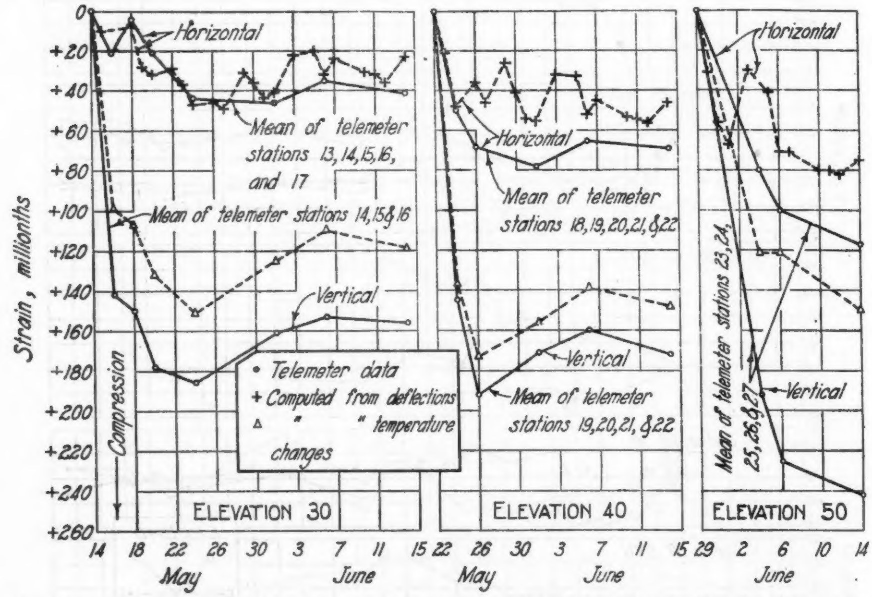


FIG. 51.—RELATIONS BETWEEN STRAINS MEASURED BY TELEMETERS AND STRAINS COMPUTED FROM TEMPERATURE CHANGES AND DEFLECTIONS.

This is as should be expected if (as may have been the case) the dam was broken loose from the abutment at the 50-ft. elevation. The tensile strains across the abutment joint were so large on the down-stream face as early as June 6 that the dam may then have been cracked loose. On June 12, a crack was observed on the down-stream side. On the up-stream side the first one observed was June 6. The strains above the 50-ft. elevation across the abutment joint are shown in Figs. 72, 73, and 74.

It was found that during the period of the hardening of the concrete, at a given point the vertical strain in the dam was greater, in general, than the

horizontal strain. It was thought at first that this might be in the nature of a continued flow added to the shrinkage. Later studies, however, showed that the vertical strains were related to the temperature changes and the lateral strains to the deflections in such a manner as to indicate that the restraint of the dam at the abutments against free movement was responsible for the difference between the vertical and the horizontal strains. The excess of the vertical over the horizontal strains was so general that there is no question as to its existence.

The curves in Figs. 52 and 53, prepared in a study of the calibration of the telemeters, give information also on the behavior of the dam under temperature changes when the concrete was from 17 to 36 days old. The strains in the vertical direction, as measured by the strain-gauge, are shown in the diagrams by the curves marked 504 *A*, 504 *U A*, and 508 *U A*, and those in the horizontal direction at the same stations by 504 *C*, 504 *U C*, 508 *C*, and 508 *U C*. The rates of expansion with temperature change (the slopes of the curves) for the vertical are nearly the same as for the horizontal direction, and in this respect conditions are different from those noted for the hardening period. The dam was free to expand or contract vertically, and since the horizontal was nearly the same as the vertical expansion (or contraction) it appears that the direct stress accompanying the expansion must have been slight.

A further examination (including thirty-five stations distributed over the dam) of the effect of the temperature rise during the drying period, showed that the difference between the vertical and horizontal strains was too small to be detected except for the stations near the abutments. For the abutment stations below the 20-ft. elevation, the strains in the vertical direction averaged about 0.000040 in. per in. greater than in the horizontal direction. For the higher elevations, the difference between vertical and horizontal strains was only about 0.000010 in. per in.

The slopes of the curves (Figs. 52 and 53) correspond quite well with the average coefficient of expansion (0.0000077 in. per in. per degree centigrade) determined in the laboratory tests at the University of California. Between Tests 2 and 3 there was at the 60-ft. elevation an average change of temperature of 4.97° cent. and an average increase in strain (including both vertical and horizontal directions) determined by the strain-gauge, of 0.00003 in. per in. Between Tests 3 and 4, there was an average drop in temperature at the 60-ft. elevation of 7.77° cent. and an average decrease in strain of 0.000071 in. per in. The corresponding coefficients of expansion were 0.0000060 and 0.0000092 for the two cases, respectively, giving an average value of 0.0000076 in. per in. per degree centigrade.

At the 60-ft. elevation, there are only three telemeter stations. At the 50-ft. elevation there are five stations and the coefficient determined for that elevation may be expected to show less variation from time to time than that for the 60-ft. elevation. For the 50-ft. elevation between Tests 2 and 3, the temperature rose 2° cent., and the average expansion was 0.000018 in. per in. Between Tests 3 and 4 there was a decrease in temperature of 6.5° and a contraction of 0.000048 in. per in. and the corresponding coefficients of expansion

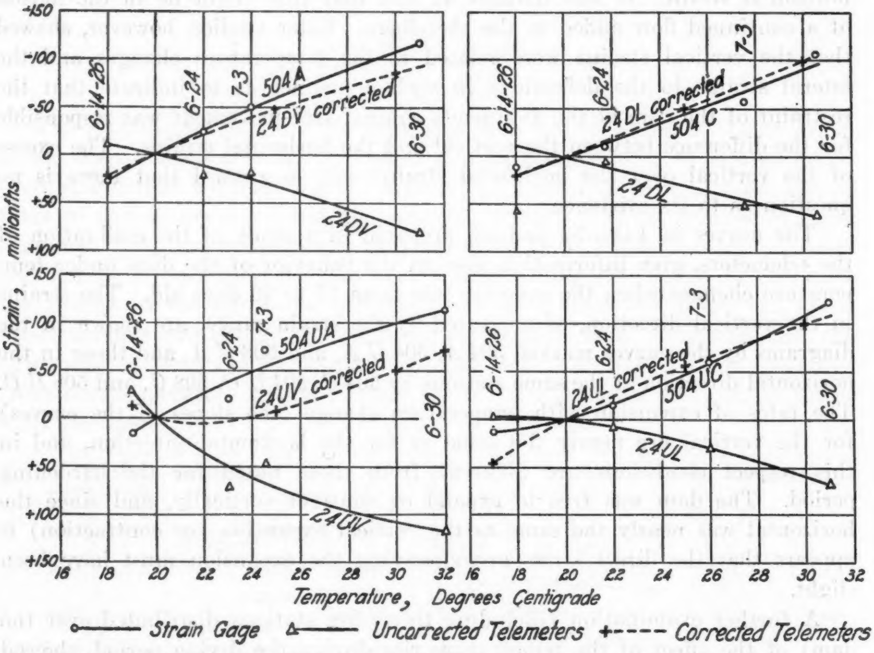


FIG. 52.—RELATION OF STRAIN TO TEMPERATURE DETERMINED BY STRAIN-GAUGE AND TELEMETER FOR STATION AT 50-FOOT ELEVATION, 24 FEET NORTH OF CENTER LINE.

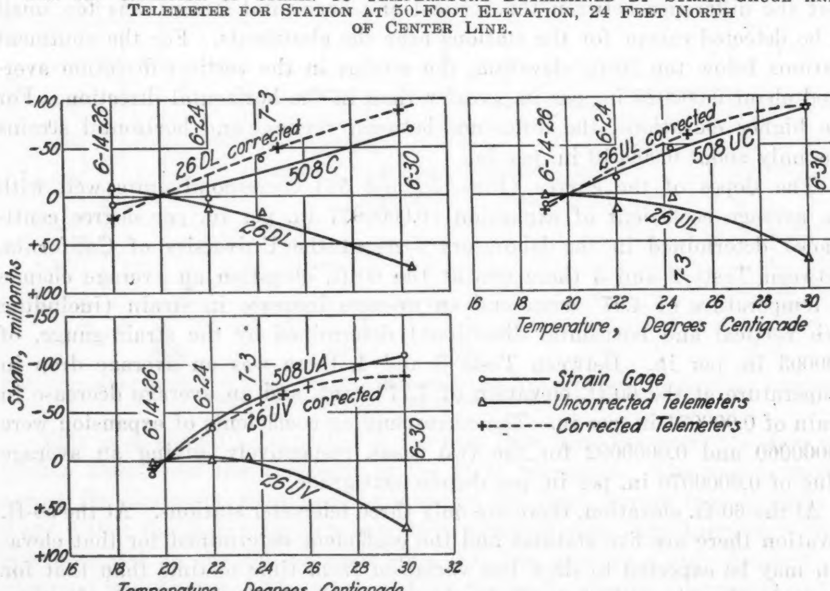


FIG. 53.—RELATION OF STRAIN TO TEMPERATURE DETERMINED BY STRAIN-GAUGE AND TELEMETER FOR STATION AT 50-FOOT ELEVATION, 24 FEET SOUTH OF CENTER LINE.

are 0.0000090 and 0.0000074, respectively, with an average of 0.0000082 in. per in. per degree centigrade. The average coefficient for the 50 and the 60-ft. elevations was 0.0000079 in. per in. per degree centigrade.

Additional information on the coefficient of expansion is furnished by tests made at the University of California on three 6 by 12-in. control specimens from the dam, and on several 3 by 3 by 40-in. specimens made from concrete of the same mix, and using some of the same aggregate as for the dam. The cylinders had telemeters embedded in them and strains were measured both with the telemeters and with dial extensometers attached externally. As the telemeters themselves were affected by the change in temperature, the values obtained by the extensometer were used in determining the thermal coefficient of expansion. The specimens were tested both dry and saturated, and the average coefficients found were 0.0000080 and 0.0000074, respectively, with an average of 0.0000077 in. per in. per degree centigrade. The 3 by 3 by 40-in. specimens at an age of 7 weeks gave an average coefficient of 0.0000074 in. per in. per degree centigrade, with the specimens successively dry and water-soaked. The agreement of the values quoted is reasonably good, and the average coefficient was about 0.0000077 in. per in. per degree centigrade. Additional discussion of the relation between temperatures and strains is found in Section 37, "Effect of Drying."

34.—Deflections and Relation Between Deflections and Temperature.— Deflections which occurred during construction, as nearly as they may be determined from the data, are shown in Fig. 12 and are discussed in Section 11. After the completion of construction it was possible to determine more exactly what the movements were. On the morning of June 5, 1926, one day after completion of the pouring of concrete, a complete set of deflection readings was taken. These were used as the datum from which to plot the isometric diagrams of deflections for several subsequent dates with the dam under no water pressure. (See Figs. 54, 55, and 57 to 62, inclusive.) Fig. 54 shows that the deflection between June 5 and June 14 was mainly at the top of the dam. Comparison of the temperatures for the various heights shown in Figs. 47 and 46 with the deflections in Fig. 54, shows that the deflections from June 5 to June 14, practically all of which were down stream, were approximately proportional to the fall in temperature for the various heights. From June 14 to June 24, the temperatures rose slightly, and Fig. 55 compared with Fig. 54 shows that there was a slight decrease in deflection in this period, but not as much as would be expected if it be assumed that the deflection was due entirely to temperature change.

On June 24, in order to study the effect of allowing the dam to dry for a period of time, the sprinklers used to keep the concrete saturated were shut off and no water was applied until July 2. Figs. 46 and 47 show that from June 24 to June 30 the temperature within the concrete rose about 8° in the upper part of the dam, and Fig. 56, in which changes of temperature and deflections at various heights are indicated graphically, shows that on June 30 the dam had almost everywhere deflected up stream by an amount which corresponded quite closely to the temperature rise. Evidently, the deflection was due mainly to the temperature rise, and since the air temperature, as shown

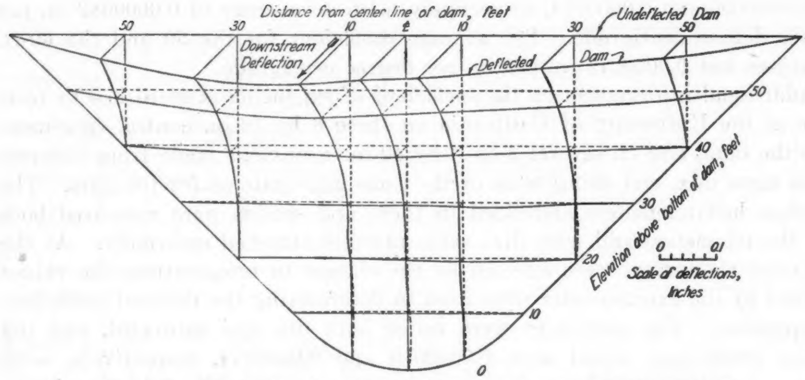


FIG. 54.—ISOMETRIC DIAGRAM OF DEFLECTIONS, JUNE 5 TO JUNE 14.

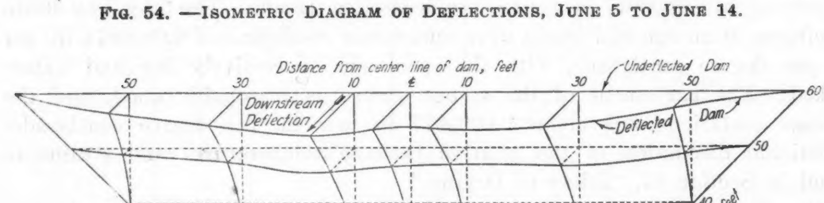


FIG. 55.—ISOMETRIC DIAGRAM OF DEFLECTIONS, JUNE 5 TO JUNE 24.

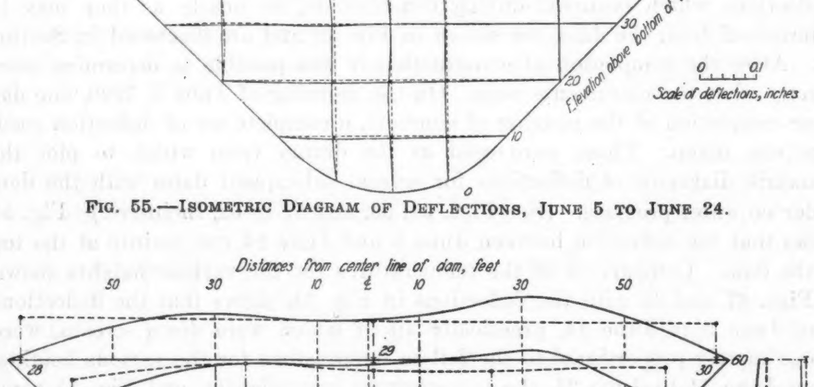


FIG. 56.—DIAGRAM OF DEFLECTIONS AND CHANGE OF TEMPERATURE, JUNE 24 TO JUNE 30.

by Fig. 42, was nearly constant during this time, the temperature rise in the concrete must have been caused by the drying out. Comparison of Fig. 57 with Fig. 55 also shows the change in deflection from June 24 to June 30.

On July 3, about 12 hours after the water had again been turned on the dam, the deflections of Fig. 58 were observed. Figs. 46 and 47 show that the temperature dropped rapidly at the top of the dam, but only slightly at the lower elevations. Likewise, Fig. 58 shows that the down-stream deflections were much greater at the top than at the lower elevations; the irregularity in the curves at the extreme right is probably due to an erroneous reading at Elevation 50, although there is no way of knowing positively that such an error was present.

The deflections in Fig. 59 were plotted from the zero readings for the first load test, that of July 12, compared with the readings of June 5. On July 12 deflections at the various elevations relative to those of June 24, were about as would be expected from the differences of temperature for these two dates.

Considerable variation in temperature occurred between Load Tests 2, 3, and 4; and the temperature differences have been used in computing the coefficients of expansion given in Section 33. Figs. 60, 61 and 62 show that the variation in deflection for this period is in reasonable agreement with the variation in the internal temperature of the dam. Data are available for showing similar no-load deflection curves corresponding to the time of each of the remaining load tests, but it is believed that those already given are sufficient to illustrate the behavior of the dam. Fig. 63 shows the crown deflection at Elevations 30 and 60, from the date of pouring to the end of the testing program.

In Fig. 64(*b*) and (*c*) are given comparisons between the difference of the crown deflections observed from June 24 to June 30 and from June 24 to July 3, with the deflections computed by the Cain formula* for the separate arch rings at various heights from the 10 to the 60-ft. elevations. The change in temperature was determined from the averages observed with the telemeters at each elevation. The coefficient of expansion used was 0.0000077 in. per in. per degree centigrade. The observed deflection from June 24 to June 30 was less than that computed for either hinged or fixed arch conditions at any elevation considered. That from June 24 to July 3 was between the deflections computed for hinged and fixed arches for most of the arch elements between the 30-ft. elevation and the top of the dam.

Coefficients are given in Fig. 64(*a*) by which the product of the temperature change and the coefficient of expansion must be multiplied in computing the crown deflection at any elevation for either given condition. This shows that a constant temperature change above the 30-ft. elevation should give nearly a constant deflection. Therefore, the deflection at any elevation should be nearly proportional to the temperature change at that elevation. It has been pointed out that this was approximately the fact with the test dam.

During the curing period, it was found that the deflections varied considerably throughout the day, probably because of the temperature variation. This

* "The Circular Arch Under Normal Loads," by William Cain, M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. LXXXV (1922), p. 233. For formula see p. 247.

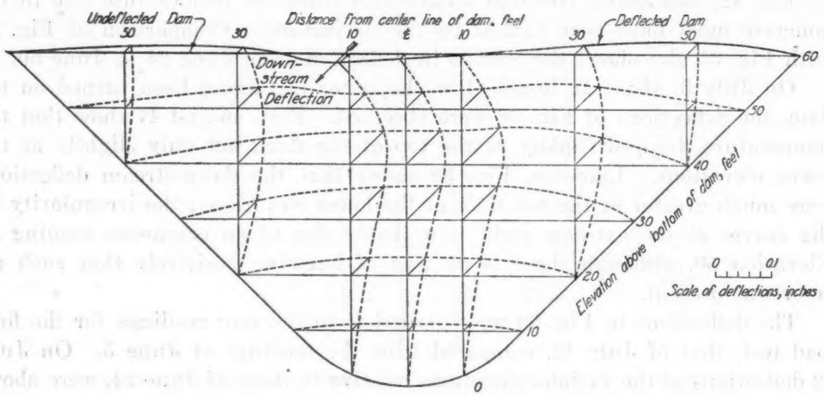


FIG. 57.—ISOMETRIC DIAGRAM OF DEFLECTIONS, JUNE 5 TO JUNE 30.

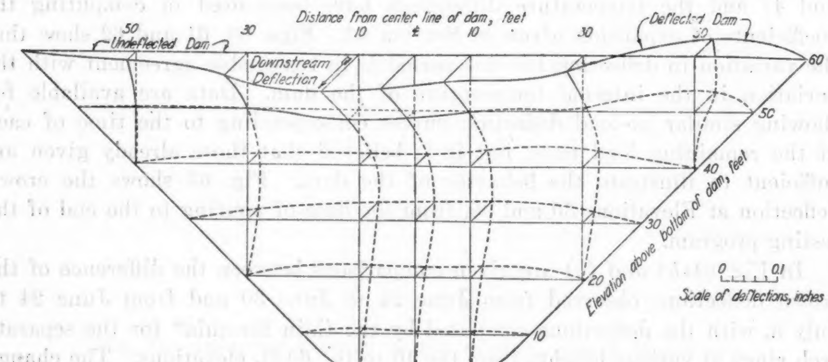


FIG. 58.—ISOMETRIC DIAGRAM OF DEFLECTIONS, JUNE 5 TO JULY 3.

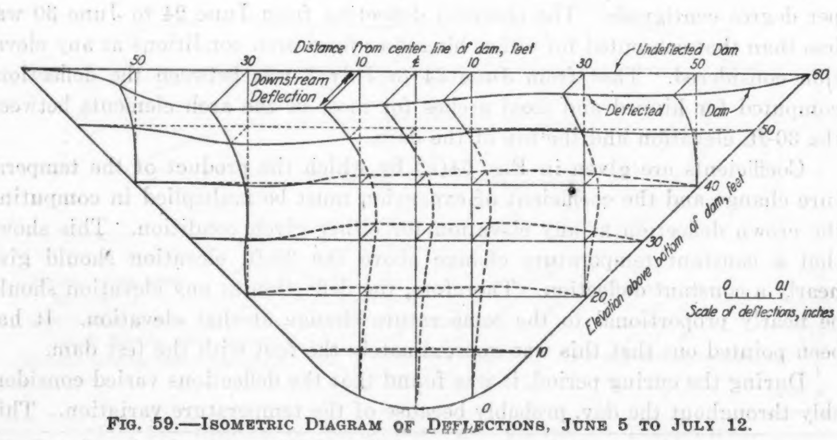


FIG. 59.—ISOMETRIC DIAGRAM OF DEFLECTIONS, JUNE 5 TO JULY 12.

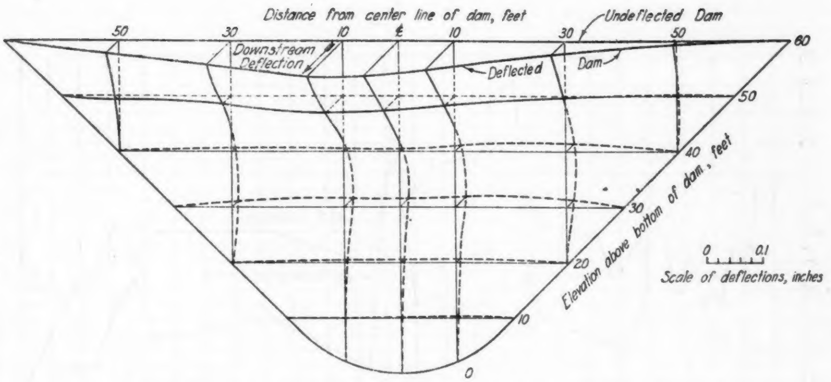


FIG. 60.—ISOMETRIC DIAGRAM OF DEFLECTIONS, JUNE 5 TO JULY 15.

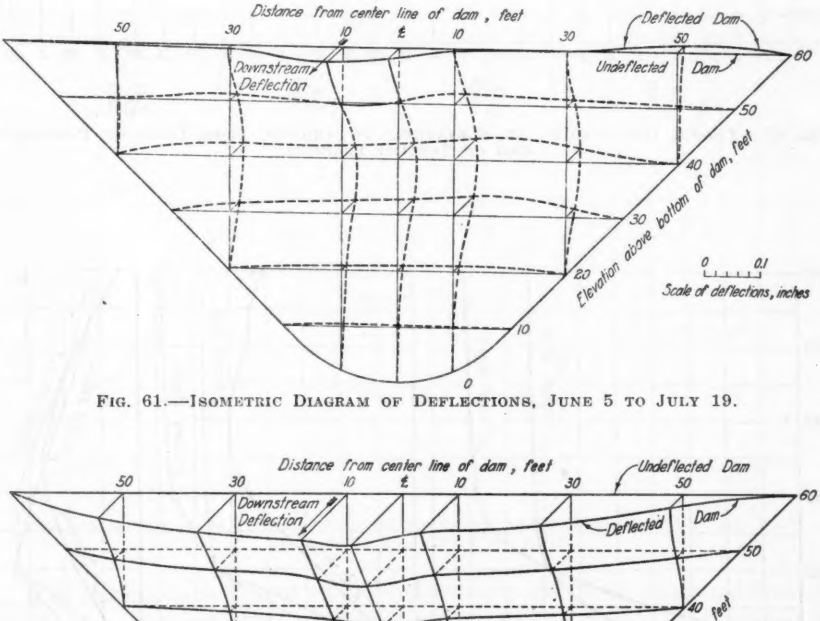


FIG. 61.—ISOMETRIC DIAGRAM OF DEFLECTIONS, JUNE 5 TO JULY 19.

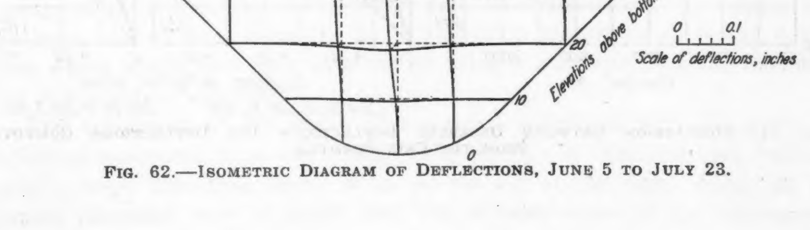


FIG. 62.—ISOMETRIC DIAGRAM OF DEFLECTIONS, JUNE 5 TO JULY 23.

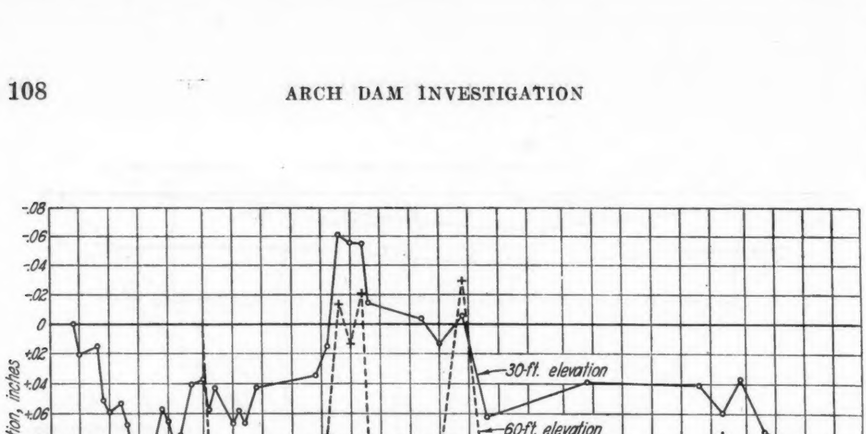


FIG. 63.—CROWN DEFLECTIONS AT ELEVATIONS 30 AND 60, FROM DATE OF POURING TO END OF TESTING PROGRAM.

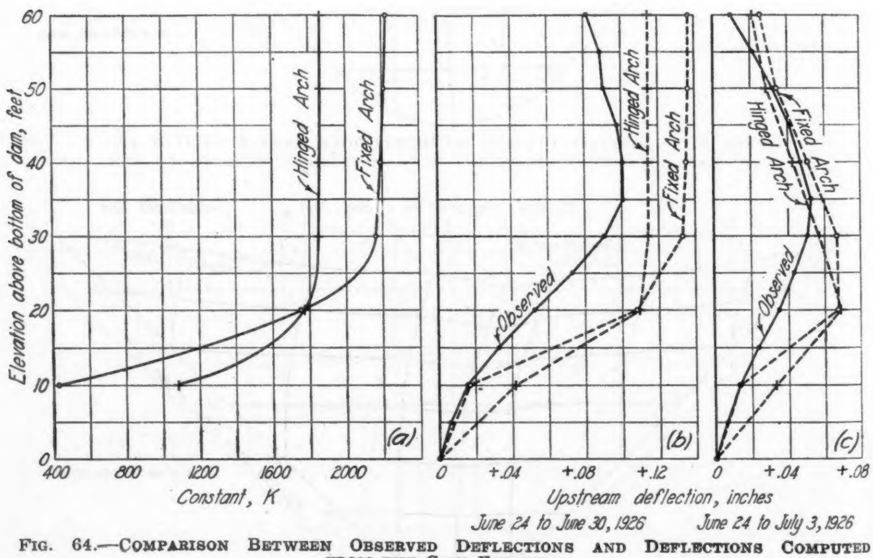


FIG. 64.—COMPARISON BETWEEN OBSERVED DEFLECTIONS AND DEFLECTIONS COMPUTED FROM THE CAIN FORMULA.

temperature variation was due partly to the fact that during the day the sun shone on only one face of the dam at a time, and partly to the fact that the water which kept the dam saturated was necessarily shut off during the observations.

In Fig. 65 the crown deflections at various elevations between 6:00 A. M. and 2:00 P. M. on June 24 are shown, also the changes of temperature and the vertical bending strains at various elevations, as measured with telemeter and strain-gauge. These bending strains are algebraic differences between the total strains measured on the up-stream and the down-stream face of the dam. The strains measured with the telemeter were available only from the bottom of the dam to the 30-ft. elevation and those measured with the strain-gauge were available only from the 30-ft. elevation to the top Fig. 65(*a*). At the 30-ft. elevation both were available. The difference between them is a large percentage of either, but it is not larger than the sum of the probable errors of the instruments.

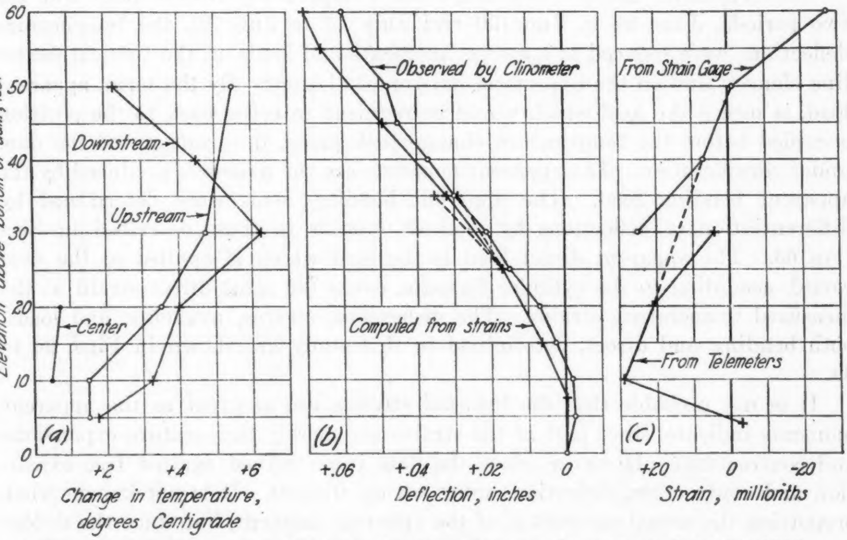


FIG. 65.—CHANGE IN TEMPERATURE, DEFLECTIONS, AND STRAINS ON JUNE 24, 1926, FROM 6:00 A. M. TO 2:00 P. M.

The deflections measured with the clinometer and those computed from the strains are shown both in the same diagram (Fig. 65(*b*)), for comparison The computed deflection curve assumes as the strain at the 30-ft elevation the average of those found by the telemeter and the strain-gauge. The upper and the lower dotted lines from the 25 to the 35-ft. elevations show the deflections which would have been obtained by using the strains from the telemeter and strain-gauge separately.

The agreement of the curve of measured deflection with the curve of deflection computed from the strains (Fig. 65(*b*)) is fair, and both indicate quite a large deflection, about $\frac{1}{16}$ in. at the top of the dam. Since all the strains measured were so small that the probable error of the instruments would be a considerable percentage of the measured strain, close agreement

may have been partly accidental, and not too much weight should be attached to it.

Temperature changes near both faces and in the interior of the dam, are shown in Fig. 65. The number of points in the diagram is too small to give full information on the distribution of the change of temperature, but, in general, it seems that the change was greatest on the up-stream face at the top of the dam and on the down-stream face at Elevation 30 and lower. The sun shone on the up-stream face during most of the time and this probably accounts for the greater change in the up-stream face at the top. It is not clear why, for the lower elevations, the change was not about the same on the down-stream face as on the up-stream face, since the platforms and framework shielded the dam from the direct rays of the sun on both faces at the lower elevations. Temperature changes in the interior of the dam were observed only at the 10 and the 20-ft. elevations, and it is clear that there the temperature change was quite small.

35.—Apparent Loads Computed from Temperature Deflections.—For the two periods, June 24 to June 30 and July 15 to July 19, the temperature deflections were reduced to apparent moments and loads on the vertical center line element and on the uppermost horizontal element. By the term, apparent load, is meant the load which would be required to bring back to the position occupied before the temperature change took place, the elements of the dam under consideration. The apparent moments are the moments produced by the apparent bending load. The apparent bending loads were determined by differentiation of deflections by methods similar to those described in Section 63. The apparent direct load is the load which if applied to the dam would, according to the cylinder formula, cause the same direct strain as the measured temperature strains. The deflections, strains, moments, and loads, both bending and direct, determined in this study are shown in Figs. 66 to 69.

It is not probable that the internal stresses are as great as the apparent moments indicate, since part of the strains are merely temperature expansions and contractions. However, since the dam is restrained against free expansion and contraction, deflections must set up stresses. It is not known what proportion the actual moment is of the apparent moment, but since the deflections and the temperature changes were much the greatest at the top of the dam, whereas the apparent moments and loads were much the greatest near the bottom, it seems likely that the actual moments were not far short of the apparent moments. The temperature changes at the top and at the bottom of the dam from June 24 to 30 were about 8° and 3.5° cent., respectively. The highest apparent bending stress resulting from the temperature changes was about 100 lb. per sq. in.

Since the dam is free to expand and contract in the vertical direction, it is assumed that the vertical direct stress due to temperature change may be neglected. In Figs. 68 and 69 the plotted points show the apparent bending loads, and the cross-hatched area shows the apparent direct loads on the uppermost horizontal element of the dam. The actual bending moment and bending stresses probably are about equal to the apparent bending moment and the stresses computed therefrom. However, for reasons given in Section 33 it is

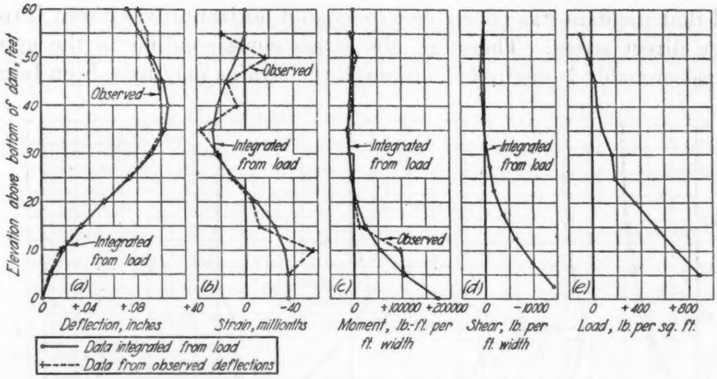


FIG. 66.—APPARENT LOAD ON VERTICAL CENTER LINE ELEMENT DUE TO TEMPERATURE DEFLECTIONS, JUNE 24 TO JUNE 30, 1926.

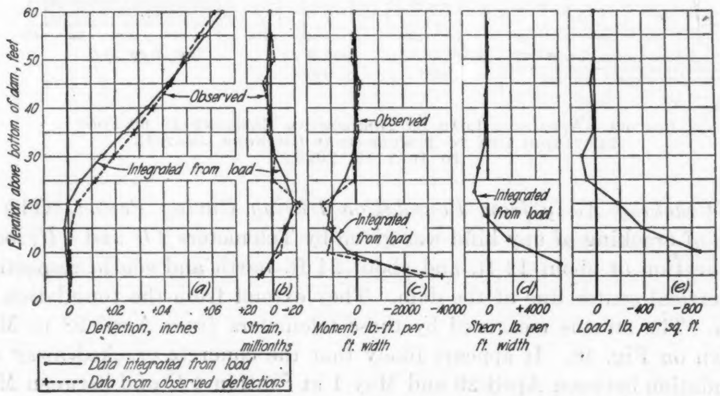


FIG 67.—APPARENT LOAD ON CENTER LINE ELEMENT DUE TO TEMPERATURE DEFLECTIONS, JULY 15 TO JULY 19, 1926.

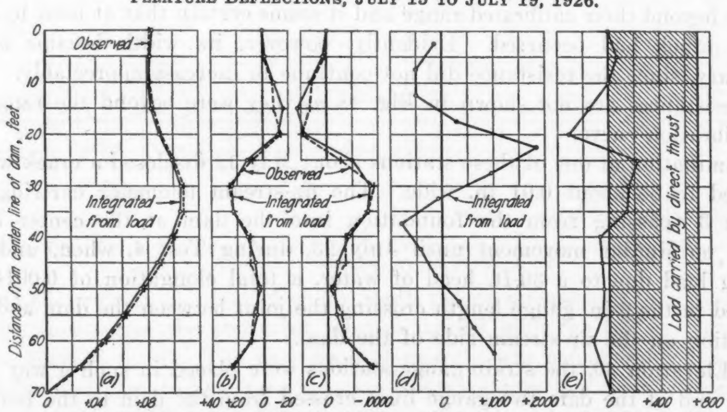


FIG. 68.—APPARENT LOAD ON HORIZONTAL ELEMENT AT 60-FOOT ELEVATION DUE TO TEMPERATURE CHANGES, JUNE 24 TO JUNE 30, 1926.

believed that the dam was nearly free to expand horizontally without introducing large direct stress. Therefore, the stress corresponding to the apparent direct load probably is negligible, although the strains may have been large.

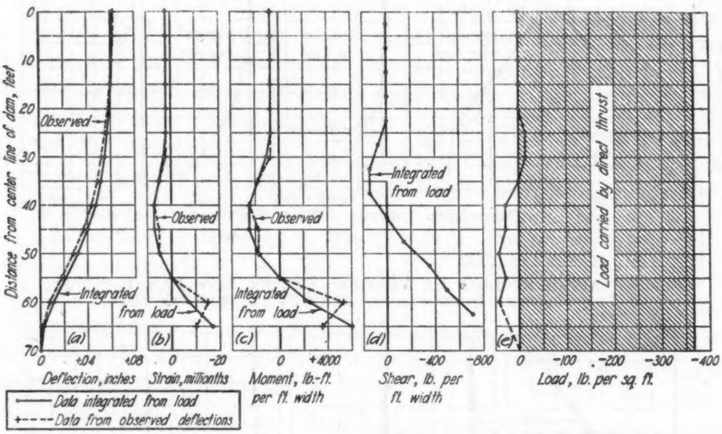


FIG. 69.—APPARENT LOAD ON HORIZONTAL ELEMENT AT 60-FOOT ELEVATION DUE TO TEMPERATURE CHANGES, JULY 15 TO JULY 19, 1926.

36.—*Cracking Away from Foundation During Curing Period.*—The first evidence of cracking of any kind was given by Telemeters 2 *U* and 4 *U*, located at an elevation of about 12 ft. and about 24 ft. north and south, respectively, of the vertical center line of the dam. They extend from the foundation into the dam. The strains measured by these telemeters from April 28 to May 3 are shown on Fig. 48. It appears likely that the concrete cracked away from the foundation between April 29 and May 1 at Station 4 *U* and between May 2 and May 3 at Station 2 *U*. After these dates the resistance in the telemeters was far beyond their calibrated range and it seems certain that at least by that time a crack had occurred. Evidently, however, its width became about stationary, since the resistance did not continue to increase appreciably. The later resistances are not shown in Fig. 48 as they were beyond the range of the calibration curve.

Examination at one of these stations about May 12 disclosed a crack which appeared to be about 0.01 in. wide. The up-stream telemeter cartridge at Station 3 (passing from the foundation into the dam at the center line) showed no similar movement until July 23, during Test 4, when, under a bending load due to a 30-ft. head of water, a total elongation of 0.0014 in. occurred in the 6-in. gauge length crossing the joint between the dam and the foundation on the up-stream side of the dam.

At Elevation 30, the strain-gauge stations were placed in such a way that at each end of the dam two gauge lines crossed from the dam to the foundation rock on the down-stream face of the dam and one on the up-stream face. The concrete was placed to a height which included these stations on May 13, and initial readings were taken on May 14 (Fig. 70). At both ends of the dam the strains on the up-stream face increased quite sharply on May 19, and

on May 22 a crack was observed at the south end. On May 22 the temperature began to fall and continued falling until the minimum temperature was reached on May 25, the date on which the maximum opening of the crack is indicated by the graphs for Gauge Lines 1 *U* and 2 *U*.

Although Fig. 70 indicates considerable elongation on the down-stream face, no crack was found at either end of the dam on that face at the 30-ft. elevation until about June 30 during the period referred to in Section 37 as the drying period. However, the principal feature of significance during this period seems to have been the internal temperature change due to shutting off the sprinkling water. The abrupt increase in strain for Gauge Lines *A* and *B* and Stations 1 *D* and 2 *D*, Fig. 70, and the sharp rises in internal temperature shown in Figs. 43 to 47, inclusive, beginning June 24, correspond perfectly with the date of shutting off the sprinkling water from the dam, whereas nothing in the air temperature diagram, Fig. 42, would account for these strains and internal temperature changes.

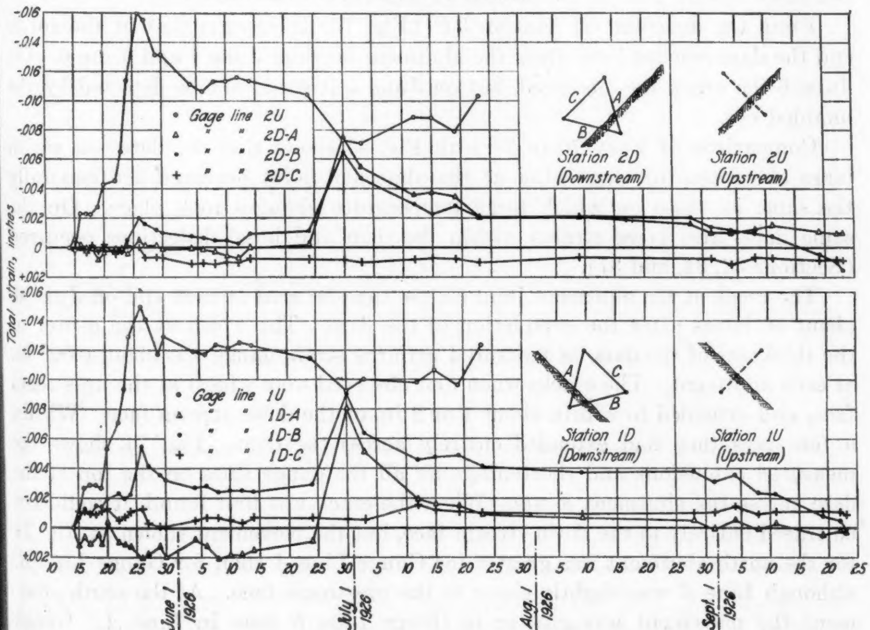


FIG. 70.—TOTAL STRAINS IN GAUGE LINES AT STATIONS 1*D*, 1*U*, 2*D* AND 2*U*, ACROSS ABUTMENT JOINTS, AT ELEVATION 30.

At the ends of the dam (Fig. 70), both the up-stream and the down-stream gauge lines across the abutment joint elongated with the fall in temperature from May 18 to 26. The up-stream elongation was considerably larger than that down stream, indicating a rotation corresponding to a down-stream deflection of the dam. Later, during the drying period, a sharp rise in the internal temperature of the dam occurred, and a corresponding shortening of both up-stream and down-stream gauge lines might have been expected. However, only the up-stream gauge lines shortened, whereas those down stream elongated.

This anomalous behavior may be accounted for if it be assumed that the dam was not entirely free to expand and that the pressure at the abutments produced an up-stream deflection resulting in a rotation of the dam at the abutments, as indicated by the strains shown in Fig. 70. Fig. 63, which gives the crown deflections at the 30-ft. elevation, shows that some such deflection as that here outlined took place as the temperatures changed. After the high internal temperature of June 30, the up-stream gauge lines elongated and the down-stream gauge lines shortened consistently with these assumptions.

The movements at the abutment joints at Elevations 44, 53, 55, and at the top of the dam, respectively, are shown in Figs. 71 to 74. A comparison of Figs. 71 to 73 shows sudden elongation of the gauge length on the up-stream face at Station 3 *U*, Elevation 55, within two days after pouring, compared with a steady change up to June 24, for Stations 3 *D* and 4 *D* on the down-stream face at Elevations 44 and 53, respectively.

From the diagram for Station 3 *U* (Fig. 73) it appears that at the south end the dam cracked loose from the abutment between June 4 and June 6. On June 6 the crack was observed, but on June 5 it could not be detected by the unaided eye.

Comparison of Figs. 70 to 74 with Fig. 47 shows that the dates on which large elongation or shortening at the abutment joint occurred are generally the same as those on which large temperature changes took place. On the same dates also large strains within the dam and large deflections occurred (Sections 33, 34, and 37).

The crack at the abutment joint on the top occurred at each end on June 6, about 36 hours after the completion of the dam. The width at the center of the thickness of the dam, as measured with the strain-gauge was about 0.008 in. at each abutment. The cracks when first observed were widest at the up-stream face, and extended to within about 1 or 2 in. of the down-stream face. Within a few days they had extended entirely across the dam. Fig. 74 shows the measured extensions and shortenings on all the gauge lines on the top of the dam across the abutment joints. When the crack was first found, it could not be traced entirely to the down-stream face, but the movement shown in Fig. 74 for the north abutment was greater for Gauge Line *A* than for Gauge Line *B*, although Line *A* was slightly closer to the up-stream face. At the south abutment the movement was greater in Gauge Line *B* than in Line *A*. Gauge Lines *A* and *B* are so close together where they cross the crack that the differences of their readings cannot be expected to tell much about the rotation of the dam at the abutment. In order to detect more exactly this rotation, Gauge Lines *E* 1 to *E* 4, inclusive, were used (Fig. 74); the movements in these gauge lines are shown clearly.

It is likely that before the load tests began on July 12, cracks extended all the way from the top of the dam down to within about 10 ft. of the bottom on the up-stream face at the abutment joint, although they were not detected at all points in this course. On the down-stream face similar cracks had been found to extend down to about the 50-ft. elevation before the drying period

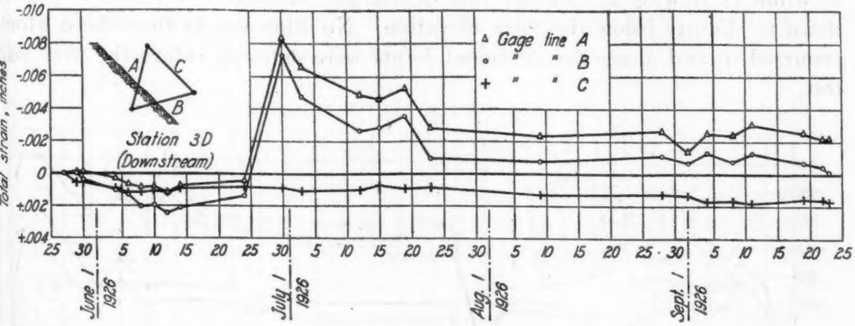


FIG. 71.—TOTAL STRAINS IN GAUGE LINES OF STATION 3*D* ACROSS ABUTMENT JOINT AT ELEVATION 44.

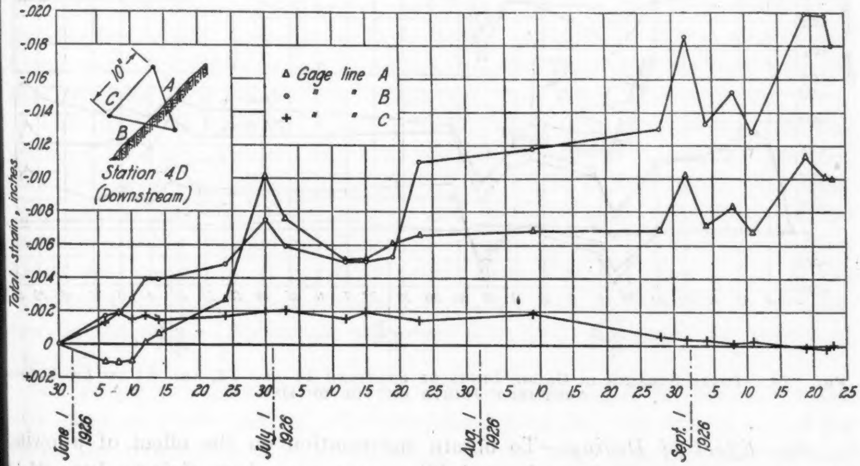


FIG. 72.—TOTAL STRAINS IN GAUGE LINES OF STATION 4*D* ACROSS ABUTMENT JOINT AT ELEVATION 53.

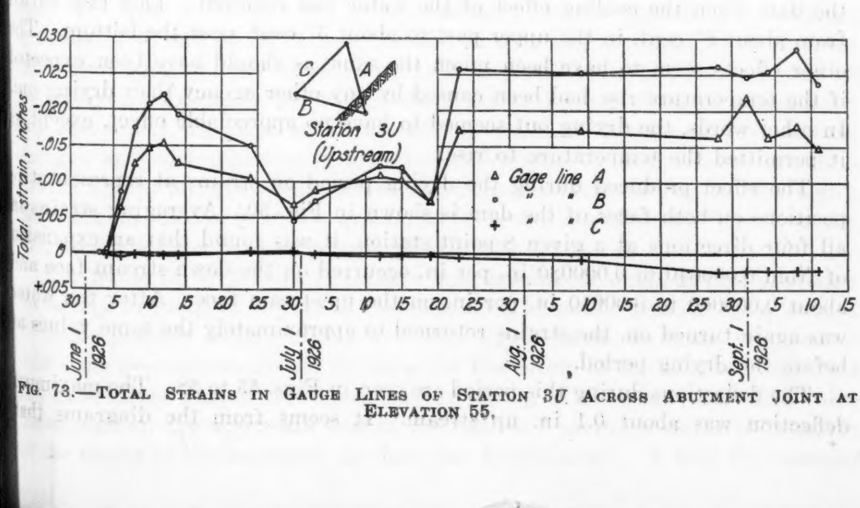


FIG. 73.—TOTAL STRAINS IN GAUGE LINES OF STATION 3*U* ACROSS ABUTMENT JOINT AT ELEVATION 55.

of June 24 to July 2. During this drying period they were found to extend down to slightly below the 30-ft. elevation. No other cracks than those which occurred in and along the abutment joints were detected before the first load test.

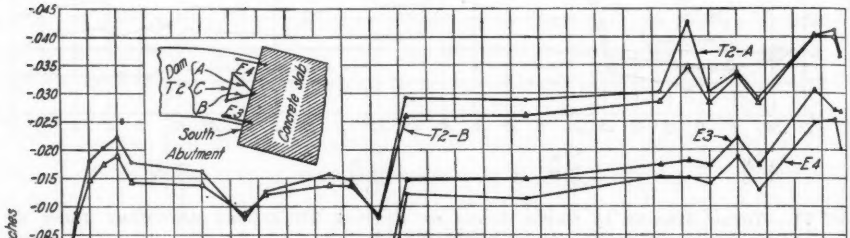


FIG. 74.—TOTAL STRAINS IN GAUGE LINES OF STATIONS $T1$ AND $T2$, AND $E1$ TO $E4$ ACROSS ABUTMENT JOINTS ON TOP OF DAM.

37.—Effect of Drying.—To obtain information on the effect of allowing the dam to dry for a time, the sprinkling water was shut off from June 24 to July 2. This caused a sharp rise in the internal temperature of all parts of the dam when the cooling effect of the water was removed. This rise varied from about 8° cent. in the upper part to about 3° cent. near the bottom. The other effects seem to have been much the same as should have been expected if the temperature rise had been caused by any other agency than drying out. In other words, the drying out seemed to have no appreciable effect, except as it permitted the temperature to rise.

The effect produced during the drying period on strains at representative positions on both faces of the dam is shown in Fig. 50. Averaging strains in all four directions at a given 8-point station, it was found that an expansion of from 0.000040 to 0.000080 in. per in. occurred on the down-stream face and about 0.000060 to 0.00010 in. per in. on the up-stream face. After the water was again turned on, the strains returned to approximately the same values as before the drying period.

The deflections during this period are seen in Figs. 55 to 58. The maximum deflection was about 0.1 in. up stream. It seems from the diagrams that

the deflections did not entirely disappear for a considerable length of time after the water was again turned on. In this respect they correspond very well with the slow disappearance of the interior temperature effect.

The change in widening of the cracks along the abutments is shown in Figs. 70 to 74, and a discussion of the cracks is given in Section 36.

H.—General Data of Tests

38.—Description of Load Tests.—Thirteen load tests were made (Table 20). The general procedure was to turn off the sprinkling water from 1 to 4 hours before the testing, and to begin zero readings about midnight. In general, the zero readings required about 2 hours, although there were considerable variations from this length of time. The water was admitted behind the dam as soon as the zero readings were completed, and with the earlier tests the reservoir was filled first to a height 10 ft. below the maximum height for that test. Readings were taken at certain key positions for comparison with previous tests under the same head of water. About ½ hour was allowed for taking these key readings and immediately more water was admitted and the head increased to the maximum for the test.

The first three tests used only the flow of water from Stevenson Creek; but so long a time was required to fill the reservoir for the 30-ft. head and greater, that there was not sufficient time during one night to take the no-load readings, fill the reservoir, and take the load readings unless the filling of the reservoir could be speeded up. For these succeeding tests the flow of Stevenson Creek was supplemented by water taken from Water Tunnel No. 3 of the Southern California Edison Company through a 6-in. outlet valve in the bulkhead in the adit. (Appearing as a dark spot near the upper left corner of Fig. 6.)

In Test 8 (made on September 4, with only the telemeter readings taken at a head of 50 ft., and all other readings at 40 ft.), the time required for filling the reservoir was so great, owing to the large quantity of water and to difficulty in operating the 6-in. valve in the tunnel, that it was necessary to omit the key readings in order to complete the test within the time available. In Tests 9, 10, and 11, it was necessary also to omit the key readings at heads of 40 and 50 ft. It was the aim in each test to complete all observations by 8:00 A. M., taking as many as time permitted after the emptying of the reservoir. In the earlier tests made in July, this was essential because the sun rose upon the dam at about 8:00 A. M., and introduced temperature changes and corresponding strains and deflections. For the later tests this did not apply; but after eight hours of continuous work throughout the night, the strain on the observers was great, and it seemed desirable not to prolong the test, except for the most urgent purposes.

For heads of 40 ft. and less, there was nothing to indicate that provision for safety in event of failure of the dam need be made, because there seemed to be no possible danger of failure. For higher heads, however, it appeared that some precautions should be taken for the safety of the men who operated the gate-valves during the filling and emptying of the reservoir. The valve stems rose on the up-stream side of the dam to a platform about 10 ft. north of the center of the length of the dam (see frontispiece). A rope life-line was

stretched across the canyon above this platform and attached at both ends to anchor-bolts in the rock. During the test the operators secured themselves by means of auxiliary ropes around their waists and looped over the life-line. During this test it seemed for a few minutes that these precautions were very timely, as a crack occurred and in the darkness it was uncertain how serious it was. This crack is described in Section 40.

At that time the water stood at an elevation of approximately 50 ft. The crack extended entirely through the dam, and to go below the dam to see how far down it extended, or to take strain-gauge and clinometer readings appeared unsafe. The station for taking the telemeter observations, shown in the frontispiece, was free from danger, and a complete set of telemeter readings was taken. These required about 1¼ hours and as soon as they were completed the water was lowered to the 40-ft. level. As the dam had carried the head of 50 ft. during this time without further indication of failure, it was considered safe to proceed with the observations, and this was done.

The readings for Test 9 were taken under a head of 50 ft. In order to establish safety, the water was first raised to an elevation of about 54 ft. and held there for about ½ hour. It was then lowered to the 50-ft. level and the readings were taken. For Test 10, again taken under a head of 50 ft., the water was raised to the 50-ft. level as rapidly as possible, and the readings were taken at once.

For Test 11, the first with a 60-ft. head, it was not possible to establish safety by raising the water to an elevation greater than that at which the readings were to be taken; but the same result was accomplished by filling the reservoir during the day and leaving it under the head of 60 ft. from about 4:45 P. M., until 10:45 P. M., when the readings were begun.

Although safety under the 60-ft. head had been sufficiently established, still the reservoir was filled during the day for Tests 12 and 13. This was necessary because of the relatively great length of time required to fill the reservoir to the 60-ft. elevation. Even if the zero readings had been taken as soon after sunset as temperature equilibrium had been established, the time remaining would not be sufficient for filling the reservoir and taking the load readings before the sun was upon the dam on the following morning. Therefore, the order was reversed, by filling the reservoir during the day, taking the load readings about the middle of the night, and, finally, taking the no-load readings after the water had been discharged. This reversed order applies to all the tests at the 60-ft. head.

In the last test, Test 13, September 23, 1926, especial emphasis was placed on getting a series of readings under heads of 60, 50, 40, and 30 ft. all in the same night. It was not feasible to take readings for all these heads with all the instruments. Consequently, all the strain-gauge readings were omitted except those measuring the change in width of cracks.

The conduct of the load tests held much interest for engineers within reach of the Test Dam. Frequently there were visitors during the day, and on two or three occasions they remained through all or part of the night to observe the tests. The first filling of the reservoir to the 60-ft. level was an event of special interest. Besides the testing crew, a number of members of the Committee on Arch Dam Investigation and a number of guests were present and

remained during the succeeding observations. The use of the electric telemeter made it feasible to take readings of strain at any place without danger to the observer. Readings at several selected stations of importance were taken at frequent intervals, and the results were watched with interest.

39.—Leakage.—Throughout the testing, there was extremely little leakage. For holding the forms together during construction, bolts extended entirely through the dam from the up-stream to the down-stream face. When the dam was under a head of water moist spots appeared around the down-stream ends of about one-quarter of these bolts due, evidently, to water following the surface of the bolts through the dam. Some leakage also occurred through the abutment joints of the dam, and some through fissures in the rock entirely away from the dam.

With a head of less than 50 ft. no leakage of any kind was discovered through the concrete or through a construction joint. Under a head of 50 ft. a small moist patch about 4 by 7 in. appeared at the horizontal construction joint 15 ft. above the base of the dam, due apparently to seepage through this joint. Under a head of 60 ft. enough moisture came through the construction joint at an elevation of 20 ft. to wet an area about 7 ft. long and 4 ft. high. Also, in the same test, four or five small patches of moisture appeared at the construction joint 55 ft. above the base of the dam. These patches were not larger than 1 ft. in diameter.

Under the head of 50 ft. rough measurements were made from which it was estimated that the total leakage was about 6 gal. per min. Of this it seems fair to estimate that 95% came through fissures in the rock. Under the head of 60 ft., the leakage was considerably larger, but the percentage which came through the dam appeared to be even smaller than under the 50-ft. head.

On September 24, after the horizontal crack at the 30-ft. elevation (Section 40) had formed, some seepage through this crack was discovered. The area covered by this seepage was quite extensive. The quantity of water coming through was very small, being greatest near the abutment.

40.—Cracking During Testing Period.—In order to facilitate the detection of cracks, a fillet of cement mortar was placed in the angle at the junction of the dam with each abutment, on the up-stream side from the bottom to the top of the dam. The smooth surface thus afforded gave a good opportunity for observation of cracks when they occurred. This fillet was placed after the formation of the first cracks due to shrinkage or temperature changes and before the first load test. It afforded a good opportunity for determining subsequent changes in the behavior of the dam.

Up to the completion of the load tests, there had been very little increase in the extent of the cracking at the abutment, and in much of the region where early cracks were formed, and later covered by the fillet, no crack was then in evidence. Near the top of the dam a distinct crack was to be found at each end on the up-stream side, for a depth at the south end of about 10 or 12 ft. At the north end the dam seemed to be free from the rock on the up-stream side for the upper 4 ft. There was then an interval of about 3 ft. with no sign of a break. Below this point a crack appeared intermittently down to an elevation of about 36 ft. Below that point to Elevation 30 there was no

crack except the old one which appeared soon after the completion of the dam. Where this was covered with fresh mortar the mortar did not crack.

The first crack due to water pressure occurred at the bottom of the dam, in the joint between the concrete and the rock on the up-stream side. After the first indication of the imminence of this crack had been given by the telemeter reading in Test 4 with a 30-ft. head of water, frequent readings were taken during the remainder of the test, as recorded in Fig. 75. A continued elongation on the up-stream side indicated a stress greater than the concrete or its bond with the rock could be expected to resist without cracking. The largest reading indicates an elongation within the 6-in. gauge length of about 0.0014 in. There was also an appreciable compression in the telemeter embedded in the rock on the down-stream side. After the reservoir had been emptied, the resistance in these telemeters returned to the previous no-load value. There is no doubt that a crack occurred before the 40-ft. head of water was reached since after that test, when the reservoir was emptied, the resistance did not return to a value even within the range of the calibration curve.

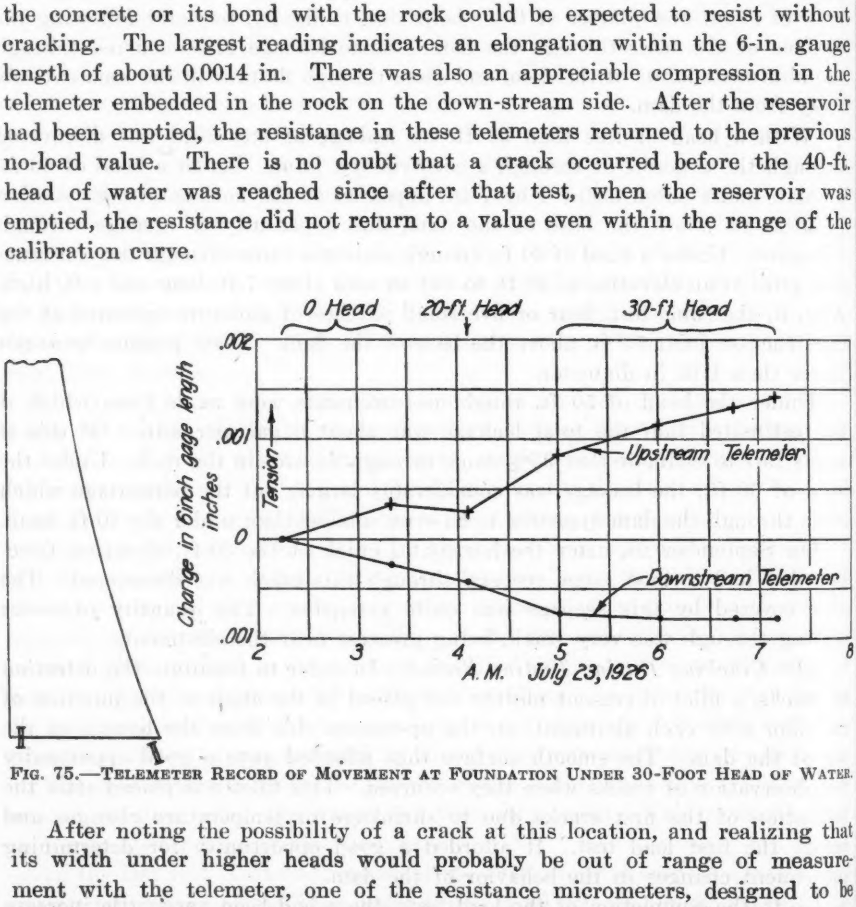


Fig. 75.—Telemeter Record of Movement at Foundation Under 30-Foot Head of Water.

After noting the possibility of a crack at this location, and realizing that its width under higher heads would probably be out of range of measurement with the telemeter, one of the resistance micrometers, designed to be used in connection with the towers for the measurement of deflections, was equipped for measuring the opening of this crack. This micrometer consisted of a coil of resistance wire, a portion of which was cut out with a decrease of the gauge length over which the instrument operates. The instrument was calibrated for movement, in inches, against resistance of the coil (Fig. 26). In adapting the instrument to the measurement of the crack under water the essential feature was its protection against leakage into the resistance coil and into the lead wires extending from it to the observation platform.

The protection consisted of an envelope made from a portion of an automobile inner tube, and it operated satisfactorily for the purpose. The method of attachment is indicated in Fig. 78, Section *D-D*. Fig. 76 shows the amount of opening of this crack for Tests 6 to 12.

Although the agreement in different tests under the same head of water is not perfect, it is sufficiently good to show that the measurements are significant of the opening of this crack. It is not unlikely that differences in temperature for successive tests would be sufficient to cause the cracks between the dam and the abutment near the top to vary in width before the application of pressure. Such a variation in the upper cracks would cause an opening of the one at the bottom to start earlier or later depending on whether the upper cracks were wider or narrower. For these reasons, the variations, shown in Fig. 76, of the width of the crack at the bottom, do not seem greater than might be expected.

Fig. 76.—Changes in Width of Crack Between Dam and Bed-Rock on Up-Stream Side at Elevation 2.

In Test 8, a vertical crack occurred very nearly on the exact center line, extending from the top of the dam downward to approximately Elevation 49 on the down-stream face and to Elevation 55 on the up-stream face. As measured with a scale and magnifying glass, the crack width on the top was found to be $\frac{1}{60}$ in. on the up-stream face and $\frac{1}{40}$ in. on the down-stream face after the water had been lowered to an elevation of 40 ft. The strain observed with the telemeter during the rise of the water was 0.000065 in. per in. (corresponding stress about 240 lb. per sq. in.) when the elevation of the water was about 47 ft.

It is not known at just what instant the crack appeared, but it does not seem likely that it occurred before the water reached a head of 50 ft. At this time the static tensile stress due to the water pressure could not have been much more than 300 lb. per sq. in. It is not unlikely that there was some initial stress in the concrete at the beginning of the test, although the dam had been kept wet by means of continuous streams of water from a sprinkler pipe up to within about 2 hours of this time. When the water level reached 50 ft. it was necessary to open the 24-in. valve to control the water level. There was evidence of considerable vibration in the dam when the valve was opened, and it is not improbable that the crack occurred during this vibration and partly on account of it. In Test 9, instruments used for measuring the change in width of the crack during filling and emptying of the reservoir, indicated vibration with an amplitude of 0.0005 in., when the 24-in. valve was opened.

At the close of Test 8, as the water was being discharged, strain-gauges held on the concrete across the crack indicated a closure of the crack on the down-stream side of 0.013 in. and on the up-stream side of 0.0031 in. With removal of the water after Test 8, the crack remained open about 0.01 in. and closed rather slowly until in the middle of the day it disappeared almost entirely. After removal of the water in Test 9 the crack immediately closed almost completely. That there was no seepage through this crack confirms the observation that it did not extend all the way through the dam, down as far as the 50-ft. elevation.

In later tests measurements were taken showing the variation in width of this crack with changing heads. Fig. 77 shows that it began opening under a head of a very few feet and continued to open until a head of from 45 to 50 ft. had been reached; and from that time on until the water reached the top of the dam the crack closed progressively. Evidently, the cause of the closure was the direct compression (arch action) which apparently extended only a short distance above the water level. The indications of a negative opening of the crack on the up-stream side of the dam under the full head (60 ft.) may be satisfactorily explained by assuming that the crack was open when the zero reading on the instrument was made. Therefore, the diagram would indicate that the crack was further closed at the top of the dam with the reservoir full than with it empty. By the end of the testing program, this crack could be traced down to an elevation of 41 ft. on the down-stream face.

During the first test to a head of 60 ft. the highest tensile stress found, based on the observed strain and a modulus of elasticity of 3 600 000 lb. per sq. in., was about 520 lb. per sq. in. This was in a horizontal direction at the down-stream side at Station 6 across the center line near the bottom of the dam. At the time the filling of the reservoir was complete the telemeter readings indicated that there was no crack at this place. The full head of 60 ft. was reached about 4:45 P. M., and this head was held until 10:45 P. M., when the deflection, strain, and other observations began. By this time the strain at the place where the stress of 520 lb. per sq. in. had been found had gone beyond the calibrated range of the telemeter, and it was evident that a

crack had occurred. Visual inspection disclosed this crack almost exactly on the center line. At this time it could be traced from about 2 in. above the bottom of the dam to a height of about 9 ft. Before the close of the testing program this crack had lengthened so that it could be traced to a height of 13 ft.

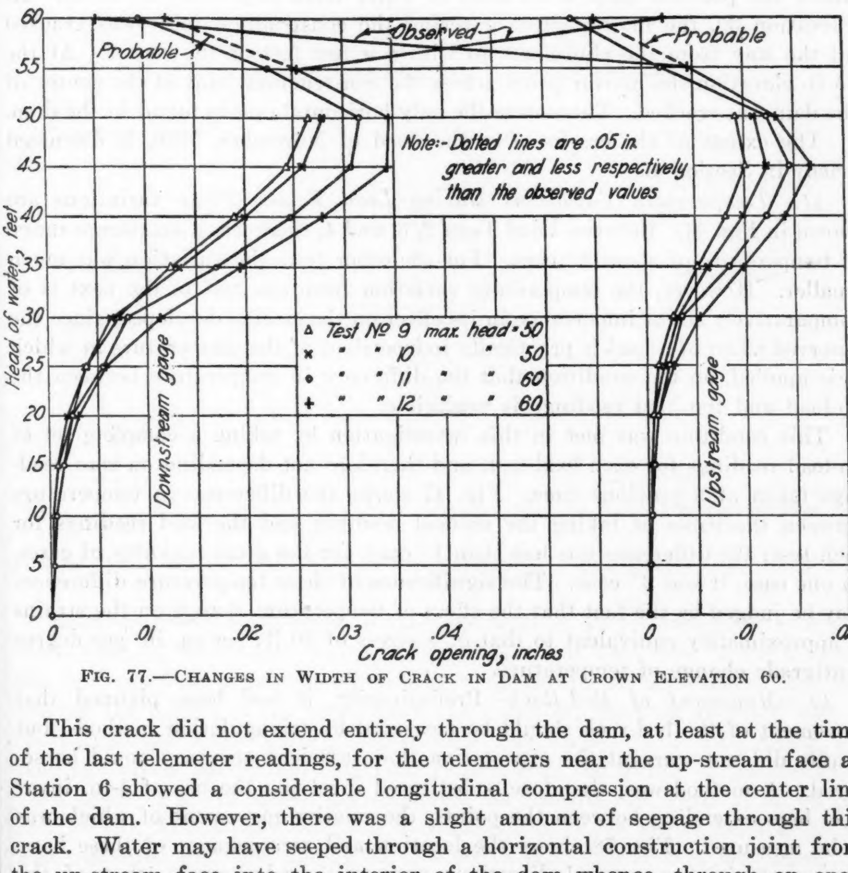


FIG. 77.—CHANGES IN WIDTH OF CRACK IN DAM AT CROWN ELEVATION 60.

This crack did not extend entirely through the dam, at least at the time of the last telemeter readings, for the telemeters near the up-stream face at Station 6 showed a considerable longitudinal compression at the center line of the dam. However, there was a slight amount of seepage through this crack. Water may have seeped through a horizontal construction joint from the up-stream face into the interior of the dam whence, through an open vertical crack, it would travel to the down-stream face. This is the more likely, since there was considerable tension in the vertical direction within this region on the up-stream face.

An attempt was made to search for cracks on the up-stream face of the dam in this region, but an effective examination could not be made because of the large quantity of falling water which could not be diverted through the over-sluice. The width of the crack on the down-stream face was not measured, but it appeared to be not more than 0.01 in. wide at the widest part.

Between Tests 5 and 6, a thin mortar patch was plastered smoothly across each construction joint on the down-stream, and also on the up-stream, face of the dam in such position that it was crossed by the vertical center line.

Most of the patches were about 4 in. square, but the one at the 30-ft. elevation extended across the full length of the dam. These patches were provided to furnish a means of detecting more accurately the time of the formation, and the location of, the cracks.

After the program of load testing had been completed, the dam was left under the pressure of a 60-ft. head of water from September 23 to 25. At Elevation 30, the mortar patch covering the construction joint was cracked all the way from the abutments to within a few feet of the center. At the 35-ft. elevation the mortar patch across the construction joint at the center of the dam was cracked. These were the only horizontal cracks found in the dam.

The extent of the cracks after the flood of November, 1926, is discussed briefly in Section 46.

41.—Temperature Variations During Load Tests.—These variations are shown in Fig. 47. Between Load Tests 2, 3, and 4, there was a maximum range of temperature of about 8° cent. For the other tests the variation was much smaller. However, the temperature variation from one test to the next is of comparatively slight importance in its effect on the strains developed, since the observed effect of a load is practically independent of the temperature at which it is applied, on the condition that the difference in temperature between the no-load and the load readings is negligible.

This condition was met in this investigation by taking a complete set of no-load readings for each load test, and therefore not depending on zero readings taken at a previous time. Fig. 47 shows the difference in temperature between the times of taking the no-load readings and the load readings for each test; the difference was less than 1° cent. for the great majority of cases. In one case, it was 3° cent. The significance of these temperature differences may be judged by the fact that the effect of temperature change on the strains is approximately equivalent to that of a stress of 30 lb. per sq. in. per degree centigrade change of temperature.

42.—Movement of Bed-Rock—Preliminarily, it had been planned that movement of the bed-rock should be measured by triangulation methods, but results did not warrant the expectation that sufficient accuracy could be secured. A method was, therefore, substituted involving the use of $\frac{3}{8}$-in. invar steel bars extending between the points, the relative movement of which was to be measured. Fig. 78 shows the location and arrangement of these bars. Movements in two general directions were measured, namely, that of the bed-rock in approximately a radial direction near the bottom of the dam, and that of the abutments in approximately the direction of a long chord, at several elevations.

For the radial movement of the bed-rock near the bottom of the dam, an invar steel bar 17 ft. long (Fig. 78) was attached at one end to a steel plug set into the rock just below the down-stream face of the dam at an elevation of about 1 ft. The bar was carried along the face of the canyon and, at its down-stream end, bore against the plunger of a 0.0001-in. Ames gauge which was attached rigidly to the face of the rock wall. The invar bar was supported at frequent intervals along the length of a 4 by 4-in. timber which spanned the gauge length. While it is possible that both points of attachment to the

rock moved, due to the pressure of the water, it seems likely that the greater part of this movement would be within the first few feet next to the dam, and that the movement at the down-stream point of attachment to the rock would be so small as to be unmeasurable.

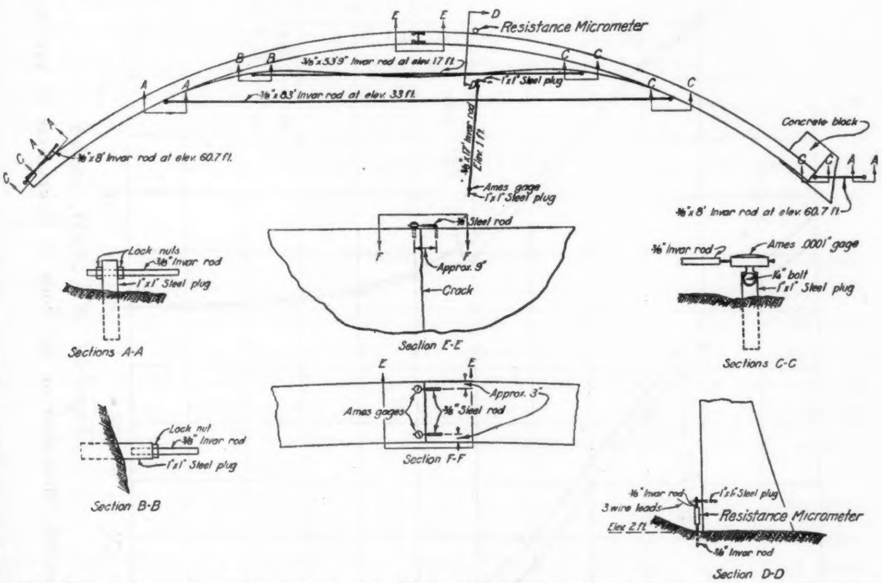


FIG. 78.—LOCATION OF STATIONS FOR MEASUREMENT OF MOVEMENT OF BED-ROCK AND CHANGES OF CRACK WIDTH.

Readings were first taken with this apparatus in Test 6, in which the maximum head of water was 40 ft. Although the bar used was of invar steel, the readings were somewhat affected by temperature changes during the test. During the release of the water from the reservoir, the time involved was so short that the temperature changes were very small. For this reason the readings taken during the emptying rather than the filling of the reservoir were used. Fig. 79 shows the measured movements for Tests 7, 9, 10, 11, and 12. In Test 6 only the total movement due to the entire head of 40 ft. was measured and this amounted to 0.00076 in. The largest movement occurred with 60-ft. head and averaged about 0.0012 in. for the two tests in which measurements were taken. Fig. 79 shows considerable divergence of measured movements for different tests, but the agreement is sufficiently good to indicate approximately the magnitude of the movement for various heads of water.

Movement of the abutments along the direction of a long chord, that is, the spreading of the canyon walls, as measured at Elevations 17 and 33, is shown in Fig. 80, with corresponding heads of water. For taking these measurements a $\frac{3}{8}$-in. invar steel bar was attached to a heavy steel plug, embedded in the bed-rock at the north end of the dam at each of these elevations (Fig. 78). Each bar extended along the direction of the long chord to a corresponding point at the south abutment where the end of the bar bore

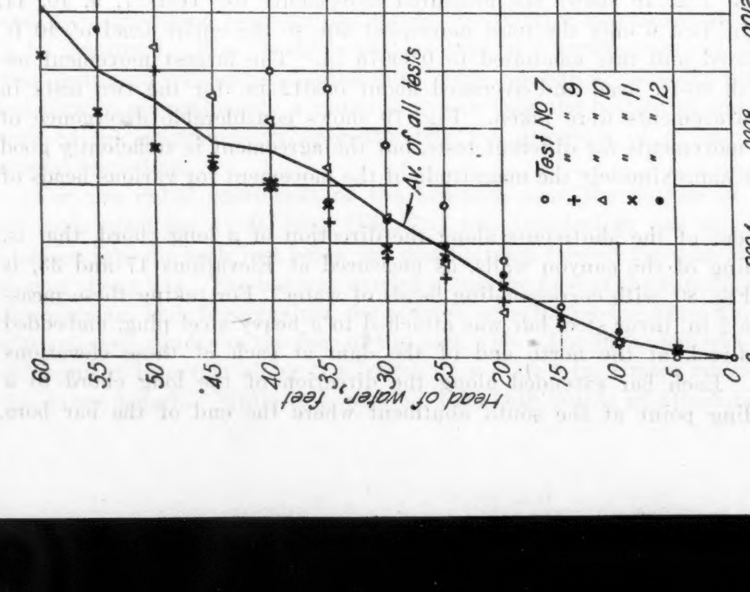


FIG. 79.—RADIAL MOVEMENT OF BED-ROCK AT 1-FOOT ELEVATION.

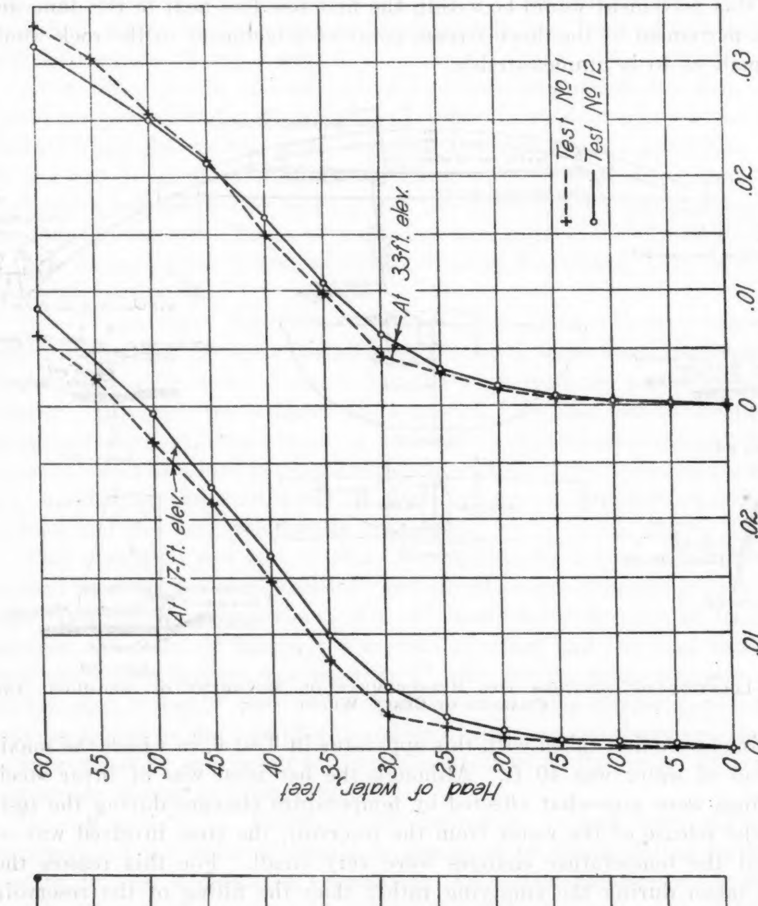


FIG. 80.—MOVEMENT OF BED-ROCK AT ELEVATIONS 17 AND 33.

against the plunger of an Ames gauge attached to another plug embedded in the bed-rock. The method of supporting the invar bar throughout its length and the end connections with the rock, were similar to those for the radial movement. Thus the total change of distance between the ends of the chord was transmitted to the Ames gauge.

The readings gave the sum of the movement at the two end points. It is this total movement along the chord length that is plotted in Fig. 80. The installation for these measurements was not made until just previous to the first test to a head of 60 ft., and the measurements were made only in Tests 11 and 12. The agreement of the measured movements for the two tests with each other is very good. The average movement for a head of 60 ft. was about 0.035 in.

At the 60-ft. elevation movements of points at the north and the south ends were measured independently. For convenience the measurement at the north end was approximately in the direction of a tangent to the dam at that point, while that at the south end was approximately along the direction of the long chord. The method of measurement was substantially the same as at the bottom. The movements of the bed-rock found at Elevation 60 are shown in Fig. 81. It will be seen that they were very small and in opposite directions at the two ends.

Although the amounts measured have not been large, they have been consistent and indicate unquestionably that movement was present. The measured spreading of abutments has been very helpful in determining from the deflections the amount of strain due to direct compression.

This spreading has been compared with corresponding values determined analytically* for the 60-ft. head of water. In making these computations the equivalent loads carried by direct compression, as shown in Fig. 118, were used to determine the end reactions at various heights of the dam. The pressures, w, per unit area corresponding to these reactions are used in Vogt's formula:

$$\Delta l = \frac{4}{a\,b}\int_0^a\int_a^b (1-\mu^2)\cdot\frac{w}{\pi E}\left\{a\log_e\frac{b+\sqrt{a^2+b^2}}{a}\right.$$

$$\left.+\,b\log_e\frac{a+\sqrt{a^2+b^2}}{b}\right\}d\,b\,d\,a = (1-\mu^2)\,\frac{w}{E}\,\frac{2}{\pi}\left\{a\log_e\frac{b+\sqrt{a^2+b^2}}{a}\right.$$

$$\left.+\,b\log_e\frac{a+\sqrt{a^2+b^2}}{b} - \frac{(a^2+b^2)^{1.5}-(a^3+b^3)}{3\,a\,b}\right\}$$

in which,

Δl = tangential movement of the abutment due to the arch thrust.
μ = Poisson's ratio = 0.15.
E = modulus of elasticity of the bed-rock = 2 000 000 lb. per sq. in.

---

* "Ueber die Berechnung der Fundamentdeformation," by Dr. Fredrik Vogt, pub. by Det Norske Videnskaps-Akademi, Oslo (1925).

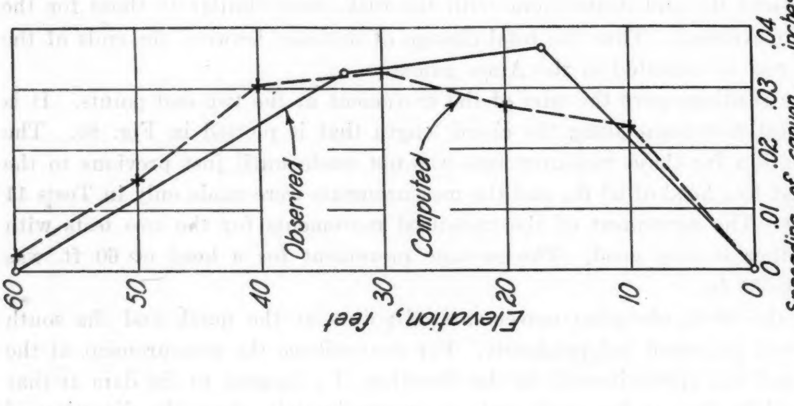


FIG. 82.—COMPARISON OF OBSERVED AND COMPUTED MOVEMENT OF BED-ROCK FOR 60-FOOT HEAD.

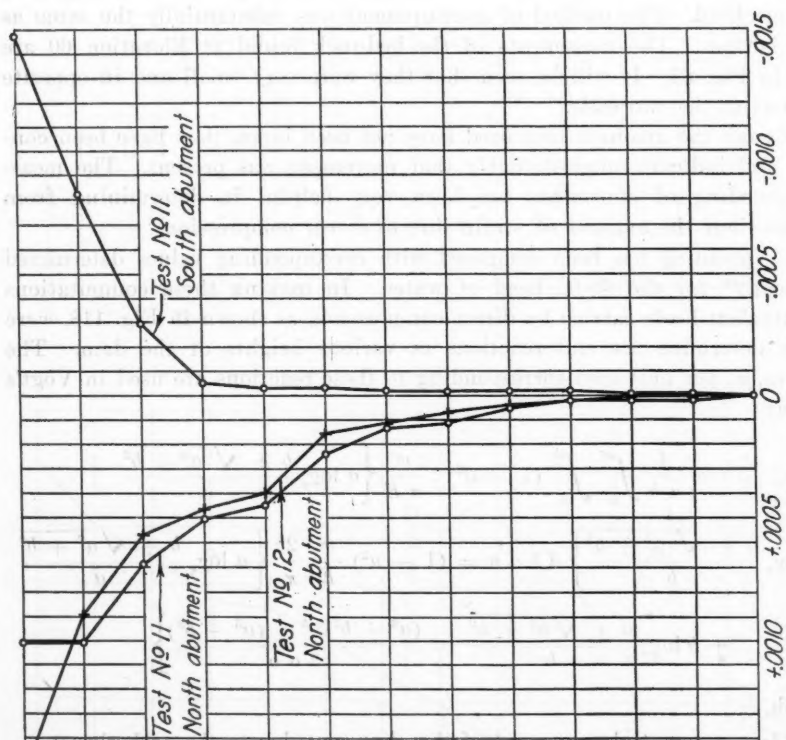


FIG. 81.—MOVEMENT OF BED-ROCK AT ELEVATION 60.

$a =$ width over which the pressure is applied; in this case a is the thickness of the dam.

$b =$ the length of the rectangle of width, a, over which the pressure, w, must be applied in order to give the movement, Δl. The length, b, has been taken as $20a$.

The value of Δl resulting from the use of the formula depends principally on the value of the pressure, w, per unit area and on the modulus of elasticity, E. A considerable variation in the assumed ratio between a and b will have relatively little effect.

The values of modulus of elasticity and Poisson's ratio given here, are based upon the tests of samples of granite taken from the foundation, as described in Sections 52 and 53. This equation for Δl gives only the movement of the bed-rock due to the direct compression. Dr. Vogt's paper includes also an equation for the movement due to the bending moment at the abutment. This movement was computed for the 30-ft. elevation and found to be negligible.

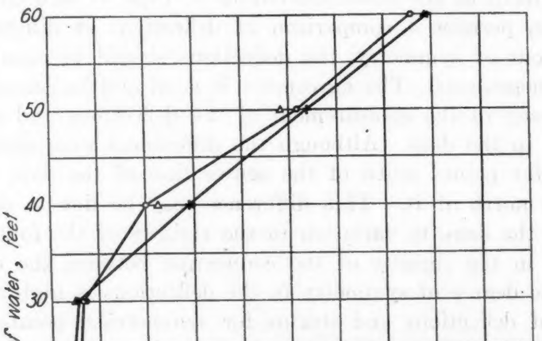


FIG. 83.—MAXIMUM DEFLECTIONS FOR VARIOUS HEADS OF WATER.

The results of the measured and computed movements of bed-rock under a head of water of 60 ft. are shown in Fig. 82. The agreement between the computed and the observed movements is sufficiently good to indicate that Dr. Vogt's analysis is of value in determining the movement of bed-rock.

43.—Deflections Due to Applied Load.—These are shown in Figs. 83 to 91. The maximum deflections, which generally occurred near mid-height of the dam, are shown in Fig. 83 for all tests. The lowest point of maximum

deflection was at Elevation 25 and the highest at Elevation 45. The most prominent feature of this deflection curve is the sharp break at the 30-ft. elevation. As discussed in Section 40, a crack between the dam and the foundation at the center line occurred when the head of water had risen to between 30 and 40 ft.

This fact may appear to explain the sudden break in Fig. 83, but closer examination shows that it is not sufficient. This is clear from the fact that the maximum deflection under a 30-ft. head was practically the same as that for the tests made after the crack occurred, and after the application of a 60-ft. head of water. In Test 13, the deflections were observed under 60, 50, 40, and 30-ft. heads, successively, and the maxima are shown in Fig. 83. If the formation of the crack had been responsible, the break would not have been found for this test, made under decreasing heads. That there is, however, some relation between the width of crack and the amount of the maximum deflection, is indicated by the similarity in the shapes of the curves in Figs. 83 and 76.

The arrangement of the deflection curves in Figs. 84 and 85 for different vertical sections, permits a comparison of deflections at different places at which, for reasons of symmetry, the deflections should be equal if the dam is perfectly homogeneous. The agreement is good and indicates both a fair degree of accuracy in the measurement of the deflections and a fair degree of homogeneity in the dam. Although the differences were slight, generally the deflections for points south of the center line of the dam were greater than for points north of it. This difference may be due to slight lack of homogeneity in the dam, to variation in the rigidity of the foundation rock, or to variation in the rigidity of the connection between the dam and the foundation. The degree of symmetry in the deflections is high and warrants the averaging of deflections and strains for symmetrical points in order to obtain more reliable values. This was done in the reduction of deflections and strains to loads.

In the test of Emigrant Creek Dam at Talent, Ore., made under the direction of D. C. Henny, M. Am. Soc. C. E., a member of the Committee, it was found that there were indications of an up-stream deflection. On account of certain inconsistencies in other parts of the data some doubt remained as to whether the up-stream deflection actually occurred. In the Stevenson Creek Dam test there were slight up-stream deflections near the ends of the dam, first under a head of water of 30 ft. As the head increased, the amount of the up-stream deflection increased, becoming as much as 0.1 in. with a 60-ft. head of water. It is shown in Part VII that the celluloid model of the Stevenson Creek Dam, tested at Princeton University, gave up-stream deflections very similar to those of the actual dam. With this double confirmation there need no longer be any reason for doubting the indication of an up-stream deflection of the Emigrant Creek Dam.

The region of down-stream deflection at the top of the Stevenson Creek Dam generally lay within the middle 50 to 60 ft. of the length of the dam, and the remaining 45 to 40 ft. at either end deflected up stream. The measurements show that up-stream deflections occurred at the 60, 55, 50, and 45-ft.

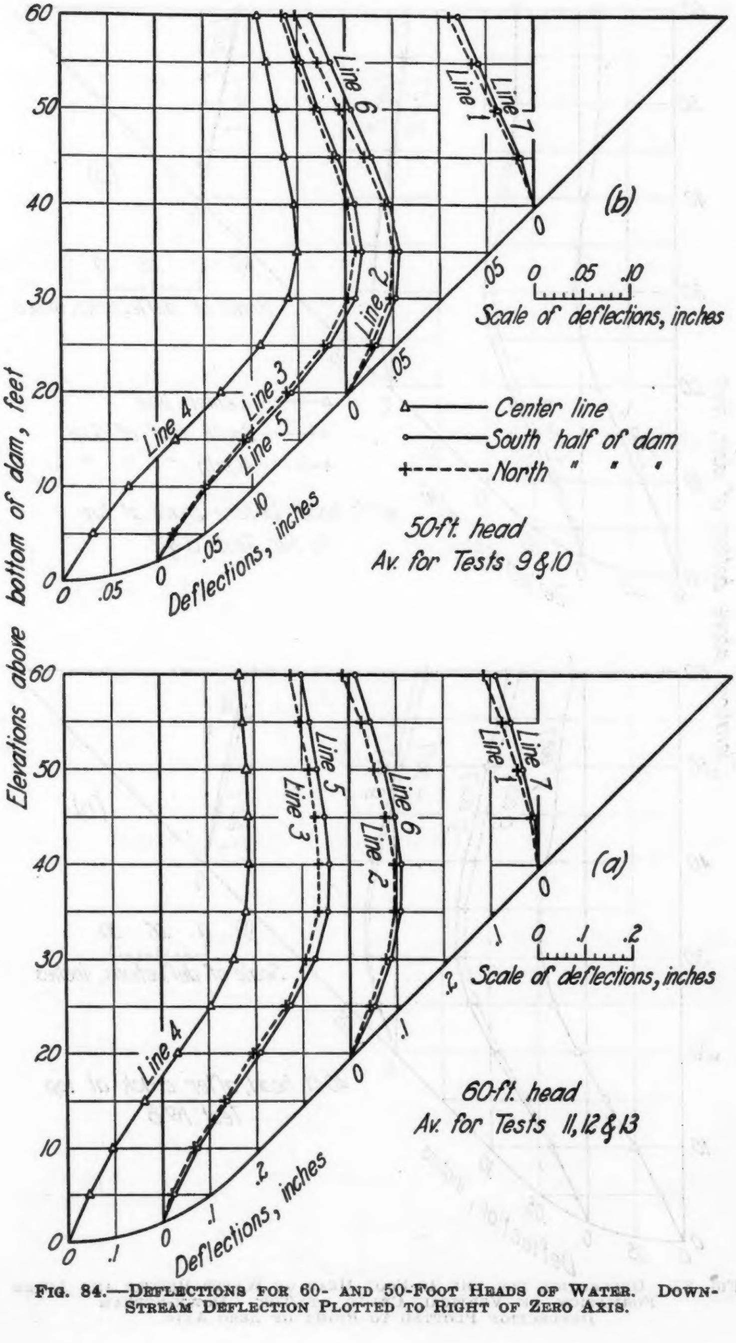


Fig. 84.—Deflections for 60- and 50-Foot Heads of Water. Down-Stream Deflection Plotted to Right of Zero Axis.

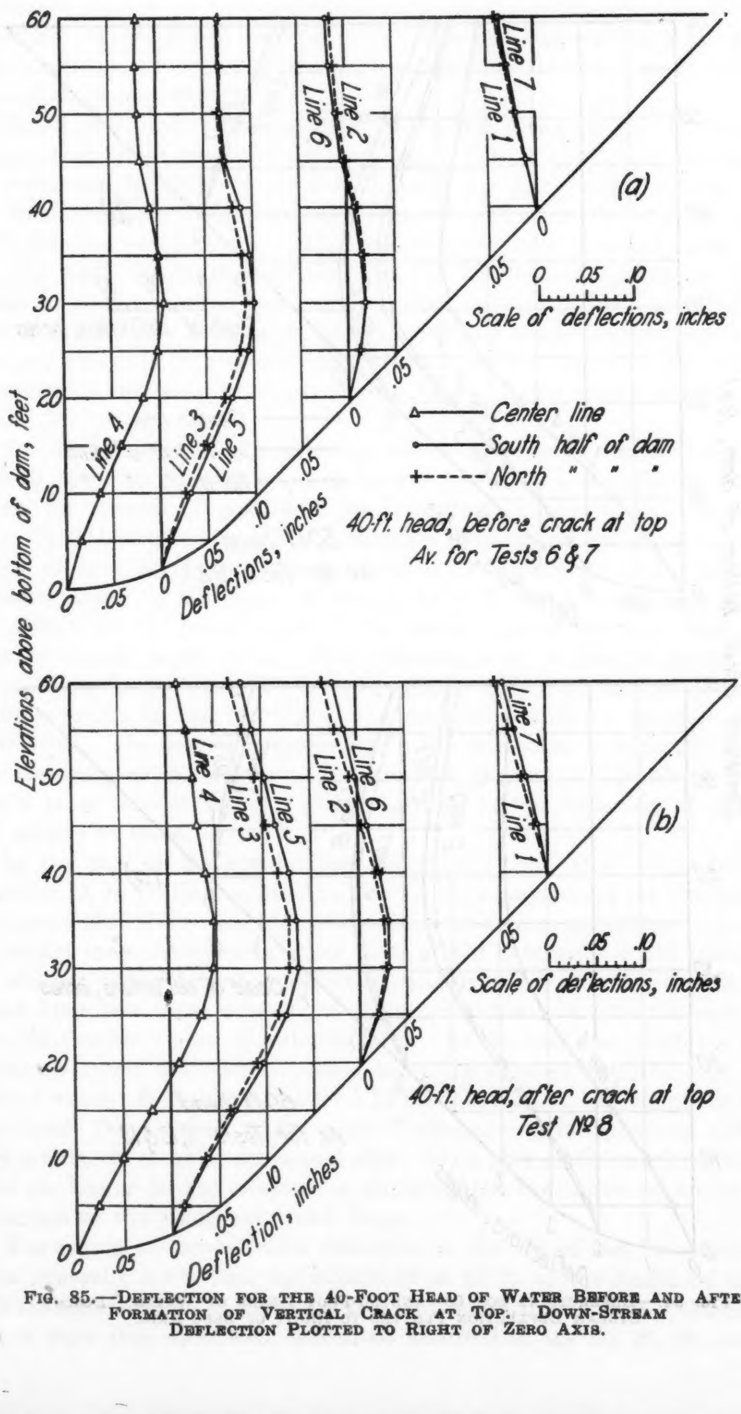


FIG. 85.—DEFLECTION FOR THE 40-FOOT HEAD OF WATER BEFORE AND AFTER FORMATION OF VERTICAL CRACK AT TOP. DOWN-STREAM DEFLECTION PLOTTED TO RIGHT OF ZERO AXIS.

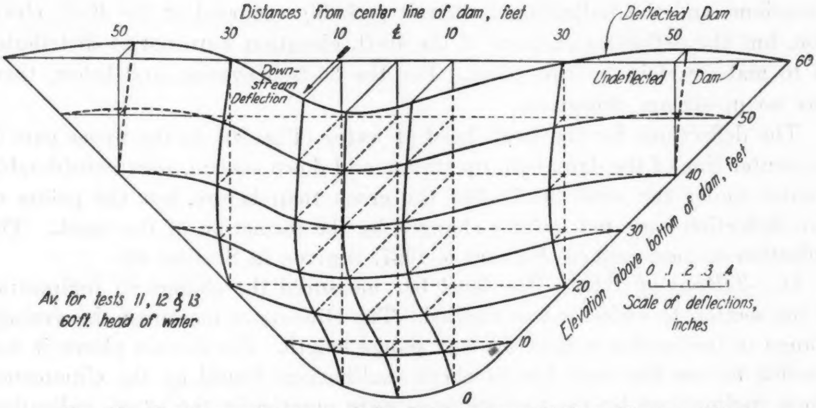


FIG. 86.—ISOMETRIC DIAGRAM OF DEFLECTIONS FOR 60-FOOT HEAD OF WATER.

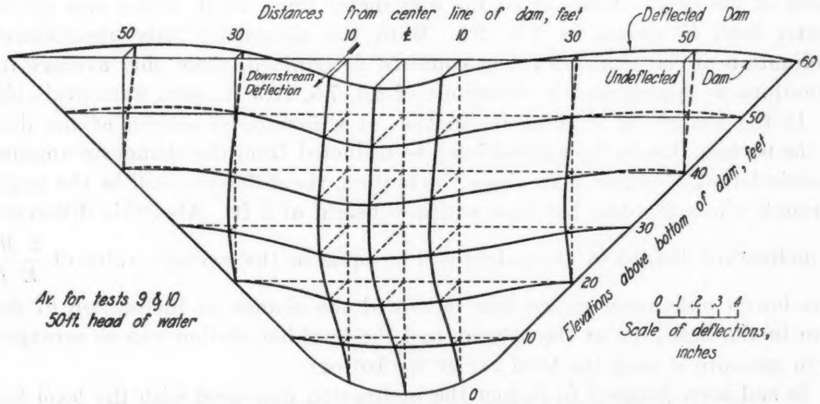


FIG. 87.—ISOMETRIC DIAGRAM OF DEFLECTIONS FOR 50-FOOT HEAD OF WATER.

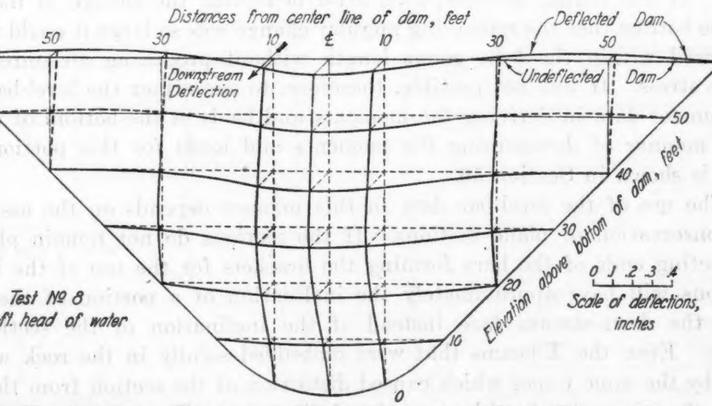


FIG. 88.—ISOMETRIC DIAGRAM OF DEFLECTIONS FOR 40-FOOT HEAD OF WATER AFTER CRACKING.

elevations, and the indication is that it probably occurred at the 40-ft. elevation, but the deflection stations at the 40-ft. elevation were not so distributed as to make certain on this point. For the 30-ft. elevation and below, there was no up-stream deflection.

The deflections for the 40-ft. head of water (Fig. 90) in the upper part of the center line of the dam, both up stream and down stream, were considerably greater under the same head after the crack than before, but the points of zero deflection were not greatly changed by the formation of the crack. The deflection as measured on January 5, 1927, is given in Section 46.

44.—Tilting of Dam.—The level bar measured the change in inclination of the section to which it was applied. The clinometer measured the average change in inclination within the 5-ft. gauge length. For certain places it was possible to use the level bar to check inclinations found by the clinometer. These inclinations by the two methods were practically the same, indicating that both instruments were behaving satisfactorily. The degree of agreement at the 10-ft. elevation on the clinometer lines, 10 ft. either side of the center line, is shown in Fig. 92. With the clinometer only approximate inclination at the 10-ft. elevation could be determined, since only average inclinations at approximately elevations of 2.5, 7.5, 12.5 ft., etc., were available.

If the change in angular inclination of a horizontal section of the dam at the bottom, due to the applied load, be deducted from the change in angular inclination of a section 5 ft. above the bottom, the difference will be the angle through which the dam has bent within a height of 5 ft. Also, this difference in inclination divided by the height will be equal to the average value of $\frac{2\,M}{E\,I}$. The lowest clinometer gauge line measured the change in inclination of the dam in the first 5 ft. at the bottom, and the level-bar station was so arranged as to measure it with the level bar at the bottom.

It had been planned to deduct the inclination measured with the level bar from that measured with the clinometer to determine the bending in the first 5 ft. It was found, however, even after deducting the change in inclination at the bottom that the remaining angular change was so large it could not have occurred within the 5-ft. gauge length without producing an unreasonably large stress. It was not possible, therefore, to use either the level-bar or the clinometer data in deriving the moments and loads at the bottom of the dam. The manner of determining the moments and loads for this portion of the dam is shown in Section 59.

The use of the level-bar data in this manner depends on the assumption of conservation of plane sections. If the sections do not remain plane, the projecting ends of the bars forming the brackets for the use of the level-bar stations will have approximately the inclination of a portion of the section near the down-stream face instead of the inclination of the section as a whole. Even the I-beams that were embedded solidly in the rock would be bent by the same forces which caused distortion of the section from the plane, thus throwing the level-bar results into error. The clinometer brackets extended only about 12 in. into the concrete and where these brackets were

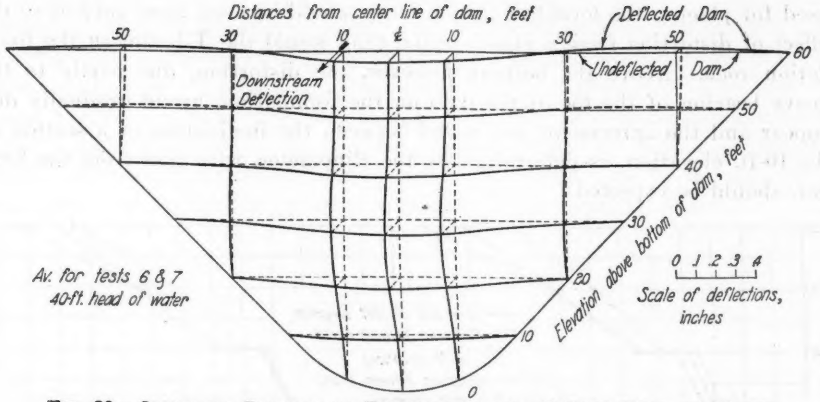


FIG. 89.—ISOMETRIC DIAGRAM OF DEFLECTIONS FOR 40-FOOT HEAD OF WATER BEFORE CRACKING.

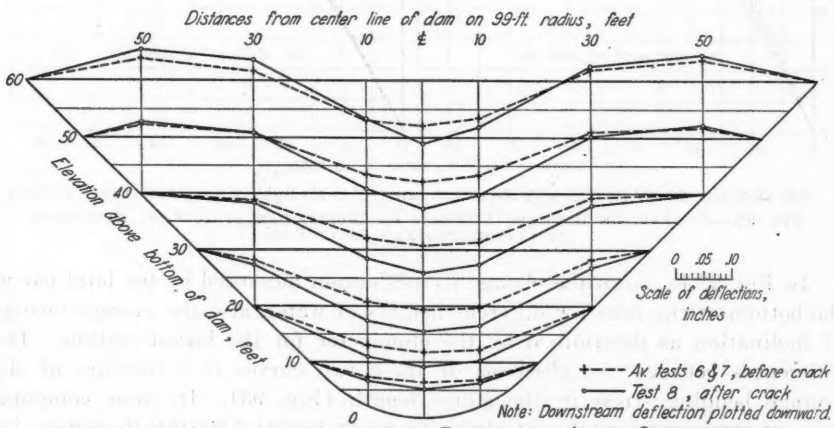


FIG. 90.—COMPARISON OF DEFLECTIONS BEFORE AND AFTER OCCURRENCE OF CRACK. HEAD OF WATER, 40 FEET.

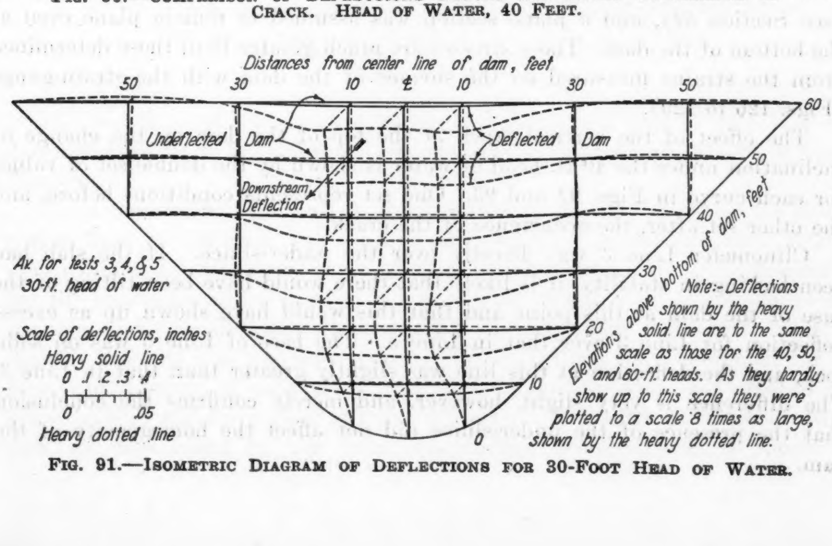
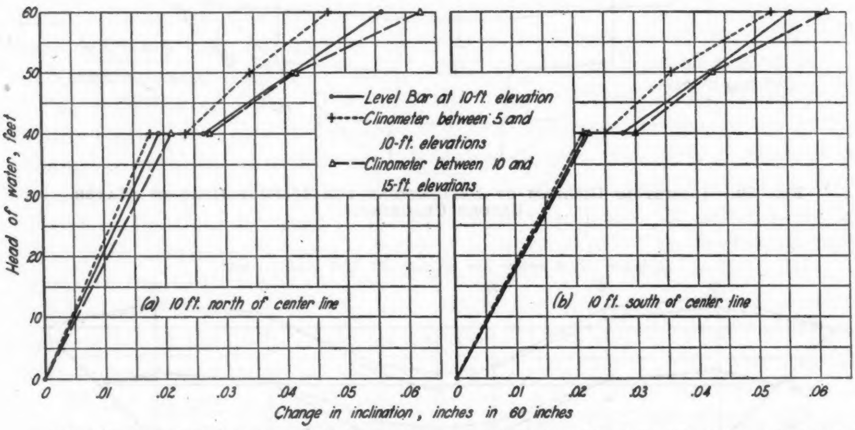


FIG. 91.—ISOMETRIC DIAGRAM OF DEFLECTIONS FOR 30-FOOT HEAD OF WATER.

used for placing the level-bar stations they would be even more subject to the effect of distortion from a plane section than would the I-beams in the foundation rock. Above the bottom, however, the distortion, due partly to the heavy bearing of the toe of the dam on the foundation, would gradually disappear and the agreement, just noted between the inclination of a section at the 10-ft. elevation, as determined by the clinometer, with that from the level bar, should be expected.



Note: Abscissas to right correspond to downstream movement of the higher of 2 points 60 in. apart vertically.
FIG. 92.—COMPARISON BETWEEN CHANGES OF INCLINATION AT 10-FOOT ELEVATION BY CLINOMETER AND LEVEL-BAR.

In Fig. 93 is shown the change in inclination measured by the level bar at the bottom of the dam for different heights of water, also the average change of inclination as determined by the clinometer for the lowest station. The differences between the abscissas of these two curves is a measure of the average bending stress in the gauge length (Fig. 93). In these computations of stress the modulus of elasticity was taken at 3 600 000 lb. per sq. in. (see Section 52), and a plane section was assumed to remain plane even at the bottom of the dam. These stresses are much greater than those determined from the strains measured on the surface of the dam with the strain-gauge (Figs. 126 to 129).

The effect of the vertical crack at the top of the dam on the change of inclination under the 40-ft. head of water is shown by the double set of values for each curve in Figs. 92 and 93. One set represents conditions before, and the other set after, the occurrence of the crack.

Clinometer Line 3 was directly over the under-sluice. If the slab had been lacking in stability, it is likely that there would have been tilting of the base of the dam at this point and that this would have shown up as excess deflection for Line 3 over that in Line 5. The base of Line 5 was on solid rock, and the deflection at this line was slightly greater than that in Line 3. The difference is very slight, however, and merely confirms the conclusion that the presence of the under-sluice did not affect the homogeneity of the dam.

45.—Strains Due to Applied Load.—The lines of equal vertical strain caused by the applied water pressure (Figs. 95, 97, 99, and 101) in general, take a horizontal direction, suggesting some degree of uniformity in the bending moment at a given height in the various vertical elements. The lines of equal horizontal strain (Figs. 94, 96, 98, and 100) show considerable symmetry about the vertical center line. They indicate that the points of greatest strain were at approximately the 25-ft. elevation and close to the abutments and show a marked variation along any horizontal or vertical element.

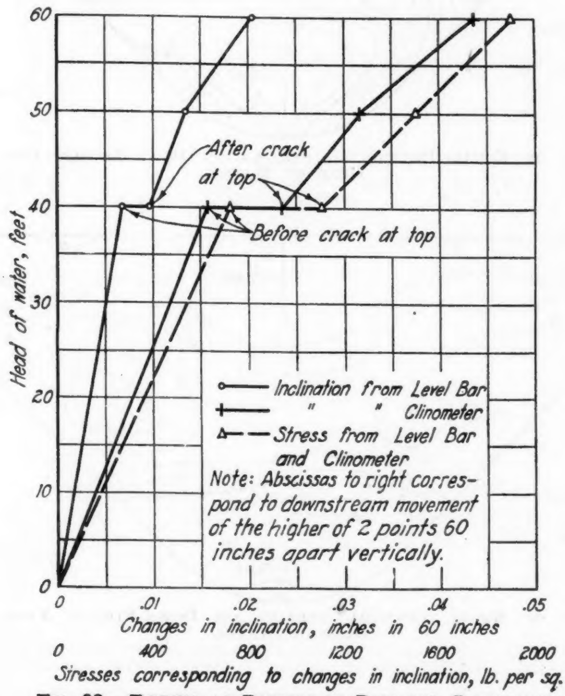


FIG. 93.—TILTING AT BOTTOM OF DAM AND CORRESPONDING STRESSES BASED ON CONSERVATION OF PLANE SECTIONS.

The locations of the vertical cracks on the center line of the dam are shown in Figs. 94 to 99, and their influence on the strains may be seen in these diagrams.

From the strains measured in the four directions, 45° apart on the downstream face of the dam, the amount and direction of the principal strains were computed. Since any three of these strains are sufficient for determining the amount and direction of the maximum strain, four combinations are possible, and four different directions for the maximum strain will result if there is any error in the observations. The directions were computed for all four combinations at each point, and the average direction and amount are given in Fig. 102 for a head of 60 ft. of water. The extreme range of direction, as determined by the four independent computations, is shown in

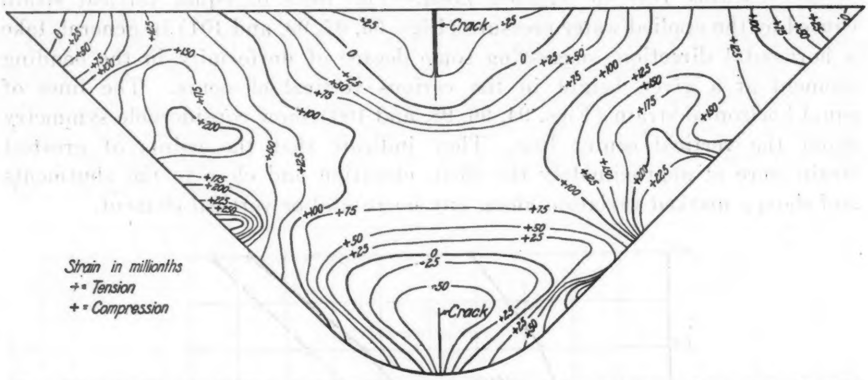


FIG. 94.—LINES OF EQUAL HORIZONTAL STRAINS ON DOWN-STREAM FACE FOR 60-FOOT HEAD OF WATER.

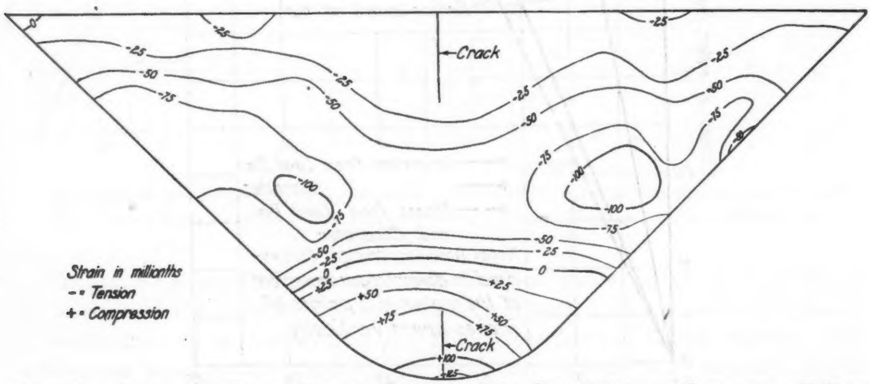


FIG. 95.—LINES OF EQUAL VERTICAL STRAINS ON DOWN-STREAM FACE FOR 60-FOOT HEAD OF WATER.

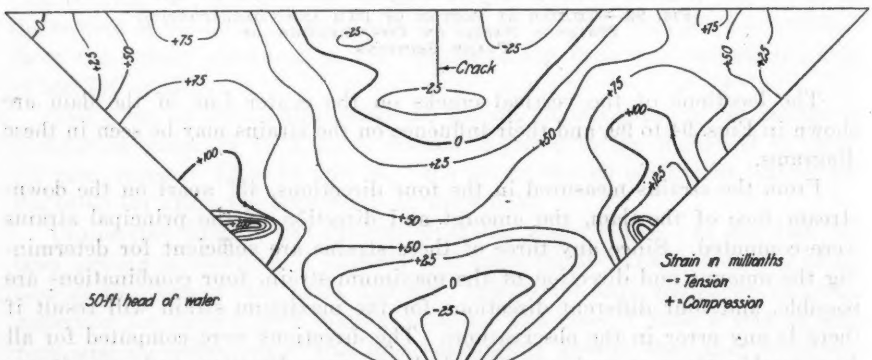


FIG. 96.—LINES OF EQUAL HORIZONTAL STRAINS ON DOWN-STREAM FACE FOR 50-FOOT HEAD OF WATER.

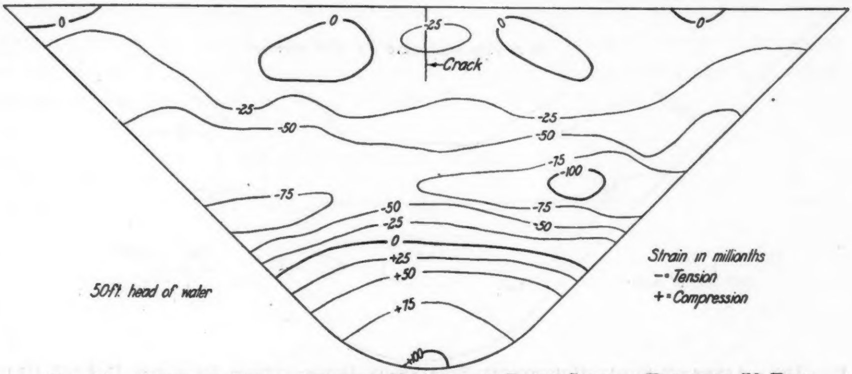


FIG. 97.—LINES OF EQUAL VERTICAL STRAINS ON DOWN-STREAM FACE FOR 50-FOOT HEAD OF WATER.

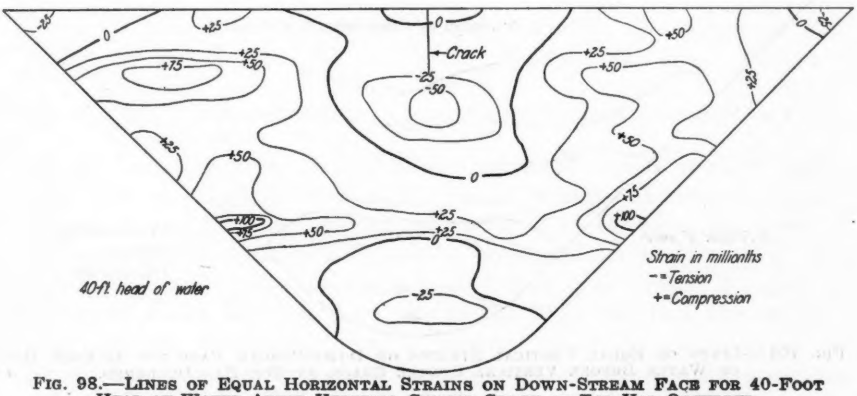


FIG. 98.—LINES OF EQUAL HORIZONTAL STRAINS ON DOWN-STREAM FACE FOR 40-FOOT HEAD OF WATER AFTER VERTICAL CENTER CRACK AT TOP HAD OCCURRED.

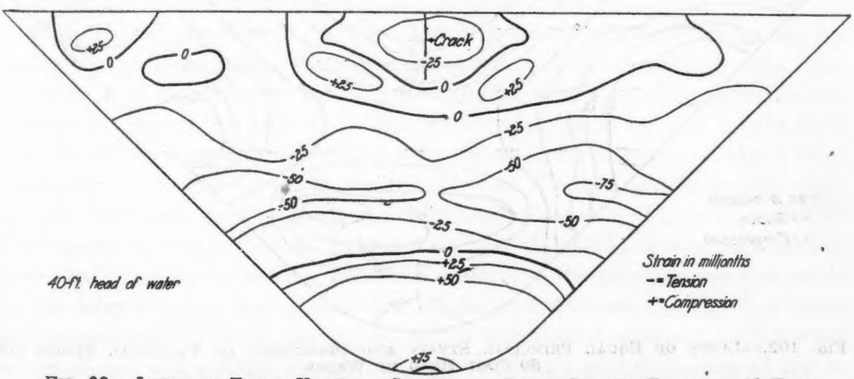


FIG. 99.—LINES OF EQUAL VERTICAL STRAINS ON DOWN-STREAM FACE FOR 40-FOOT HEAD OF WATER AFTER VERTICAL CENTER CRACK AT TOP HAD OCCURRED.

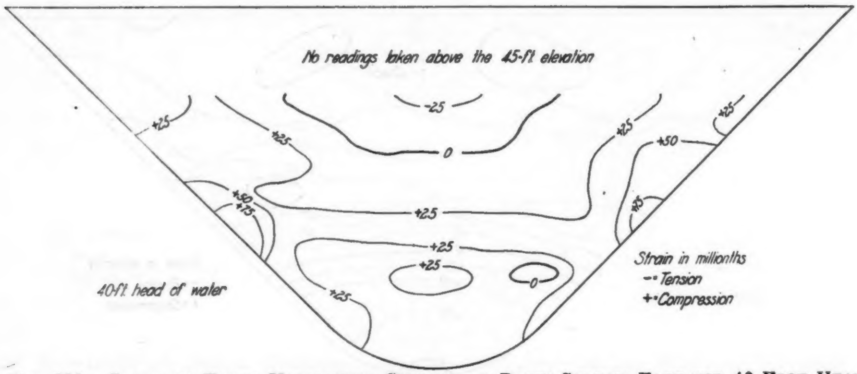


FIG. 100.—LINES OF EQUAL HORIZONTAL STRAINS ON DOWN-STREAM FACE FOR 40-FOOT HEAD OF WATER BEFORE VERTICAL CENTER CRACK AT TOP HAD OCCURRED.

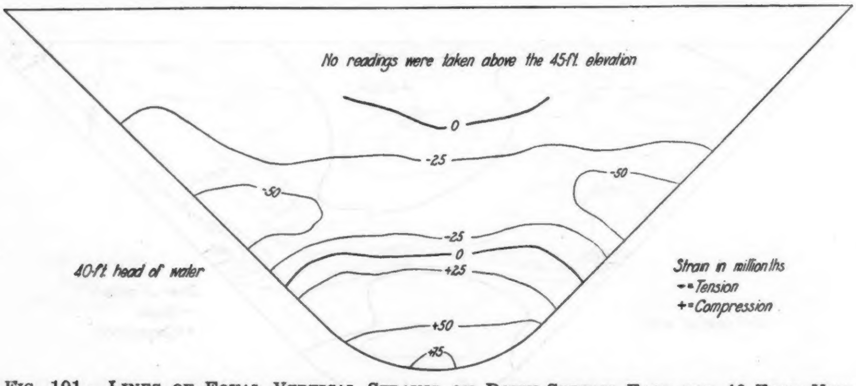


FIG. 101.—LINES OF EQUAL VERTICAL STRAINS ON DOWN-STREAM FACE FOR 40-FOOT HEAD OF WATER BEFORE VERTICAL CENTER CRACK AT TOP HAD OCCURRED.

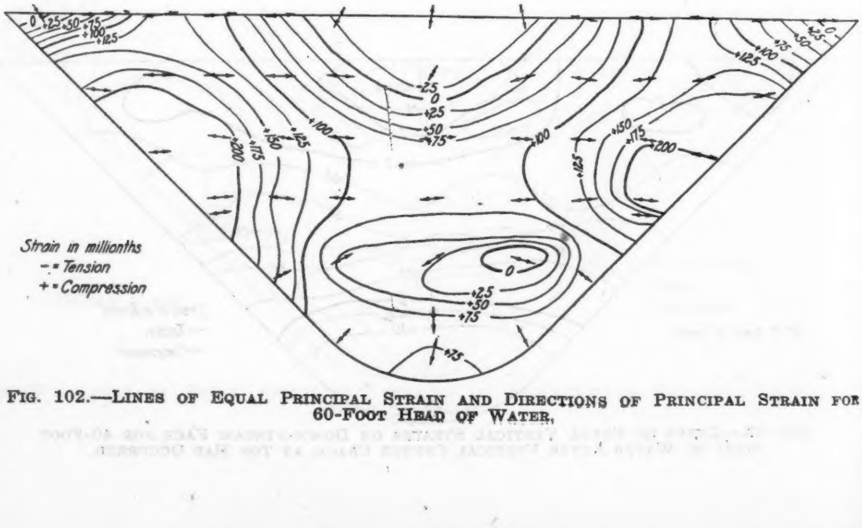


FIG. 102.—LINES OF EQUAL PRINCIPAL STRAIN AND DIRECTIONS OF PRINCIPAL STRAIN FOR 60-FOOT HEAD OF WATER.

Fig. 103 for each point. The points to which the strains apply are the stations shown in Fig. 27, which consist of four gauge lines, *A, B, C,* and *D.* These stations generally are at the elevations that are multiples of 10 ft. The arrow at any point in Fig. 102 shows the average of the directions for the individual values of the larger principal strain found for Tests 11 and 12.

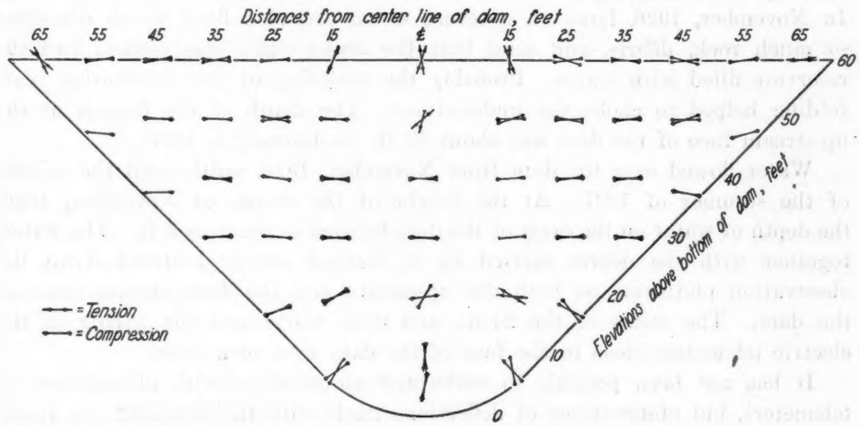


FIG. 103.—EXTREME RANGE IN DIRECTION OF PRINCIPAL STRAIN FOR 60-FOOT HEAD OF WATER.

At any point there are, of course, two principal strains at right angles. For the sake of simplicity in Fig. 102, the directions and amounts of only the larger ones of these are shown. Above the 20-ft. elevation, the direction of the larger principal strain is, in general, horizontal. Therefore, its amount should be nearly the same as the measured horizontal strain. Comparison with Fig. 94 shows that this is so. Below the 20-ft. elevation, the principal strains near the foundation were, in general, nearly perpendicular to the abutment line. At interior points they approach a horizontal direction, the principal exception being on the vertical center line of the dam near the top and bottom. Here vertical cracks have occurred and the larger of the principal strains approach the vertical instead of the horizontal.

Since the larger principal strains generally took a horizontal direction, the smaller must have taken a vertical direction. Thus, it will be seen that the division of the dam into imaginary vertical and horizontal elements for the purpose of analysis, not only served the ends of simplicity, but also for the most part furnished a means of determining the maximum stresses. As a matter of fact, the largest principal strain, shown in Fig. 102, is only 0.0002, while the largest strain in the horizontal direction, shown in Fig. 94, is 0.0003. The reason for this is that the horizontal strain of 0.0003 occurred at a station where only horizontal strains were measured, and the principal strains could not be determined at that point. The strain of 0.0003 corresponds to a stress of approximately 1100 lb. per sq. in. with a modulus of elasticity of 3 600 000 lb. per sq. in., and a Poisson's ratio of 0.15. Adding a probable stress of 100 lb. per sq. in. due to temperature change, as discussed in Section 35, indicates

that about 1 200 lb. per sq. in. was the maximum stress present. Comparison of this stress with the strengths of concrete (see Table 18 and Fig. 11) shows that the concrete was not near failure by compression.

46.—Effect of Flood.—After completion of the program of tests the stoplogs closing the sluice beneath the dam were removed with the expectation that all water entering the reservoir would be drained through this opening. In November, 1926, however, a severe storm caused a flood which deposited so much rock, débris, and sand that the under-sluice was choked and the reservoir filled with water. Probably the wrecking of the observation scaffolding helped to choke the under-sluice. The depth of the deposit at the up-stream face of the dam was about 40 ft. on January 5, 1927.

Water flowed over the dam from November, 1926, until about the middle of the summer of 1927. At the height of the storm, of November, 1926, the depth of water on the crest of the dam became as great as 3 ft. The water, together with the débris carried by it, washed out or battered down the observation platforms on both the up-stream and the down-stream faces of the dam. The stems of the 24-in. and 6-in. valves and the wiring of the electric telemeters close to the face of the dam were torn loose.

It has not been possible to make any observations with clinometers or telemeters, but observations of deflections made with the theodolite on January 5, 1927, on points placed at the crest of the dam, indicate an increase in the down-stream deflection, compared with that for the 60-ft. head on September 22, 1926, of about 0.1 in. at the center line and 0.2 in. at points 30 ft. away from the center line.

An inspection of the dam on January 5, 1927, when about ½ in. of water was flowing over the crest, indicated that the cracks had not extended appreciably in length, nor had the crack in the upper part of the dam increased noticeably in width. The crack near the bottom of the dam, however, was wider and whereas on October 1, 1926, it stopped about 2 in. from the bottom, on January 5 it had extended all the way down. On the whole, it may be said that the dam had been little affected by the severe test given it by this storm.

In February, 1927, another storm visited the vicinity of the dam and the water was again at a high elevation. An inspection of the dam on July 14, 1927, disclosed a crack on the down-stream face, starting at the foundation at an elevation of about 8.5 ft., 18 ft. south of the center line and extending upward and toward the center line to an elevation of about 21 ft. On February 26, 1928, a similar crack was found north of the vertical center line. It is possible that the latter crack was present at the time of the examination in July, 1927. These cracks are indicated in Fig. 104. It is not known what caused them. They may have resulted from a blow from a heavy boulder thrown against the up-stream face by the force of the flood, or they may have been caused by the pressure of the débris with which the reservoir was filled nearly to the top. No other damage to the dam than these cracks and an increased amount of leakage is evident. The profiles of the débris in the reservoir at its line of contact with the up-stream face of the dam as determined on January 5, and again on July 16, 1927, are shown in Fig. 104.

I.—Factors Influencing Reduction of Data

47.—Sources of Error.—In determining the moments from the observed data, the data were reduced, first, to the form of strains, and then to stresses, through the use of a term involving the modulus of elasticity and Poisson's ratio. Sometimes short-cuts have been used in the operations, but these steps are involved directly or indirectly. The moment in a member when the stress is known, is given by the formula, $M = \frac{\sigma I}{z}$, in which, M is the resisting moment in the member, σ is the stress; I is the moment of inertia; and z is the distance from the neutral axis to the surface in which the stress, σ, is known. This formula at best is an approximation, but a very close approximation for the case to which it applies, that is, a straight beam with parallel tension and compression faces. If the faces of the beam are not parallel, or if the axis is curved, this formula does not apply and it becomes important to ascertain how great are the errors introduced by its use in determining the moments from the observed strains.

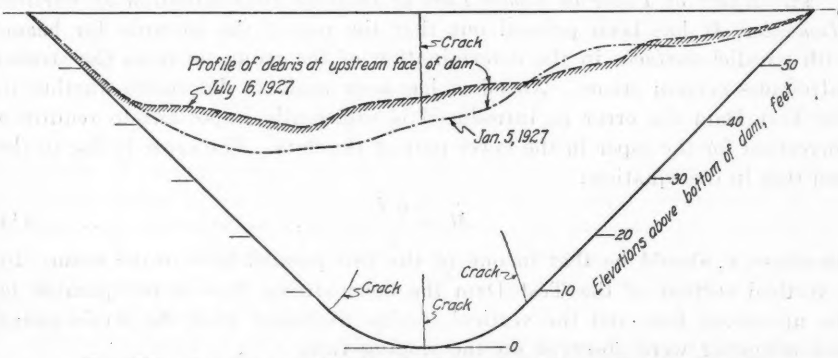


Fig. 104.—Condition of Stevenson Creek Dam After Floods of 1927.

In the determination of moments from the deflection data of the clinometer, differentiations of the deflections are made in the direction in which the moment is to be determined. The deflection, however, includes whatever part is due to shear. Consequently, the resulting moment is in error by the amount due to shear.

In the determination of loads from deflections and strains, differentiations performed along vertical and horizontal elements, will give bending loads which would be carried by these elements if there were no physical contact between them, that is, if the cantilevers were entirely separated fom each other and from the arch ribs. However, because the cantilevers and arch ribs are not free from each other, torsions are set up between adjacent cantilevers and adjacent arch ribs which transfer moment from one element to the next. Neglecting the possibility of this transfer of moment may introduce appreciable errors into the resulting loads. A study of the amount of torsion in the dam indicates, however, that for all places where information is available, the torsion was small and may be neglected in the interpretation of the data. This is discussed in Section 51.

The term involving modulus of elasticity and Poisson's ratio is an essential part of the determination of the stress and the moment at any point. However, it is almost impossible to determine exactly the value of this modulus. Even if there were no chances for error in determining the stress-strain curve, because of the fact that this curve is not a straight line, the modulus is not constant and the assumption that it is introduces some error into the results. It is not practicable to make determinations of stress at all points taking account of the variability of the modulus of elasticity. Moreover, it was found that the results were affected only slightly by the differences in the modulus for different portions of the stress-strain curve. To keep the work within bounds, it was, therefore, expedient to assume a constant modulus of elasticity, and the determination of the most probable value for this modulus is of the greatest importance in the complete interpretation of the test results. Section 52 shows how the modulus may be determined from the test of the dam itself. These comments on the modulus of elasticity apply to Poisson's ratio also.

48.—*Effect of Taper in Lower Part of Dam on Determination of Vertical Moments.*—It has been pointed out that the use of the formula for beams with parallel surfaces in the determination of the moments from the strains introduces certain errors. An effort has been made to determine whether in the Test Dam the error so introduced is sufficiently important to require a correction for the taper in the lower part of the dam. The error is due to the fact that in the equation:

$$M = \frac{\sigma I}{z} \qquad (1)$$

the stress, σ, should be that in one of the two parallel faces of the beam. In a vertical section of the Test Dam the down-stream face is not parallel to the up-stream face and the vertical strains measured with the strain-gauge and telemeter were observed on the sloping face.

The strains determined from the clinometer data, however, are strains normal to the horizontal section of the beam and no correction for the taper need be applied to these data.

It may be shown that with a beam of any homogeneous material in which stress is proportional to strain, the moment on any plane section that remains plane after bending is given by the expression:

$$M = \frac{1}{6}\, \sigma_n\, b\, t^2 \qquad (2)$$

in which, σ_n is the "normal" stress in the extreme fiber, that is, the stress normal to the section considered. However, the maximum stress at the surface must be parallel to the surface and both the shear and the stress at right angles to the surface must be zero.* For any particle under this condition of stress, the stress, σ_a at any angle, α, with the maximum stress, σ_m, is,

$$\sigma_a = \sigma_m \cos^2 \alpha$$

Therefore, if in Equation (2) the normal stress, σ_n makes an angle, α, with the maximum stress, that equation may be written:

$$M = \frac{1}{6}\, \sigma_m\, b\, t^2 \cos^2 \alpha \qquad (3)$$

* "Stresses in Wedge-Shaped Reinforced Concrete Beams," by William Cain, M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. LXXVII (1914), p. 745.

In Professor Cain's paper* the section is taken so that the stress, σ_c, makes an angle, α, with the normal to the section and there results the equation:

$$M = \frac{1}{2}\ \sigma_c\ k\ j\ b\ t^2 \cos^2 \alpha \ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

Equation (4) is for a reinforced beam. For a homogeneous beam, $k = \frac{1}{2}$ and $j = \frac{2}{3}$, and Equation (4) reduces to the form of Equation (3).

There may seem to be inconsistencies in taking the section so that it makes an angle other than 90° with the neutral axis. Hence a different assumption was made using a circular cross-section (Fig. 105) normal to both surfaces of the beam and to the neutral axis, which lies in the center of the section. The moment of the stresses everywhere normal to the circular section was considered. The maximum value of the angle, θ, occurs at the bottom of the dam and is about $\frac{3}{16}$ radian, or about 10 degrees. Neglecting terms involving the third and higher powers of θ, as being very small in proportion to the value of M, the formula becomes:

$$M = \frac{1}{6}\ \frac{\sigma_n\ b\ t^2}{\cos^2 \theta}\left(1 + \frac{\theta^2}{6}\right) \ldots\ldots\ldots\ldots\ldots\ldots\ldots (5)$$

in which, M is the moment at the center of gravity of the circular section; t, the thickness of the dam at the heighe of the down-stream end of the section; σ_n, the bending stress in the extreme fiber normal to the circular section; and θ, one-half the angle between the up-stream face and the tangent to the down-stream face, that is, one-half the angle, α, of Equations (3) and (4). The angle, θ, is in radians.

There is also a direct stress due to the inclination of the section assumed. The moment in terms of the total stress, σ, the sum of bending and direct stresses, is given by the approximate equation:

$$M = \frac{1}{6}\ \sigma\ b\ t^2\ \frac{1}{\left(1 - \frac{7}{6}\ \theta^2\right) + \frac{t}{6\,y}\left(\theta - \frac{2}{3}\ \theta^3\right)} \ldots\ldots (6)$$

The y of Equation (6) is the vertical distance from the section under consideration to the point of application of the resultant force. It was determined by dividing the average moments found in Tests 11, 12, and 13 at the various heights by the shears found from the same tests at the same heights.

The coefficient, $\cos^2 \alpha$, of Equation (3) and the coefficient,

$$\frac{1}{\left(1 - \frac{7}{6}\ \theta^2\right) + \frac{t}{6\,y}\left(\theta - \frac{2}{3}\ \theta^3\right)},$$

of Equation (6) have been computed (Table 21).

* "Stresses in Wedge-Shaped Reinforced Concrete Beams," by William Cain, M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. LXXVII (1914), p. 745.

TABLE 21.—COEFFICIENTS FOR CORRECTION OF MOMENTS DETERMINED FROM MEASURED STRAINS.

Elevation, in feet.	Angle, a, in radians.	Angle, a, in degrees and minutes.	$\cos^2 2\theta = \cos^2 a$	$\dfrac{1}{\left(1 - \frac{7}{6}\theta^2\right) + \frac{t}{6y}\left(\theta - \frac{2}{3}\theta^3\right)}$
30	0	0 = 0	1.00000	1.00000
25	0.0295	1 = 41.3	0.99652	1.00177
20	0.0588	3 = 22.1	0.96625	1.00404
15	0.0892	5 = 06.5	0.96851	0.99960
10	0.1194	5 = 50.6	0.94403	1.00503
5	0.1502	8 = 36.2	0.91248	1.01174
0	0.1763	10 = 06.6	0.88052	....

At the 5-ft. elevation of the dam, the angle, θ, is approximately 0.15 radian. For this elevation, Equation (3) as corrected gives a value for the moment about 9% less than that obtained by neglecting the effect of the taper. Equation (6) as corrected gives a value only about 1% less than the uncorrected value. However, the center of gravity for the section to

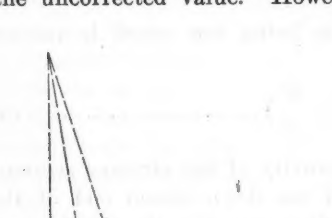


FIG. 105.—CURVED SECTION USED IN DETERMINING EFFECT OF TAPER.

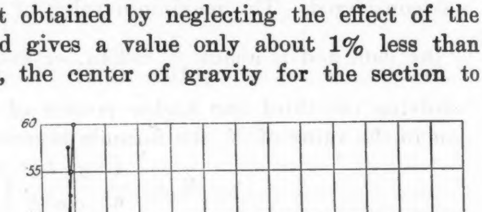


FIG. 106.—MOMENT AND SHEAR FOR VERTICAL CENTER LINE UNDER 60-FOOT HEAD. MOMENTS CORRECTED FOR TAPER OF VERTICAL ELEMENT.

which the moment of Equation (6) applies, is lower than the section to which Equation (3) applies (Fig. 105). The moments determined from the strains by means of Equation (2), without correction for the taper, are shown as a solid line in Fig. 106. The corrected moments, as given by Equation (3), are

indicated by triangles. The corrected moments for Equation (6) are indicated by plus signs. It will be seen that the dotted line fits both sets of corrected moments equally well. The indication, therefore, is that the correction of the moments found by Equation (6) is equivalent, for this case at least, to the similar corrections given in Equation (3). While the corrections for Equation (3) are not large, they are easily made and have been applied to the moments determined from the strain-gauge data (Fig. 106).

49.—Effect of Curvature of Dam on Determination of Horizontal Moments.—The ordinary analysis of stresses and moments in a beam assumes that a plane section before bending remains plane during bending, so that the stress is proportional to the strain. Various analyses have been made on the stresses and moments in a curved beam. For the purpose of this examination, Slocum and Hancock's analysis* is used. This analysis assumes for the curved, as well as for the straight member, that plane sections remain plane and that stress is proportional to strain. However, since the curvature of the section makes fibers on the concave side of the neutral axis shorter than those on the convex side, any given stress, although creating the same strain at all points, will cause a smaller change of length in a given element on the inner side of the curve than on the outer side. Consequently, the neutral axis will be nearer the inside of the curved beam than the outside.

The resulting formulas, when applied to a curved beam of homogeneous material with a rectangular section, may be reduced to the following approximate form:

$$M = \frac{1}{6} \sigma b t^2 \frac{\frac{1}{2} \frac{t}{R} - 1}{\frac{1}{6} \frac{t}{R} \mp 1} \quad \text{(7)}$$

in which, R is the mean radius.

Evidently, $\dfrac{\frac{1}{2} \frac{t}{R} - 1}{\frac{1}{6} \frac{t}{R} \mp 1}$ is the coefficient by which the moment for a straight beam must be multiplied to obtain the moment for a curved beam giving the same extreme fiber stress.

The minus sign in the denominator of Equation (7) applies to the extrados, and the plus sign, to the intrados (in the test the strains were measured on the intrados). The maximum value of $\dfrac{t}{R}$ for the Test Dam (at the bottom) is about 0.077. Using this value with the plus sign in the equation:

$$M = 0.95 \frac{1}{6} \sigma b t^2 \quad \text{(8)}$$

This indicates an error of about 5% at the bottom of the dam, due to neglect of the curvature in determining the moments from the measured strains. At the 10-ft. elevation the corresponding error is about 3 per cent. Evidently,

* "Strength of Materials," Second Edition, p. 191.

this is too small to be considered in view of much larger uncertainties below the 10-ft. level (see Section 60) and in view of the fact that the error is much smaller than this for all other positions. At the 30-ft. level, and higher, the error due to the use of the formula for straight beams is about 1 per cent.

50.—Effect on Stress Determinations of Neglecting Deflection Due to Shear.—The tensile and compressive strains have been determined from the observed deflections on the assumption that all the deflection was due to bending. If some of the deflection was due to shear, then the strains found are larger than the true values by an amount equal to the bending strain which would be computed from the deflection due to shear. A possible method for approximating the errors introduced by the inclusion of the shearing deflection with the bending deflection may be outlined as follows:

(1) From the observed deflection curve, determine the moment and shear curves, assuming the deflection to be due entirely to moment.

(2) From the shear curve so obtained, determine the deflection due to this shear.

(3) Deduct this shear deflection from the total deflection, obtaining the corrected deflection curve.

(4) Re-determine the shears from the corrected deflection, and if they differ from the values obtained from the total deflections by an appreciable amount, determine a new shear deflection curve.

(5) When a shear deflection curve has been sufficiently established, assume temporarily that these are bending deflections, and deduce from it by double differentiation the corresponding moments and strains. These moments and strains will be the corrections to be applied to the moments derived on the assumption that the deflection was due entirely to bending.

These operations, which have been outlined for clearness, may be materially shortened due to the fact that, whereas the shear curve is integrated once in obtaining the shear deflection and this curve then is differentiated twice to obtain the moment, one differentiation of the shear curve gives the same result immediately.

The shearing deflection, $d\,z$, within a height, $d\,y$, may be expressed as:

$$d\,z = \frac{V}{A}\frac{d\,y}{G} \qquad (9)$$

in which,

V = the total shear on the section.
A = the area of the section.
G = the shearing modulus of elasticity.
y = distance upward from the bottom of the dam.

$$\frac{d\,z}{d\,y} = \frac{V}{A\,G} \qquad (10)$$

Since $V = \dfrac{d\,M}{d\,y}$,

$$\frac{d^2\,z}{d\,y^2} = \frac{d}{d\,y}\left(\frac{V}{A\,G}\right) = \frac{d}{d\,y}\left(\frac{1}{A\,G}\cdot\frac{d\,M}{d\,y}\right) \qquad (11)$$

Expressing the derivative indicated in Equation (11):

$$\frac{d^2 z}{d y^2} = \frac{A\ G\ \dfrac{d^2 M}{d y^2} - \dfrac{d M}{d y} \cdot \dfrac{d\ A\ G}{d y}}{A^2\ G^2} \qquad (12)$$

From general relations,

$$\frac{d^2 z}{d y^2} = \frac{M'}{E\ I} = \frac{2\ \sigma'\ I}{E\ I\ t} = \frac{2\ \sigma'}{E\ t} \qquad (13)$$

in which, M' is the moment which would cause a deflection equal to the shearing deflection; σ' is the corresponding flexural stress in the extreme fiber; and t is the thickness of the dam. If a section of unit thickness be considered, A of Equation (12) becomes t. Therefore, from Equations (12) and (13):

$$\frac{2\ \sigma'}{E\ t} = \frac{t\ \dfrac{d^2 M}{d y^2} - \dfrac{d M}{d y} \cdot \dfrac{d\ t}{d y}}{t^2\ G} \qquad (14)$$

$$\sigma' = \frac{E}{2\ G}\left(\frac{d^2 M}{d y^2} - \frac{1}{t} \cdot \frac{d M}{d y} \cdot \frac{d t}{d y}\right) \qquad (15)$$

The significance of the various terms in Equation (15) will be of interest, in estimating the magnitude of the apparent stress, σ', since σ' is a correction which if applied to the bending stress computed from the measured deflections will give the true bending stress.

It is shown in mechanics that $\dfrac{E}{2\ G} = 1 + \mu$. Since μ for this concrete is about 0.15, $\dfrac{E}{2\ G}$ is about 1.15. The term, $\dfrac{d^2 M}{d y^2}$, is the load carried by bending. The magnitude and sign of the load, for vertical elements, are given at the extreme right in Figs. 126, 127, 128, and 129. The largest possible value is 3 750 lb. per sq. ft., or 26 lb. per sq. in. This is at the bottom of the dam, and from there upward the value decreases rapidly. At an elevation of 5 ft., this load is 11.9 lb. per sq. in.

The term, $\dfrac{1}{t}\ \dfrac{d M}{d y}$, is the average shearing stress at the section considered. The shearing stress may be obtained by dividing by $144t$ the total shear per foot of width given in Fig. 126. The term, $\dfrac{d\ t}{d\ y}$, is 0 within the upper 30 ft. of the dam where the thickness is constant. From the bottom to the 30-ft. elevation the profile of the down-stream face of the dam is an arc of a circle the radius of which is 84.57 ft., and this term becomes,

$$\frac{d\ t}{d\ y} = -\frac{30 - y}{\sqrt{84.57^2 - (30 - y)^2}}$$

At the 5-ft. elevation the term, $\dfrac{1}{t}\ \dfrac{d\ M}{d\ y}\ \dfrac{d\ t}{d\ y}$, is approximately -1.0 lb. per sq. in. Substituting these values of $\dfrac{E}{2\ G}$, $\dfrac{d^2 M}{d\ y^2}$, and $\dfrac{1}{t}\ \dfrac{d\ M}{d\ y}\ \dfrac{d\ t}{d\ y}$ in Equation (15)

the apparent stress, σ', at the 5-ft. elevation becomes $-1.15\,(11.9 - 1.0) = -12.5$ lb. per sq. in. The stress at this elevation found by differentiation of deflections without regard for error due to shearing deflections, is about -255 lb. per sq. in. The error due to neglecting the shearing deflection is, therefore, about 5% at this point. The uncertainty surrounding the interpretation of results at the bottom of the dam is so much greater than this that it is not profitable to undertake to make corrections for this error. At higher elevations where there is less uncertainty about interpretation of results the error due to neglect of shearing deflections is much smaller and, therefore, it is still not worth while to make the corrections. Fig. 107 shows the apparent bending stresses for the vertical center line of the dam for a head of 60 ft. of water against the dam.

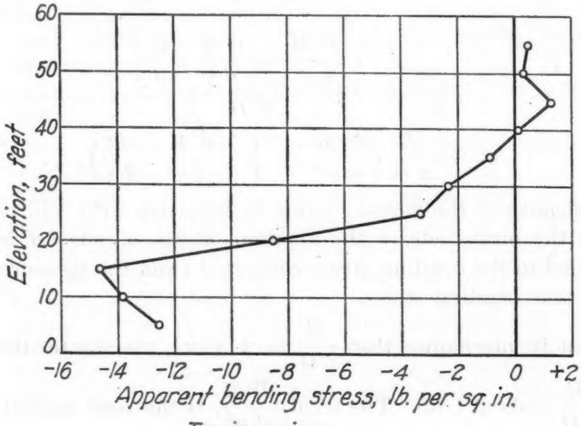


FIG. 107.—INFLUENCE OF SHEAR ON DETERMINATION OF BENDING STRESSES.

Equation (15) may be applied to a horizontal element by substituting x for y in that equation. Here, $\frac{d\,t}{d\,x} = 0$, since for a given elevation the thickness, t, is constant. Equation (15) then becomes:

$$\sigma' = \frac{E}{2\,G}\,\frac{d^2\,M}{d\,x^2} \qquad (16)$$

In this case, $\frac{d^2\,M}{d\,x^2}$ is the equivalent bending load in a horizontal element. Its maximum value, as given in Fig. 124, is -500 lb. per sq. ft., or -3.5 lb. per sq. in., at an elevation of 40 ft. under a head of 60 ft. Substituting this value in Equation (16), the greatest error in the stresses in the horizontal elements, due to the inclusion of shearing deflections as bending deflections, is about $\sigma' = -1.15 \times 3.5 = -4.0$ lb. per sq. in.

51.—Torsion.—In Equation (23) appears the term, $2\,\frac{\delta^4\,z}{\delta\,x^2\,\delta\,y^2}\,\frac{E\,I^*}{1-\mu^2}$. This term enters because of the torsion, but it is not a load carried by torsion,

*The symbol, δ, is used in this report instead of ∂, which is preferred by mathematicians.

although its form might so indicate. The load, w_x, carried in the x-direction is the first derivative of the shear, V_x, for that direction. The shear, however, has a term involving the derivative of the torsional moment, M_z, that is,

$$V_x = \frac{\delta M_x}{\delta x} + \frac{\delta M_z}{\delta y}\text{*} \qquad (17)$$

and, therefore,

$$w_x = \frac{\delta^2 M_x}{\delta x^2} + \frac{\delta^2 M_z}{\delta x \, \delta y} \qquad (18)$$

The torsional moment, M_z, is,

$$M_z = \frac{\delta^2 z}{\delta x \, \delta y} \frac{E I}{1 - \mu^2} \qquad (19)$$

from which,

$$\frac{\delta^2 M_z}{\delta x \, \delta y} = \frac{\delta^4 z}{\delta x^2 \, \delta y^2} \frac{E I}{1 - \mu^2} \qquad (20)$$

Equation (18) shows how the expression involving torsional moment comes into the equation for the load, and Equation (20) shows how this portion of the load is related to the deflections. The same expression as that of Equation (20) enters the equation for load in the y-direction also, and since the total load is the sum of the x-load and the y-load the torsion term in Equation (23) is twice the expression given in Equation (20).

Just as the loads carried by the horizontal element have been separated into parts called "equivalent load carried by direct compression", and "equivalent load carried by bending," this term may be called "equivalent load carried by torsion". It is, in fact, a correction, half of which should be applied to the load carried in the horizontal direction and half to the load carried in the vertical direction in order to obtain the total load, w.

The form of the expression for the equivalent torsional load shows that this load may be determined by successive differentiations of the deflection, z, times EI. As long as the moment of inertia and the modulus of elasticity are constant, the order in which the differentiations with respect to x and y take place is immaterial. However, below the 30-ft. elevation the thickness of the dam varies with the elevation, y, and, therefore, the coefficient, EI, is not constant. Hence in this portion of the dam the differentiations must be performed alternately with respect to x and y. The equivalent torsional load determined by such differentiations of the deflections are given in Table 22, under the caption of Method z. Because of the great horizontal distance between points at which the deflections were measured, it was necessary to interpolate between the measured deflections to obtain deflections at closer intervals. The errors entering into such operations are large and the resulting torsional loads are correspondingly inaccurate. It will be seen, however, that the loads so determined are not large.

* "Moments and Stresses in Slabs," by H. M. Westergaard and W. A. Slater, Members. Am. Soc. C. E., *Proceedings*, Am. Concrete Inst., Vol. 17, p. 426 (1921).

The moments in the vertical and the horizontal directions may also be used to determine the torsional moments and the equivalent torsional loads. Since,

$$\frac{E\,I}{1-\mu^2}\,\frac{\delta^2 z}{\delta y^2} = M_y$$

the equivalent torsional load may be written:

$$\frac{E\,I}{1-\mu^2}\,\frac{\delta^4 z}{\delta x^2\,\delta y^2} = \frac{\delta^2}{\delta x^2}\left(\frac{E\,I}{1-\mu^2}\,\frac{\delta^2 z}{\delta y^2}\right) = \frac{\delta^2 M_y}{\delta x^2}$$

The derivatives, in the x-direction, of the bending moments, M_y, have been determined for the head of 60 ft. and the results are shown in Table 22, as equivalent torsional loads, under the caption of Method M_y. The moments, M_y, were determined from the strains measured with the strain-gauge and since the strain-gauge measurements were taken at closer intervals in the horizontal direction than the deflections, a correspondingly larger number of points were available for determination of the equivalent torsional loads by this method than from the deflections. As previously, the loads so determined are quite erratic, but again, also, none of these loads is large.

In the same manner the moments in the horizontal direction would give the equivalent torsional loads. Due to the possibility of some confusion between bending and direct strains in the horizontal direction, there is greater chance for error in this case than in the use of the vertical moments. Hence the horizontal moments have not been used.

The simplest and most direct source of information on the equivalent loads carried by torsion is found in the results of the strain-gauge measurements taken in the diagonal directions. The torsional moment in a horizontal plane is* equal to one-half the difference of the bending moments in the two directions at 45° with the horizontal. The strains measured in the 45° directions contain components of the direct thrust in the horizontal direction, and, therefore, the total stress at 45° multiplied by the arm of the resisting couple would not give correctly the moment in that direction. However, since the moment desired is the difference between the moments in the two 45° directions, and since the component of the thrust in one of these directions is the same as that in the other, the moment of the thrust will be eliminated completely in the difference of these moments. The torsional moments have been determined in this manner from the bending moments in the two 45° directions and differentiation with respect to the x-axis and with respect to the y-axis, successively, has yielded the equivalent loads carried by torsion, shown in Table 22, under the caption of Method M_{xy}.

The equivalent torsional loads determined by the three methods outlined, and their averages are given in Table 22, also the ratio of the average torsional load to the total water pressure for the points considered.

It will be seen from Table 22 that by all the methods the equivalent torsional loads are small. In determining loads due to bending, it is believed that the error involved may be as large as ± 100 lb. per sq. ft. in most

* "Moments and Stresses in Slabs," by H M. Westergaard and W. A. Slater, Members, Am. Soc. C. E., *Proceedings*, Am. Concrete Inst., Vol. 17, p. 426 (1921).

cases; only a few values of the equivalent torsional loads are larger than this. The error of the equivalent torsional load in pounds per square foot is of more significance than the error in terms of percentage of the total load. For example, close to the top of the dam a very small error in pounds per square foot would be a very large percentage of the total pressure at the point considered and yet it would not be of great significance. However, it is of some interest to know the ratio of the equivalent torsional load to the total water pressure; generally, these ratios (Table 22) are small. Only a few large percentages occur and these are near the top of the dam where the total water pressure is small.

TABLE 22.—EQUIVALENT TORSIONAL LOAD.*

Elevation, y, in feet.	Method.	Horizontal Distance from Vertical Center Line, in Feet.										
		0	7.5	10	15	20	25	30	35	40	45	50
55	z	0	....	+6	....	+20	....	+6	....	−17	....	....
	M_y	....	....	−13	−16	....	−32	+24	+20	....	0	−144
	M_{xy}	....	−23	....	....	−36	....	−36	....	+54	....	−144
	Ave.	0	−23	−4	−16	−8	−32	−2	+20	+18	0	−144
	Ave. ÷ Total =	....	−0.07	−0.01	−0.05	−0.03	−0.01	−0.01	+0.06	+0.06	0	+0.46
50	z	0	....	+6	....	+20	....	+6	....	−17	....	....
	M_y	−12	....	−1	−27	....	−60	−158	−28	....	+63	....
	M_{xy}	....	....	....	....	....	....	....	....	....	....	....
	Ave.	−6	....	+3	−27	+20	−60	−76	−28	−17	+63	....
	Ave ÷ Total =	+0.01	....	−0.05	−0.04	+0.03	−0.10	−0.11	−0.04	−0.03	−0.10	....
45	z	....	....	....	....	....	....	....	....	....	....	....
	M_y	0	....	+5	−12	....	+76	−192	+104	....	+27	....
	M_{xy}	0	+34	....	....	+36	....	−36	....	....	....	....
	Ave.	0	+34	+5	−12	+36	+76	−114	+104	....	+27	....
	Ave. ÷ Total =	0	+0.04	+0.01	−0.01	+0.04	+0.08	−0.12	+0.11	....	+0.03	....
40	z	0	....	+32	....	−37	....	−35	....	....	....	....
	M_y	−8	....	−5	−8	....	+48	−112	+79	....	....	....
	M_{xy}	....	....	....	....	....	....	....	....	....	....	....
	Ave.	−4	....	+14	−8	−37	+48	−74	+79	....	....	....
	Ave. ÷ Total =	−0.00	....	+0.01	−0.01	−0.03	+0.04	−0.06	+0.06	....	....	....
35	z	....	....	....	....	....	....	....	....	....	....	....
	M_y	....	....	−32	−16	....	−56	−128	....	....	....	....
	M_{xy}	....	+12	....	....	−72	....	+90	....	....	....	....
	Ave.	....	+12	−32	−16	−72	−56	−19	....	....	....	....
	Ave. ÷ Total =	....	+0.01	−0.02	−0.01	−0.05	−0.04	−0.10	....	....	....	....
30	z	−12	....	....	....	....	....	....	....	....	....	....
	M_y	−12	....	0	0	....	−88	−24	....	....	....	....
	M_{xy}	....	....	....	....	....	....	....	....	....	....	....
	Ave.	−12	....	0	0	....	−88	−24	....	....	....	....
	Ave. ÷ Total =	−0.01	....	....	....	....	−0.05	−0.01	....	....	....	....
25	z	....	....	....	....	....	....	....	....	....	....	....
	M_y	....	....	−1	0	+144	....	....	....	....	....	....
	M_{xy}	....	+46	....	....	....	....	....	....	....	....	....
	Ave.	....	+46	−1	0	+144	....	....	....	....	....	....
	Ave. ÷ Total =	....	+0.02	....	....	+0.06	....	....	....	....	....	....
20	z	....	....	....	....	....	....	....	....	....	....	....
	M_y	−60	....	−16	−88	....	....	....	....	....	....	....
	M_{xy}	....	....	....	....	....	....	....	....	....	....	....
	Ave.	−60	....	−16	−88	....	....	....	....	....	....	....
	Ave. ÷ Total =	−0.02	....	−0.01	−0.03	....	....	....	....	....	....	....

* Loads are given in pounds per square foot for three methods, also the average and the ratio of the average to the total pressure (62.5 $(60 - y)$) per unit of area.

Considering the smallness of the equivalent torsional load, and the difficulty of obtaining accurate values for it, this load has been neglected in the study of the data of the test (Chapters J to N).

52.—Modulus of Elasticity, Concrete and Granite.—During the construction of the dam, control specimens were made at frequent intervals for the

purpose of determining the modulus of elasticity of the concrete. Generally these were 6 by 12-in. cylinders. In addition, however, a beam, 12 in. square and about 8 ft. long, was made for determining the modulus of elasticity. The beam was tested by measuring the deflection under a known load and computing the modulus of elasticity from the relation between modulus, load, and deflection. Most of the control cylinders used for determining the modulus of elasticity were sent to the University of California for test; however, telemeters were embedded in four of the cylinders and these were tested at the dam site.

The testing of the cylinders yielded a variety of results. The one thing common to all the tests was the shape of the stress-strain curve. In the effort to make the most of these tests, the most probable values of the strain were determined from all cylinder tests, including those with telemeters embedded. A most probable stress-strain curve was thus obtained (Fig. 108), giving an initial modulus of elasticity of 3 800 000 lb. per sq. in.; a secant modulus of 2 700 000 lb. per sq. in. at the maximum stress of 900 lb. per sq. in.; and a secant modulus of 3 000 000 lb. per sq. in. at a strain of 0.0002. This latter strain represents approximately the highest found in the dam, so that the range of modulus to be expected would be from about 3 800 000 to 3 000 000 lb. per sq. in. if this curve is used as the criterion. The values of the secant modulus of elasticity are also given in Fig. 108.

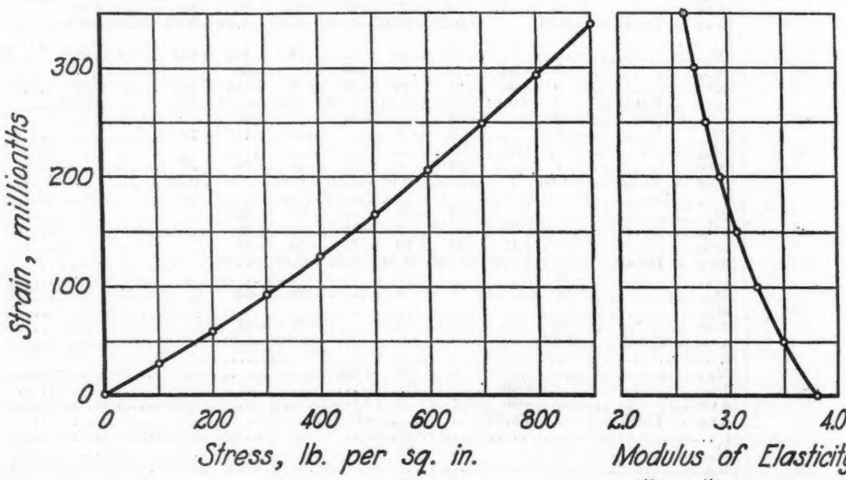


FIG. 108.—STRESS-STRAIN RELATION AND MODULI OF ELASTICITY FROM CYLINDER TESTS.

The test beam under a stress of 216 lb. per sq. in. gave a secant modulus of elasticity of 4 000 000 lb. per sq. in. for the earlier tests and of 6 000 000 for the later tests. Because the modulus of 6 000 000 at the later ages seemed too high, check tests were made on the beam, and this value was confirmed. It is difficult to reconcile the variations in modulus of elasticity between the beam and the test cylinders. However, as shown in Fig. 109, it was found from tests of the cylinders in which telemeters had been embedded that

under a stress of 500 lb. per sq. in., the secant modulus determined immediately upon application of the load was about 5 000 000 lb. per sq. in., but that with the load maintained constant on the specimen, the modulus fell rapidly and consistently to a value of about 2 900 000 lb. per sq. in., within about 4 hours.

Maintaining the load constant for about an hour gave a modulus of about the same magnitude as that determined from the cylinders tested at Berkeley. In testing the beam, the deflection was determined almost instantaneously upon application of the load and it would be expected, from the information obtained from the cylinders having telemeters embedded in them, that the modulus of elasticity determined from the beam would be higher than it was from the cylinders tested at Berkeley.

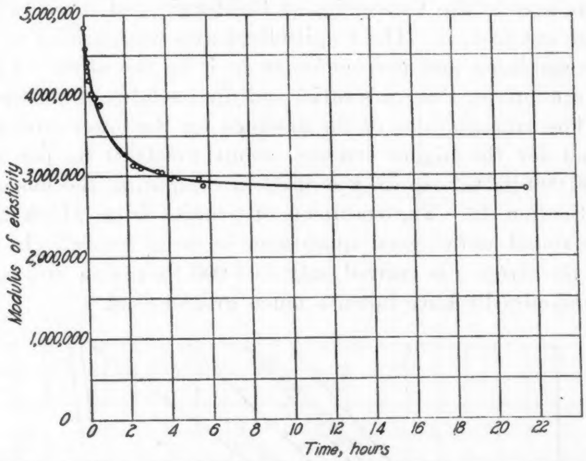


FIG. 109.—EFFECT OF TIME UNDER LOAD ON MODULUS OF ELASTICITY.

Because of the importance of a knowledge of the modulus of elasticity in the interpretation of the results, a further effort was made to obtain information on its value. As shown by Equation (25), Section 54, the load on the dam, as determined from the deflections and strains, is given in terms of the modulus of elasticity. Since the load actually applied by the water pressure is known, it is possible for any given case to use a modulus of elasticity which would make the computed load agree with that known to have been applied. However, the wrong modulus, if used to bring about this agreement at one point, will not give the correct value of the load at some other point. Accordingly, after determining the loads in terms of modulus of elasticity for a large number of points, it was found that the value which gave the closest agreement between computed loads and applied loads for a large number of points was 3 600 000 lb. per sq. in., with Poisson's ratio taken at 0.15. While this is not a perfect agreement with the results of the cylinder tests, it is sufficiently close to give confidence in the general concordance of the data.

A modulus of elasticity of 3 600 000 lb. per sq. in. was used for the reduction of all strains to stress whether in compression or tension. No determination of the modulus in tension was made. Other investigations have given

various conclusions, but, in general, the modulus of elasticity in tension has been found to be equal to, or slightly less than, that in compression. There seems to be little or no greater reason for using different values of the modulus in tension and in compression, than for using a different modulus for each different degree of compression. The effects of using a variable modulus were studied; as they were very small it was concluded that any possible gain in accuracy would be more than offset by the confusion and complexity attending the use of a variable modulus in working up the test data.

In order to interpret the measured yielding of foundation and abutments, it became necessary to know the modulus of elasticity of the granite of which they were composed. A block of granite was cut from the foundation rock at a place about 20 ft. down stream from the dam at an elevation of about 15 ft. It was sent to the University of California and there two 6 by 12-in. cylinders were cut from it. The longitudinal axis was parallel to the bedment plane in one specimen and perpendicular to it in the other. Fig. 110 gives the stress-strain curves, Poisson's ratio, and the modulus of elasticity for these specimens. The average value of the modulus for the lower stresses was about 2 000 000; and for the higher stresses, about 2 500 000 lb. per sq. in. The value of 2 000 000 lb. per sq. in. was used in computing the movement of the bed-rock in Section 42. The specimen of granite from which the cylinders were cut, was sound and of good appearance in every respect. In determining the modulus the stress was carried only to 3 000 lb. per sq. in., but the specimen would undoubtedly have taken a much greater load.

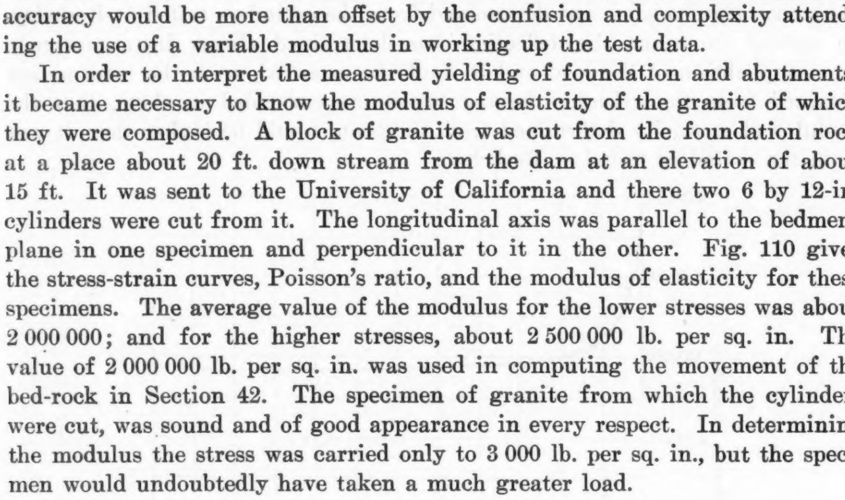


Fig. 110.—Lateral and Longitudinal Strains, Modulus of Elasticity, and Poisson's Ratio for Granite Taken from Foundation.

53.—*Poisson's Ratio.*—Equation (25) shows that Poisson's ratio, μ, the ratio of the lateral to the longitudinal strain in a member under longitudinal stress, enters, along with the modulus of elasticity, into the expression for

load, as determined from deflections and strains. Values of μ were determined for all the cylinders sent to the University of California for determining the modulus of elasticity. Fig. 111 gives longitudinal and lateral stress-strain diagrams and corresponding diagrams of Poisson's ratio for Control Specimens 173 and 174. The results for these cylinders are representative of all the control specimens from the dam. These specimens were taken from between Elevations 35 and 40.

In Fig. 111 Poisson's ratio varied from 0.17 to 0.29. The extreme range of all values was from 0.1 to 0.3 except for one value of 0.4 at a stress of 200 lb. per sq. in. This includes values determined from tests of specimens made at the University of California from materials similar to those used in the Test Dam. The proportions used in making these specimens were as nearly as possible identical with those used in the Test Dam. The results of these later tests are given in Part V.

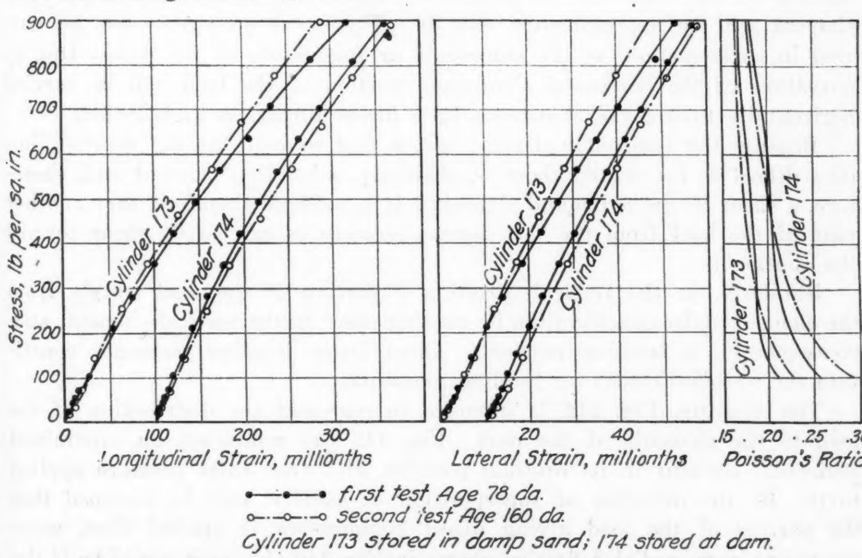


FIG. 111.—LATERAL AND LONGITUDINAL STRAINS AND POISSON'S RATIO FOR CYLINDERS 173 TO 174.

With a stress of 500 lb. per sq. in., the average value of Poisson's ratio for all the tests was 0.17. In preparing this report, when it became necessary to decide on a value for the reduction of the data of the Test Dam, the available information on the lateral and longitudinal strains gave an average value of 0.15 and this was used. This value differed so little from the final average of 0.17, the difference had so slight an effect on the results, and the variation of individual values from each other was so great, that there was no reason for changing to the final average and recalculating the stresses and loads.

Poisson's ratio for a sample of the granite taken from the foundation rock is shown in Fig. 110. The values vary rather uniformly from as low as 0.1 at low stresses to as high as 0.2 at high stresses.

J.—General Method of Analyzing Test Data

54.—General Outline.—In carrying the load, there are in the arch dam several distinct, although interdependent, units of structural action. The dam may be assumed to be made up of a system of horizontal strips (arches) and a system of vertical strips (beams, usually termed cantilevers) intersecting each other and acting at the same time and in combination with each other. If it were known how much load is carried by each element of each system, the design of the dam would be simple; therefore, it seems that the principal aim in analyzing the test data should be that of determining how much load is carried by each of these elements.

The conception of the behavior may be simplified as follows: Assume a 1-ft. square element of the dam at a mean depth of 10 ft. below the surface of the water in the reservoir. The total water pressure on this element will be approximately 625 lb. This total pressure must be carried in some manner to the abutments or foundation of the dam. Due to curvature of the horizontal element, a portion of the load will be carried horizontally toward the abutments by a direct thrust, as arch action.

Because the horizontal element is not free to move at the support, but does deflect at the center, there is, obviously, a bending moment and, therefore, a shear in the horizontal element. It is evident, therefore, that another part of the load from the 1-ft. square element is carried by shear toward the abutments.

Similarly, in the vertical direction a portion of the load travels from the element under consideration by causing shear in the vertical element, and, consequently, a bending moment. These latter bending moments constitute the so-called cantilever bending moments.

The diagram, Fig. 112, is intended to represent the distribution of the load to the elements of the dam. Fig. 112 (*a*) represents an unstrained horizontal element in its original position with full water pressure applied to it. By the principle of superposition of loads it may be assumed that the portion of the load giving direct compression is applied first, causing the uniform radial deflection shown in Fig. 112 (*b*), as it would be if the dam were a part of a complete cylinder and free to deflect radially at all points. The uniform water pressure represented as causing this constant direct compression is also shown in Fig. 112 (*b*).

The equivalent bending load, which pulls the ends back to the original position and causes a bending deflection, as shown in Fig. 112 (*c*), may then be assumed to be applied independently of the load causing direct compression. Not all the load is accounted for by the equivalent load carried by direct thrust and that carried by bending. The remainder, carried by vertical bending, also is shown in Fig. 112 (*c*). Fig. 112 (*d*) represents an unstrained vertical section of the dam in its original position with the full water pressure applied to it. Fig. 112 (*e*) represents the same element deflected by the portion of the load carried by vertical bending, and shows the corresponding load causing the bending.

The following analysis is aimed at determining from the data of the test the values of the different components of the load illustrated in Fig. 112.

It is possible to separate completely from other components the load carried by the bending in the vertical elements of the dam. It is not possible, however, to distinguish completely between the load carried by the bending of the horizontal elements and that carried by their direct compression, since a load which at one point causes only a direct compression will in general cause a bending moment at another point and *vice versa*.

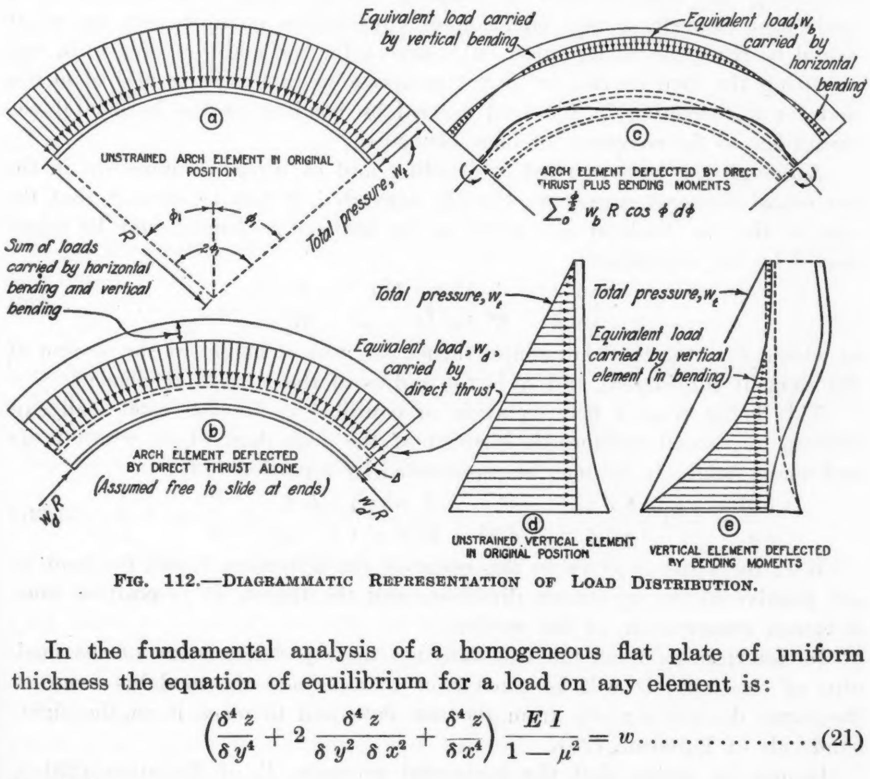


FIG. 112.—DIAGRAMMATIC REPRESENTATION OF LOAD DISTRIBUTION.

In the fundamental analysis of a homogeneous flat plate of uniform thickness the equation of equilibrium for a load on any element is:

$$\left(\frac{\delta^4 z}{\delta y^4} + 2\,\frac{\delta^4 z}{\delta y^2\;\delta x^2} + \frac{\delta^4 z}{\delta x^4}\right)\frac{E\,I}{1-\mu^2} = w \qquad \text{.............. (21)}$$

in which,

x, y, and z = distances in the directions of the axes of a rectangular co-ordinate system, z being the deflection of the plate.

E = the modulus of elasticity of the material.

I = the moment of inertia of the section.

μ = Poisson's ratio for the material, that is, ratio of lateral to longitudinal strain, sometimes used as $\frac{1}{m}$.

w = the total unit pressure on the plate at any given point.

In Equation (21) the term, $\frac{E\,I}{1-\mu^2}\frac{\delta^4 z}{\delta\, y^4}$, represents the load carried by bending in the y-direction. The term, $\frac{E\,I}{1-\mu^2}\frac{\delta^4 z}{\delta\, x^4}$, represents the load carried by bending in the x-direction, and the term, $2\frac{E\,I}{1-\mu^2}\frac{\delta^4 z}{\delta\, y^2\,\delta\, x^2}$, represents the equivalent load carried by torsion.

The arch dam may be considered to have the combined properties of a flat plate and an arch in direct compression. If it were possible to separate completely the loads carried by bending and by direct compression in the horizontal element, a new equation of equilibrium could be set up which would be the same as Equation (21), except for the addition of a term representing the load carried by direct compression in the horizontal elements, and for a correction in the load carried by bending in the horizontal elements due to the curvature of these elements.

Although the loads carried by bending and by direct compression in the horizontal elements cannot be entirely separated, it can be shown* that the sum of the two loads at any point in the horizontal element may be represented by the expression:

$$\left(\frac{\delta^4 z}{\delta\, x^4}+\frac{1}{R^2}\frac{\delta^2 z}{\delta\, x^2}\right)\frac{E\,I}{1-\mu^2}+\frac{P}{R}\quad\ldots\ldots\ldots\ldots\ldots\ldots(22)$$

in which, P is the total horizontal thrust per unit of height at the section of the element considered; and R is the radius of the horizontal element.

This being true, a new equation of equilibrium for the total load carried on any small rectangular element of the arch dam whose width is dx and whose height is dy, may be represented by Equation (23):

$$\left\{\frac{\delta^4 z}{\delta\, y^4}+2\,\frac{\delta^4 z}{\delta\, y^2\,\delta\, x^2}+\frac{\delta^4 z}{\delta\, x^4}+\frac{1}{R^2}\frac{\delta^2 z}{\delta\, x^2}\right\}\frac{E\,I}{1-\mu^2}-\frac{P}{R}=+\,w\ldots.(23)\dagger$$

With the signs as given in this equation the deflection, z, and the load, w, are positive in the up-stream direction, and the thrust, P, is positive when it causes compression on the section.

In determining from the test data the most probable value of the modulus of elasticity, it will be convenient to eliminate the modulus from all the terms derived directly from the test data and to place it on the right-hand side of Equation (23).

It may be shown that the horizontal pressure, P, of Equation (23) is given by the expression:

$$P=+\left\{\frac{\varepsilon_x+\varepsilon_y\cdot\mu}{1-\mu^2}\right\}E\,A\ldots\ldots\ldots\ldots\ldots\ldots(24)$$

* See "Technische Elastizitätslehre," von Hans Lorenz, p. 241 (1913), and "Applied Elasticity," by John Prescott, p. 288 (1924).

† Throughout the report the plus (+) sign is given to (1) vertical distance upward from the base of the dam; (2) horizontal distance along the center line of the dam from left to right when facing up stream; (3) horizontal radial distance from center line of the thickness of the dam in a general up-stream direction, that is, away from the center of curvature of the dam; (4) deflection in a radial up-stream direction, that is, away from the center of curvature of the dam; (5) load in an up-stream direction; (6) compressive stress or compressive strain; and (7) bending moment causing compression on the up-stream face and tension on the down-stream face of the dam. The significance of the minus (—) sign will be obvious from the conventions given for the plus (+) sign. The sign of shear is as obtained by differentiation of the moment curve proceeding in a positive direction along the moment curve.

in which, ε_x and ε_y are the horizontal and vertical strains, due to the axial force, P, and A is the area of the section on which P acts. Substituting this value of P in Equation (23), and dividing by $\frac{E\,I}{1-\mu^2}$,

$$\frac{\delta^4 z}{\delta y^4} + 2\,\frac{\delta^4 z}{\delta y^2\,\delta x^2} + \frac{\delta^4 z}{\delta x^4} + \frac{1}{R^2}\,\frac{\delta^2 z}{\delta x^2} - \frac{A}{R\,I}\left\{ \varepsilon_x + \varepsilon_y\,\mu \right\} = + w\,\frac{1-\mu^2}{E\,I} \quad ..(25)$$

From the test data, from the properties of the section of the dam, and from the head of water, all the terms of Equation (25), except E, are known. It is possible, therefore, to solve the equation for E and thus determine the value that best fits the conditions at the various sections of the dam. The values so obtained correspond well with the modulus of elasticity determined from the test cylinders.

The portion:

$$\left\{ \frac{\delta^4 z}{\delta x^4} + \frac{1}{R^2}\cdot\frac{\delta^2 z}{\delta x^2} \right\} \frac{E\,I}{1-\mu^2}$$

of Equation (23), may be said to represent the equivalent load from the element under consideration that is carried by bending in the horizontal direction.

Of this expression the term, $\frac{1}{R^2}\,\frac{\delta^2 z}{\delta x^2}$,* enters because of the curvature of the horizontal elements. In the test the maximum value of this term represents a load of only 2 lb. per sq. ft. This load is represented, in the portion of the dam having a thickness of 2 ft., by a strain of only 0.000003 and this is too small to be measured accurately.

The term, $\frac{P}{R}$, may be said to represent the equivalent load carried by direct compression in the horizontal element. It will be desirable to define what the terms representing the equivalent bending load and the equivalent load carried by direct compression are, since it has been stated that they do not actually represent the bending load, and the load carried by direct compression, respectively.

The equivalent bending load is an imaginary load, which, if integrated twice with respect to distance along the arc, will give the moment at all points along the arc; or, it may be described as the load which, if distributed along a straight beam of the same length as that of the arc, will give the same moments at all points as in corresponding positions along the arc.

The expression, $\frac{P}{R}$, has been called the equivalent load causing direct compression. It will be seen from its make-up that it is the radial load per unit of length of arc, which, if applied to a complete, relatively thin, circular ring of a uniform cross-section, would cause the same direct compression in the ring as that actually found in the horizontal element under consideration.

* See "Applied Elasticity," by Timoshenko and Lessels, p. 230 (1925).

Equation (25) can be used as given only for a structure having a constant moment of inertia. In the Test Dam, due to the varying thickness, there is a corresponding variation in the moment of inertia and a modification of the procedure of Equation (25) was necessary.

The deflections were differentiated twice to obtain the values of $\frac{M}{E\,I}$. Using the known values of $E\,I$ at various positions, the values of the moment were obtained and a double differentiation of these moments in the directions of the proper axes gave the equivalent bending loads in the x and y-directions (see Section 63).

55.—Interpretation of Strain Diagrams.—It is shown in works on elasticity that the relations between stress, strain, modulus of elasticity, and Poisson's ratio (σ, ε, E, μ), are given by the equations:

$$\frac{\sigma_v}{E} = \frac{\varepsilon_v + \mu\,\varepsilon_h}{1 - \mu^2} \qquad \text{(26)}$$

and,

$$\frac{\sigma_h}{E} = \frac{\varepsilon_h + \mu\,\varepsilon_v}{1 - \mu^2} \qquad \text{(27)}$$

In Fig. 113, the dotted lines give values of the ratio, $\frac{\sigma_v}{E}$, of Equation (26). This expression may be thought of as strain corrected for Poisson's ratio, although it is not a strain. Generally, however, the strains plotted in the diagrams of this report are the observed strains, and in order to determine the stresses with exactness from the given strains, it is necessary to take into account Poisson's ratio and the strains mutually at right angles with each other. The stresses are given by the equations:

$$\sigma_v = \frac{E\,\varepsilon_v}{1 - \mu^2}\left(1 + \mu\,\frac{\varepsilon_h}{\varepsilon_v}\right) \qquad \text{(28)}$$

$$\sigma_h = \frac{E\,\varepsilon_h}{1 - \mu^2}\left(1 + \mu\,\frac{\varepsilon_v}{\varepsilon_h}\right) \qquad \text{(29)}$$

Using μ equal to 0.15 and E equal to 3 600 000 lb. per sq. in., $\frac{E}{1 - \mu^2}$ is equal to 3 680 000 lb. per sq. in. Therefore, if the strain in either direction is zero, the stress in the other direction (at right angles to the former strain) is the same as would be found by using a modulus of elasticity of 3 680 000 lb. per sq. in. and neglecting Poisson's ratio. Where ε_v and ε_h are equal, the stress determined for either direction is about 15% greater or less (greater if the signs of the two strains are the same, or less if they are opposite) than if Poisson's ratio is neglected.

The horizontal strains for the 30-ft. elevation are shown in Figs. 114 to 117. A few horizontal strains for other elevations are shown in Fig. 124. For other portions of the dam the strains in either the vertical or horizontal direction may be estimated from Figs. 94 to 101 which give lines of equal strain. With the aid of these diagrams and Equations (28) and (29), an estimate of the stress in either the vertical or the horizontal direction at any portion of the dam may be made.

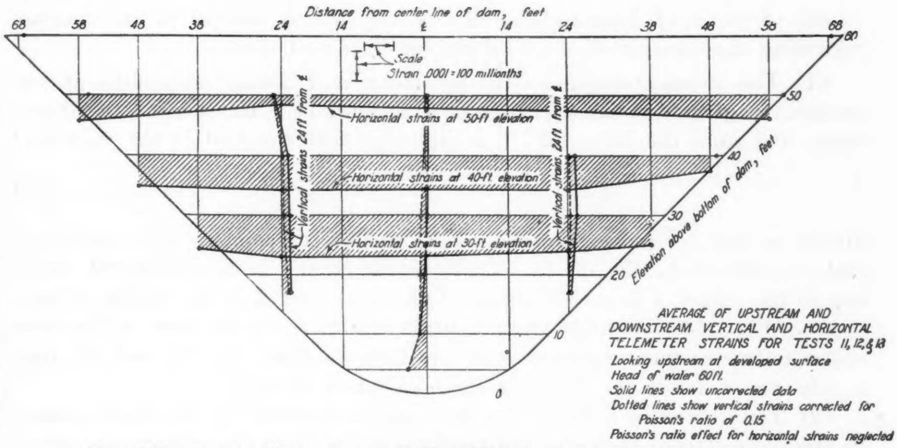


FIG. 113.—DIRECT STRAINS FOR 60-FOOT HEAD; AVERAGE OF UP-STREAM AND DOWN-STREAM TELEMETER STRAINS.

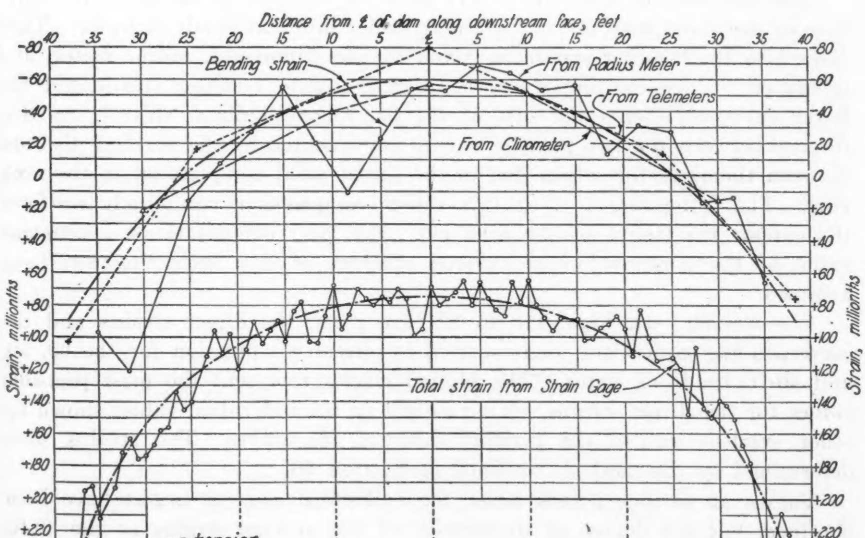


FIG. 114.—COMPARISONS OF BENDING STRAINS FROM CLINOMETER, TELEMETER, AND RADIUS METER DATA WITH TOTAL STRAINS FROM STRAIN-GAUGE, FOR HORIZONTAL ELEMENT AT ELEVATION 30, UNDER HEAD OF 60 FEET.

K.—Equivalent Load Carried by Direct Thrust

56.—Sources of Information.—There are several sources of information regarding the amount of the load carried by direct thrust:

(1) The average strain for the up-stream and down-stream sides at any section, as given by the telemeter observations, is a measure of the direct strain, and since the thrust, $E\,\varepsilon\,t$, is connected with the load by the equation:

$$w = -\frac{E\,\varepsilon\,t}{R} \qquad (30)$$

known as the "cylinder formula", it is possible to compute the equivalent load, w, resisted by the thrust. In Equation (30), ε is the measured strain due to the thrust, t is the thickness of the dam, and R is the radius of curvature. Fig. 113 shows the average direct strain, ε, for all parts of the dam where telemeter measurements were available in Tests 11, 12, and 13, that is, with the water in the reservoir at an elevation of 60 ft.

(2) The difference between the total strain observed by the strain-gauges on the down-stream face and the bending strain from the clinometers gives the strain, ε, due to direct thrust at various points of the arch; and from Equation (30), the load, w, may be determined.

The diagrams in Figs. 114 to 117 show the total strain at the 30-ft. elevation as measured with the strain-gauge under different heads of water. They show also the bending strains as given by the clinometer, radius meter, and telemeter. Since the upper curves represent only bending strain and the lower curve represents the sum of the bending and direct strains, the difference between the two average curves (represented by the vertical distance between them) is the strain due to the direct axial compression in the arch ring. The indication is that this direct compression was nearly uniform throughout the length of the arch rib. For later computations, a constant value of the compression at various portions of any arch ring has been assumed.

57.—Results.—In Figs. 118 to 121 are given the direct strains and the corresponding equivalent loads carried by direct compression for the 60, 50, and 40-ft. heads of water. The plotted circles represent the most probable values for the direct strains, giving weight to the individual values shown by other symbols and to the general shape of the curves. The strains were determined by the methods outlined in Section 56.

Values at various points along the horizontal element might have been obtained, but the degree of uniformity of the average strains at the 30-ft. elevation (see Figs. 114 to 117) is considered sufficient to warrant using an average for these elevations also. This is fortunate, since the analysis of the data is simplified by assuming a constant direct strain in the horizontal elements, and since, for other than the 30-ft. elevation, the data are not sufficiently complete to warrant using other than the average strain. The agreement of the strains derived by different methods is generally fair.

The water pressure (or equivalent direct load) necessary to produce the direct strains, shown in Figs. 118 to 121, is indicated in the right-hand portion of each diagram. For the 60-ft. head (Fig. 118), the equivalent load carried

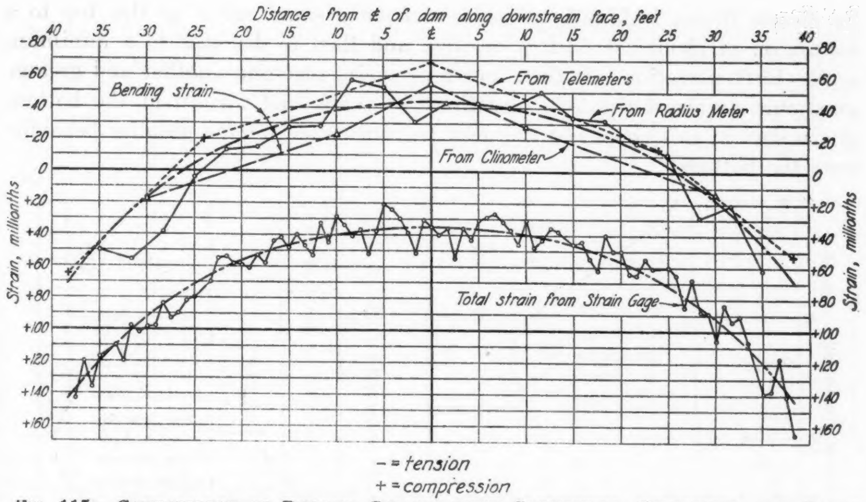


FIG. 115.—COMPARISONS OF BENDING STRAINS FROM CLINOMETER, TELEMETER, AND RADIUS METER DATA, WITH TOTAL STRAINS FROM STRAIN-GAUGE, FOR HORIZONTAL ELEMENT AT ELEVATION 30 UNDER 50-FOOT HEAD.

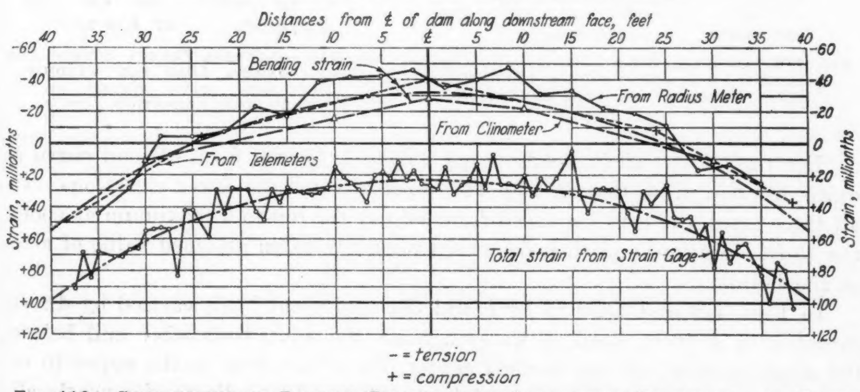


FIG. 116.—COMPARISONS OF BENDING STRAINS FROM CLINOMETER, TELEMETER, AND RADIUS METER DATA, WITH TOTAL STRAINS FROM STRAIN-GAUGE, FOR HORIZONTAL ELEMENT AT ELEVATION 30 UNDER 40-FOOT HEAD AFTER CRACKING.

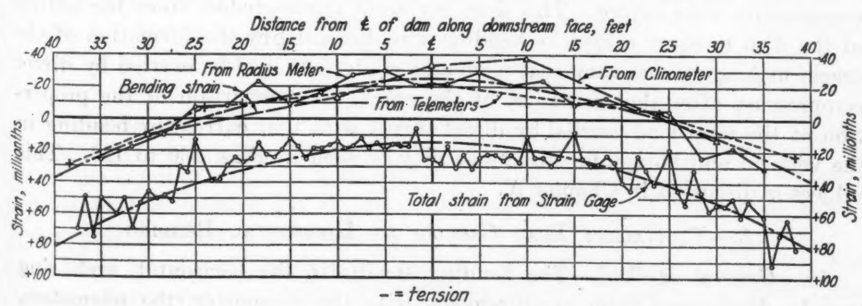


FIG. 117.—COMPARISONS OF BENDING STRAINS FROM CLINOMETER, TELEMETER, AND RADIUS METER DATA, WITH TOTAL STRAINS FROM STRAIN-GAUGE, FOR HORIZONTAL ELEMENT AT ELEVATION 30 UNDER 40-FOOT HEAD BEFORE CRACKING.

by direct thrust is found to increase from a small value at the top to a maximum at about the 35-ft. elevation, and then to decrease to a minimum at the 15-ft. elevation. It then increases again, reaching another and greater maximum at about the 5-ft. elevation. A crack had formed at the bottom of the dam at this time, and this may be responsible for the peculiar behavior near the bottom.

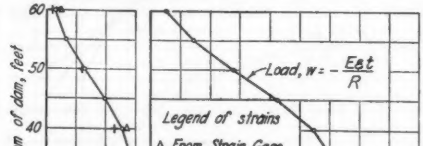


FIG. 118.—AVERAGE DIRECT STRAIN AND CORRESPONDING LOAD FOR VARIOUS ELEVATIONS WITH WATER AT 60-FOOT ELEVATION.

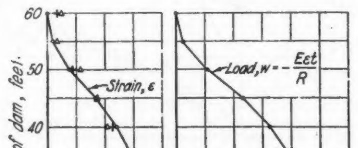


FIG. 119.—AVERAGE DIRECT STRAIN AND CORRESPONDING LOAD FOR VARIOUS ELEVATIONS, WITH WATER AT 50-FOOT ELEVATION.

For the 50 and 40-ft. heads (Figs. 119 to 121), the form of load curve is quite different from that in Fig. 118, especially for the lower elevations. In all three cases, the load increased from zero at the top to a maximum at about the 15-ft. elevation, and then decreased regularly to an assumed value of zero at the bottom.

In Figs. 120 and 121 will be found the equivalent loads carried by direct thrust with a 40-ft. head of water against the dam, both after and before the crack occurred on the vertical center line of the dam in the upper 10 to 20 ft. of its height. Comparison of Figs. 120 and 121 indicates for nearly all elevations that after this crack occurred more load was carried by direct compression than before. This does not seem unreasonable, since the ability of the dam to carry a load by bending was decreased by the formation of the crack, making it necessary that a greater equivalent load be carried by direct compression after the formation of the crack. A comparison of the proportion of the total load carried by direct thrust with that carried by bending in the vertical and horizontal directions may be seen in Figs. 130 to 133. This subject is discussed in Chapter *N*.

L.—EQUIVALENT LOAD CARRIED BY HORIZONTAL BENDING

58.—General Method.—The bending strains in the horizontal arch ring may be determined from measurements with the clinometer, the telemeters, and the radius meter; also from the total strains measured by the strain-gauges by subtraction of the direct strain. The radius meter, however, was

used only at the 30-ft. elevation of the dam and, therefore, will not give any information on the bending except at this elevation. The telemeters were placed at every 10-ft. elevation, but so far apart in horizontal direction that detailed information regarding the bending along the arch ring could not be obtained from them. They gave valuable information, however, regarding the bending at the places where they were located.

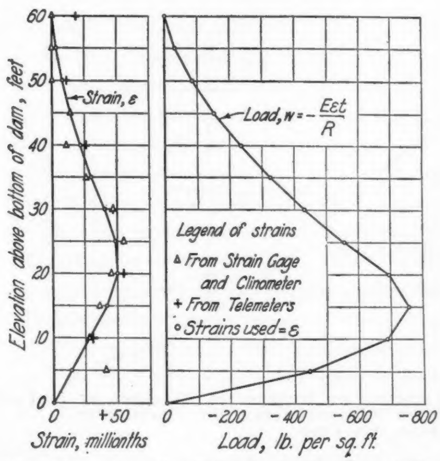


FIG. 120.—AVERAGE DIRECT STRAIN AND CORRESPONDING LOAD FOR VARIOUS ELEVATIONS WITH WATER AT 40 FEET AFTER CRACKING.

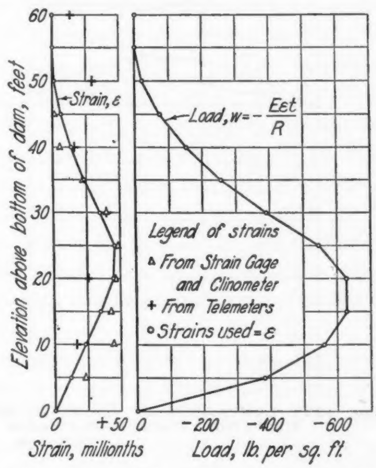


FIG. 121.—AVERAGE DIRECT STRAIN AND CORRESPONDING LOAD FOR VARIOUS ELEVATIONS WITH WATER AT 40 FEET BEFORE CRACKING.

By double differentiation of the measured deflections of the horizontal elements, bending strains may be obtained for every 5-ft. elevation; but the strains obtained will be the average between any two stations and, as the clinometer stations were placed rather far apart (10 and 20 ft. in a horizontal direction), the results from the clinometer data thus obtained will give little information on the variation of the bending strains along the arch ring. However, in order to determine whether the strain-gauge and the clinometer data harmonized with each other, a study was made, which is illustrated by Figs. 122 and 123. In the lower part of these diagrams the average deflections of one-half the dam for the 50 and 30-ft. elevations, with a head of 60 ft. of water, are shown as crosses. The horizontal strains measured with the strain-gauge at 10-ft. intervals along the horizontal element are shown in the upper part of the diagram as circles. They include both the direct (axial) and the bending strains. Since the direct strain has been shown to be nearly constant (see Section 56) the curve of bending strains should be parallel to the curve determined by the strain-gauge.

By a process of "cutting and trying" the intermediate deflections have been so selected that when the resulting deflection curve has been differentiated twice to obtain the bending strains, the strain curve will lie, as nearly as possible, parallel to the strain curve determined by the strain-gauge. The bending strains so obtained are shown in the upper part of Figs. 122 and 123.

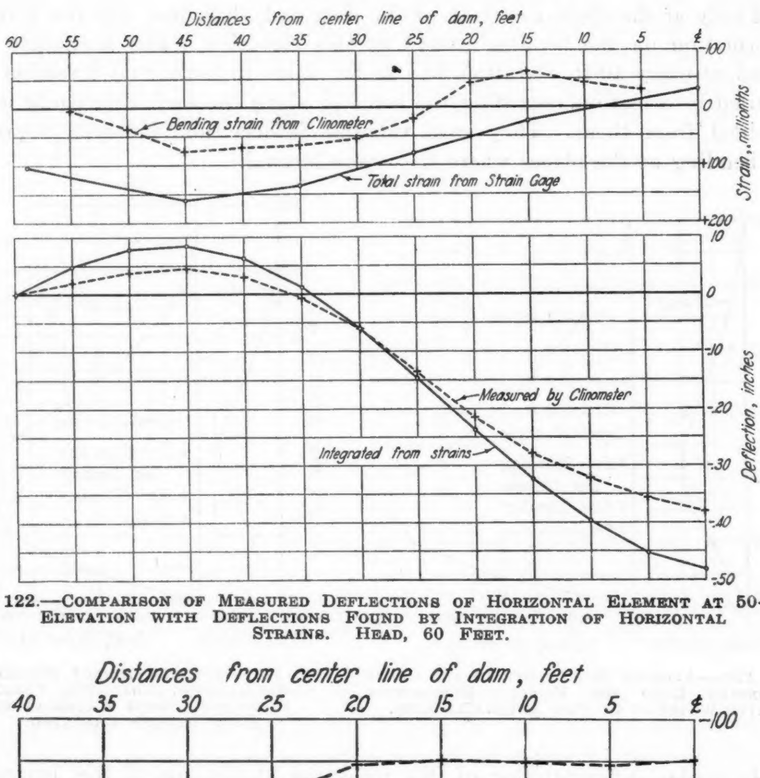


FIG. 122.—COMPARISON OF MEASURED DEFLECTIONS OF HORIZONTAL ELEMENT AT 50-FOOT ELEVATION WITH DEFLECTIONS FOUND BY INTEGRATION OF HORIZONTAL STRAINS. HEAD, 60 FEET.

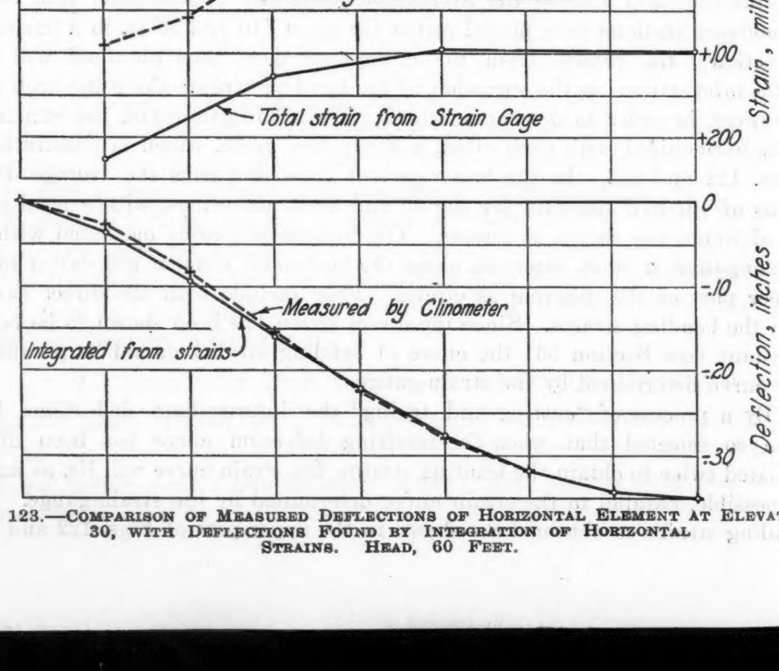


FIG. 123.—COMPARISON OF MEASURED DEFLECTIONS OF HORIZONTAL ELEMENT AT ELEVATION 30, WITH DEFLECTIONS FOUND BY INTEGRATION OF HORIZONTAL STRAINS. HEAD, 60 FEET.

For the 60-ft. head this was done for each horizontal element the elevation of which was a multiple of 10 ft. The fact that it was possible to find deflection curves which passed through the points determined by measurement and which gave a strain curve approximately parallel to the curve of strains measured by the strain-gauge, is a good indication of the agreement between the strain-gauge and the clinometer.

The deflections of the horizontal elements at the 50 and 30-ft. elevations, as measured with the clinometer, and the deflections determined by double integration of the measured strains at these elevations are shown in Figs. 122 and 123. Both deflections are for a height of water of 60 ft. The agreement between deflections deduced by the two methods is much better for the 30-ft., than for the 50-ft., elevation. This may be accounted for partly by the fact that at the 50-ft. elevation the strains were measured at stations 10 ft. apart horizontally, while at the 30-ft. elevation a continuous row of strain-gauge measurements only 10 in. apart was available. The greater length at the 50-ft. than at the 30-ft. elevation will also increase the error in the deflections obtained by integration of strains for the former elevation. On the whole, the agreement between the measured deflections and those determined from the strains seems fair.

For determining the loads the strain-gauge data were used in preference to the clinometer data. A smooth curve was drawn through the points of measured strain, and strains, read from this curve at intervals of 5 ft. along the element, were used in a tabular differentiation as illustrated for the vertical elements in Table 24. Since the direct compression was nearly constant, its effect on the values obtained in the differentiation would be negligible. The strains, shears, and loads were obtained in this way for the elements with elevations in multiples of 5 ft., down to and including the 15-ft. elevation. As an illustration, the results so obtained are shown in Fig. 124 for the elements at elevations of 60, 50, 40, and 30 ft. (see, also, the loads in Fig. 125).

59.—Determination of Loads on Lower Part of Dam.—In the lower part of the dam the strains in the vertical direction were so affected by influences such as the rotation of the toe, causing an excessive stress at the down-stream face of the dam for a considerable height, that the conservation of plane sections could not be assumed with reasonabe expectation of determining with sufficient accuracy the loads carried by the vertical elements. The strains and deflections in the horizontal direction also were not taken close enough together for satisfactory use in determining the loads carried by the horizontal elements in bending. Besides this, the vertical crack that occurred at the bottom of the dam under the 60-ft. head before the strains had been observed, vitiated the data for such use.

It was necessary, therefore, to arrive at the loads in the lower part of the dam in another way. At the 10-ft. and the 5-ft. elevations the strain was observed on the up-stream face and at the center of the thickness of the dam with telemeters. Strains on the down-stream face were of no value in this study because of the presence of the crack previously noted, and the depth of the crack was not known. If the section be assumed to remain plane during bending, the assumption of a given strain in the concrete at the base of the

crack is equivalent to assuming a depth of the crack and, consequently, the depth of section remaining intact, that is, the depth of the "secondary arch" which is assumed to carry the load. With this information and by making use of the measured strains on the up-stream face and at the center of the thickness of the dam, it is possible to compute the horizontal bending moment

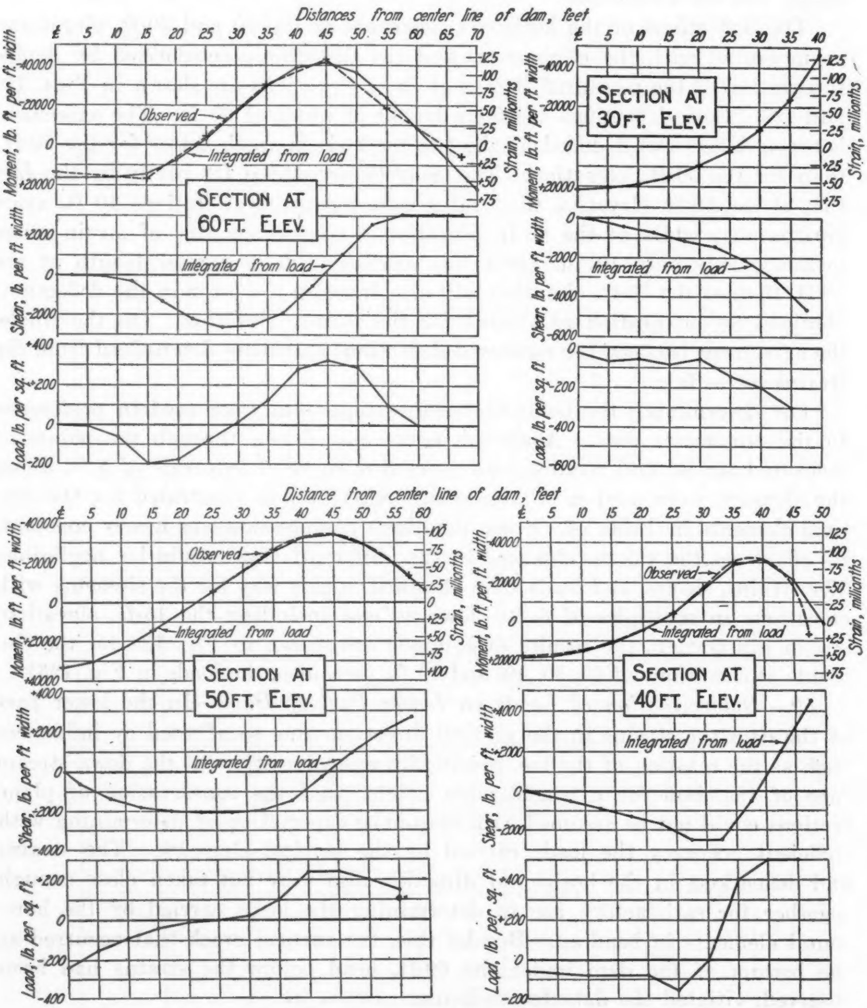


Fig. 124.—Bending Strains, Shears, and Bending Loads in Horizontal Elements at Elevations 30, 40, 50, and 60, Under a 60-Foot Head.

resisted by the section. The telemeter data give a basis for computing the bending moment and direct strain at the abutments. In spite of the conflict between the assumptions the errors introduced appear to compensate each other to a considerable extent and the results of such a study were used as the basis for determining the loads carried by the secondary arch.

So little is known as to how much strain may be resisted at the base of a crack that there is much uncertainty as to what strain should be assumed. As a first trial, the tensile strain at the 10-ft. elevation was assumed as 0.000050 at the base of the crack, and, in the second trial, as 0.000010. The loads obtained for the 5 and 10-ft. elevations for the two different assumptions were as given in Table 23.

It will be seen from Table 23 that, although the variation in the assumed value of the strain at the base of the crack has an important influence on both the equivalent bending load and the equivalent load causing direct thrust, the differences compensate each other and the total loads carried are nearly the same for the two cases. Hence, the average value was taken as the load carried by the horizontal element. In view of the approximate nature of the analysis, an assumption of the manner of variation of the load from crown

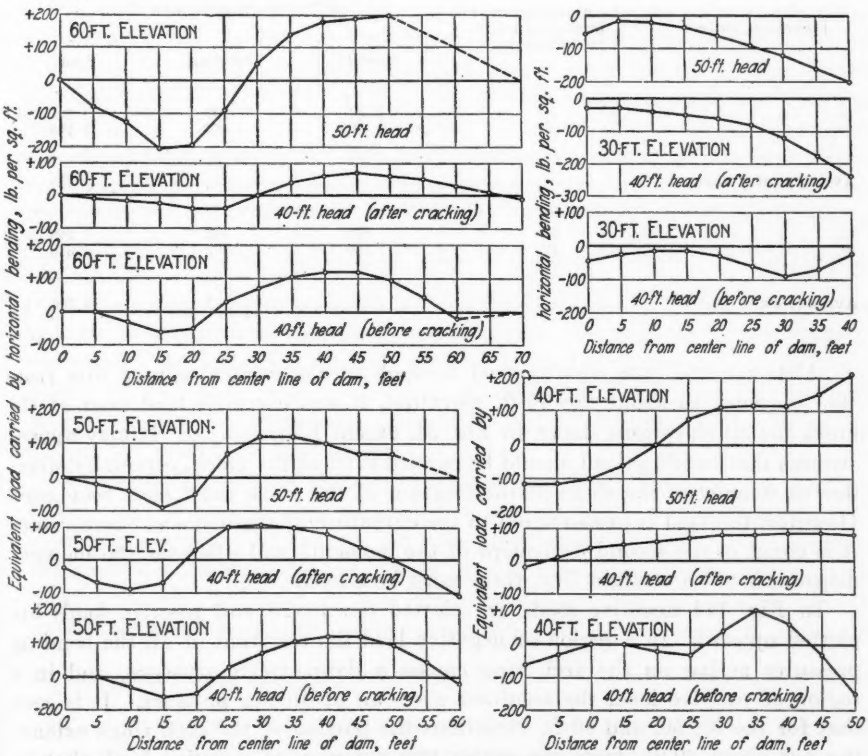


FIG. 125.—SUMMARY OF LOADS CARRIED BY HORIZONTAL BENDING ON HORIZONTAL SECTIONS AT ELEVATIONS 30, 40, 50, AND 60.

to abutment would not be warranted, and so the load has been assumed to be uniform throughout the length of the arch. For the vertical elements, the average loads given in Table 23, subtracted from the total water pressure at corresponding elevations, have been used instead of the loads determined from the measured strains in these elements.

60.—Results.—In Fig. 124 specimen results for the equivalent load carried by horizontal bending are given. This diagram shows the distribution of strains and of corresponding moments, and the shears and loads obtained therefrom by differentiation of the moments for the 60, 50, 40, and 30-ft. elevations, with a 60-ft. head of water against the dam. An idea of the confidence to be placed in the results may be gained by comparing the agreement between the strains observed and those obtained by double integration of the load curve. The agreement checks the differentiation by means of which the load was derived.

TABLE 23.—APPROXIMATE LOADS CARRIED ON HORIZONTAL ELEMENTS AT 5 AND 10-FOOT ELEVATIONS.

Elevation, in feet.	Assumed strain at base of crack, in millionths.	Load Carried by Horizontal Element in Pounds per Square Foot.		
		Direct.	Bending.	Total.
5	50	1 575	132	1 707
5	10	2 135	—382	1 753
Average total load				1 730
10	50	818	462	1 280
10	10	1 126	207	1 333
Average total load				1 306

Although the dam was cracked through at the vertical center line from the top down to about the 40-ft. elevation, it was carrying load even at the crack for all elevations shown in Fig. 54, except Elevation 60. It may appear strange that bending load should be carried even at the crack, especially since, due to symmetry, the shear at the location of the crack must have been zero. However, the load is proportional to the curvature of the moment curve (since it is equal to the second derivative of the moment) and wherever the moment diagram is not a straight line, there must be a load.

In Fig. 124 negative loads are plotted downward and positive loads are plotted upward. In a region of negative load the resultant of all the bending pressures acting on the arch ring causes a down-stream pressure, and in a region of positive load, the resultant gives an up-stream pressure. It is seen that for the 60, 50, and 40-ft. elevations the portion of the arch rings extending out about 30 ft. from the center was a region of negative load, that is, was carrying load, and that the portion from there on to the abutments was a region of positive load, that is, was applying load to the vertical elements. For the 30-ft. elevation, however, within the entire distance from the center line to the abutment, load was carried by horizontal bending.

The portions of the total load carried by direct compression, and by bending in the vertical and horizontal directions, may be seen in Figs. 130 to 133. This subject is discussed in Chapter *N*.

The point of greatest bending moment in the horizontal elements is indicated in Fig. 124 to have been from 40 to 45 ft. away from the center line for the 60-ft. head of water. The strain accompanying this bending moment was 0.000125 in. per in. at the 60-ft. elevation. As the direct compressive strain was only 0.000012 in. per in., the resultant tensile strain was 0.000113 in. per in. With a strain as great as this, it seems that the concrete must have been close to tension failure at approximately the one-quarter point of the upper element. The point of zero moment at the same time was close to 25 ft. from the center line for all elevations. Previous to the formation of the vertical cracks, it is probable that the greatest bending moment was at the center line.

M.—Load Carried by Vertical Bending

61.—General Method.—The deflections on the seven vertical lines indicated in Fig. 27 were determined by the clinometer in 5-ft. steps. These deflections are proportional to the average slopes in the 5-ft. gauge lengths. Due to symmetry of the sections measured, each slope curve has its counterpart on the opposite side of the center line and a more accurate curve was obtained by using the average for each symmetrical pair. The average slope curve was integrated, giving the deflections shown as plotted points and dotted curves in Figs. 126 to 129. A single differentiation of the slope curve gave the bending strains at each point which were multiplied by the quantity, $\frac{E\,I}{t}$, to give the moments in the vertical element 1 ft. wide. The value used for the modulus of elasticity, E, was 3 600 000 lb. per sq. in. The thickness, t, and the moment of inertia, I, apply to the point at which the strain is measured. As indicated by the method of reducing strains to moments, plane sections have been assumed to remain plane, although this probably is not the true condition near the bottom, where foundation influences will change the conditions considerably.

In the differentiation of experimental curves, slight inaccuracies are magnified, and where several successive differentiations are performed, it is generally necessary to smooth out the curve at some stage. The moments determined as described were plotted and a smooth curve was fitted to the plotted values. In order to test the correctness of this modification in the moment curve, these moments were reduced back to deflections by a reversed process; that is, the moments were divided by $\frac{2\,E\,I}{t}$ to reduce them to strains, and the strains were integrated successively to obtain the slope and the deflection curves. The graphs of strains, slopes, and deflections obtained by this integration, were then drawn to test the agreement of the derived results with the original data. Having thus checked the substantial correctness of the modified moment curve, the shear and load curves were obtained from it by two successive differentiations.

All differentiations and integrations were performed by taking tabular differences or sums, as illustrated in Table 24, so there was no chance for inaccuracies due to errors in mechanical operation. Consequently, double

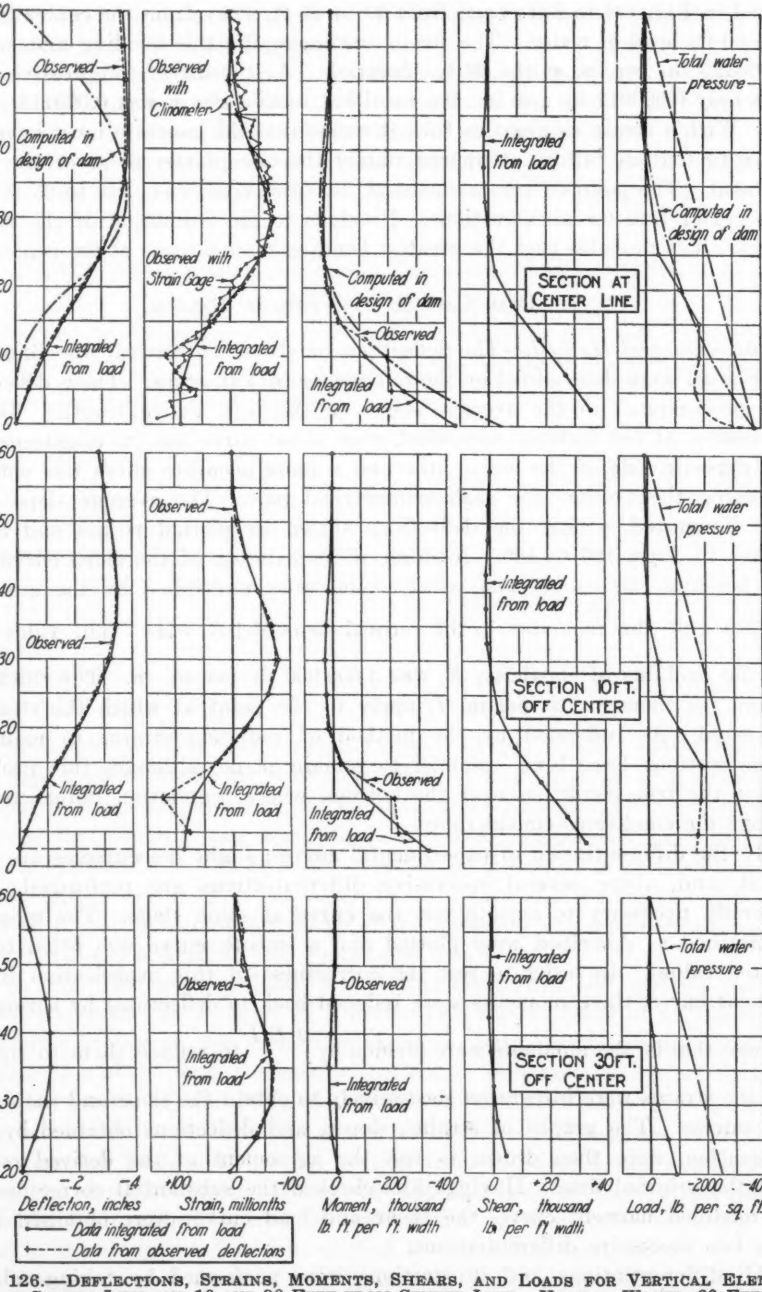


FIG. 126.—DEFLECTIONS, STRAINS, MOMENTS, SHEARS, AND LOADS FOR VERTICAL ELEMENTS AT CENTER LINE AND 10 AND 30 FEET FROM CENTER LINE. HEAD OF WATER, 60 FEET, WITH DAM CRACKED AT TOP AND BOTTOM FOR TESTS 11, 12, AND 13.

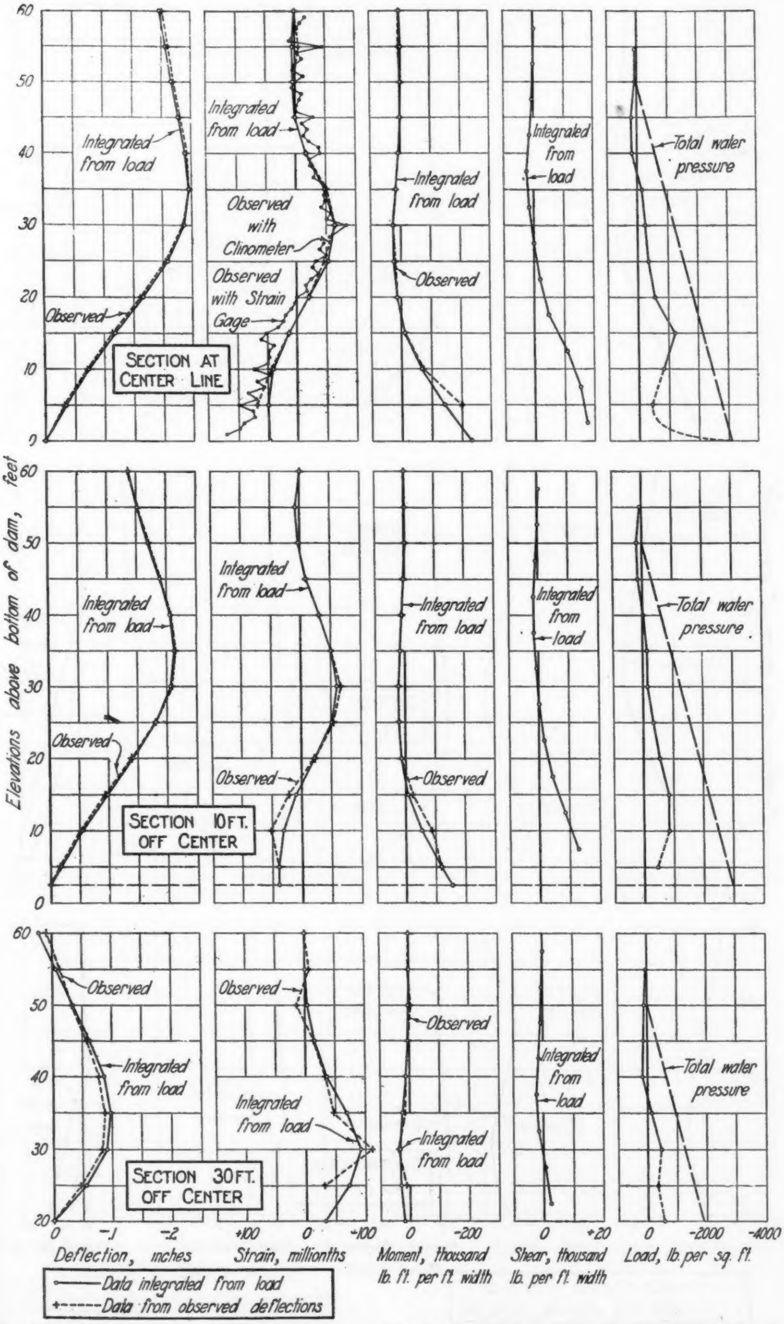


FIG. 127.—DEFLECTIONS, STRAINS, MOMENTS, SHEARS, AND LOADS FOR VERTICAL ELEMENTS AT CENTER LINE AND 10 AND 30 FEET FROM CENTER LINE. HEAD OF WATER, 50 FEET, WITH DAM CRACKED AT TOP. AVERAGE FOR TESTS 9 AND 10.

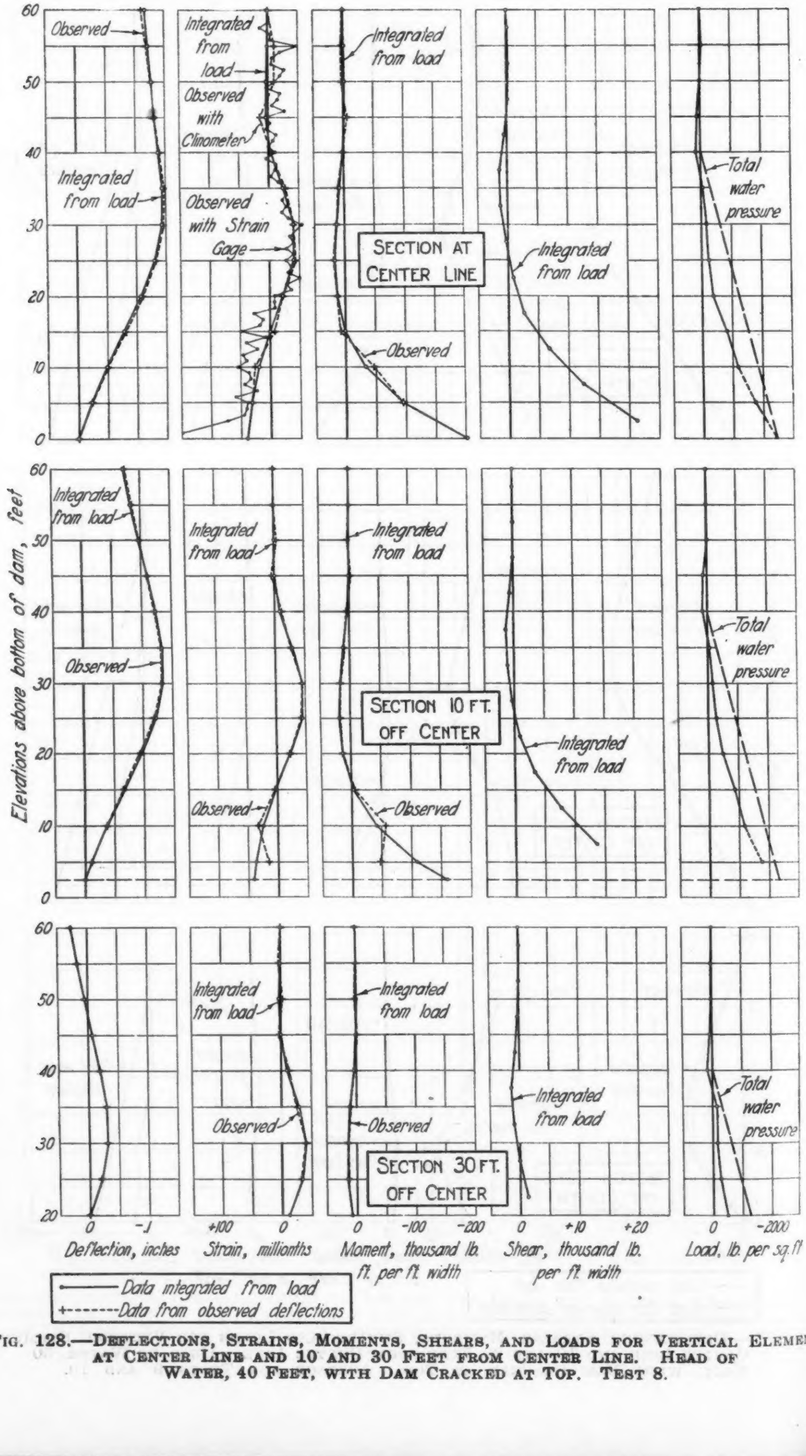


FIG. 128.—DEFLECTIONS, STRAINS, MOMENTS, SHEARS, AND LOADS FOR VERTICAL ELEMENTS AT CENTER LINE AND 10 AND 30 FEET FROM CENTER LINE. HEAD OF WATER, 40 FEET, WITH DAM CRACKED AT TOP. TEST 8.

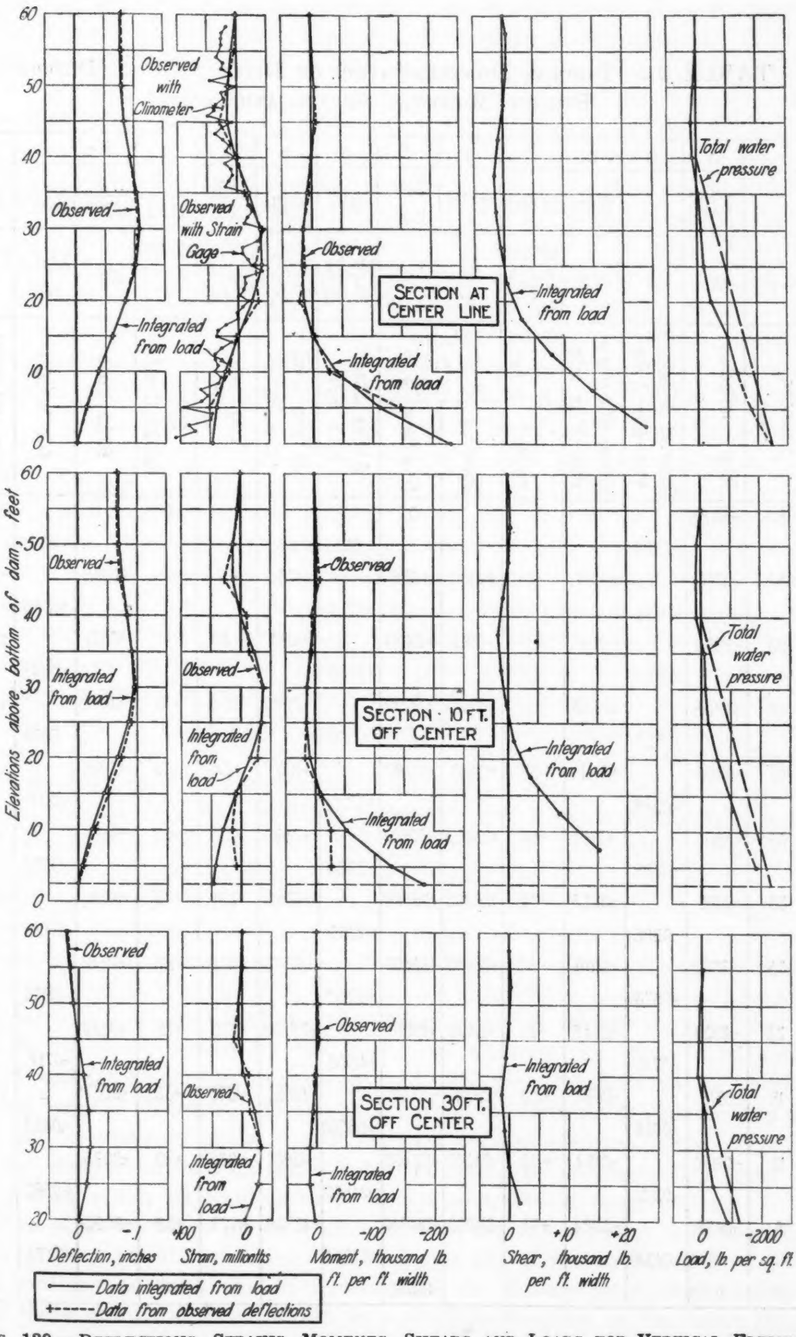


FIG. 129.—DEFLECTIONS, STRAINS, MOMENTS, SHEARS AND LOADS FOR VERTICAL ELEMENTS AT CENTER LINE AND 10 AND 30 FEET FROM CENTER LINE. HEAD OF WATER, 40 FEET. DAM UNCRACKED. AVERAGE FOR TESTS 6 AND 7.

TABLE 24.—TABULAR DIFFERENTIATION OF DEFLECTIONS TO DETERMINE STRAINS, MOMENTS, SHEARS, AND LOADS.

	a	b	c	d	e	f	g	h	i	j	k	l	m
	$\Sigma_0^y(b)$		$\Delta(b)$	$138.89(c)t$	$1023 I(c)$	See text	$\Delta(f)$	$\Delta(g)$	$\frac{\Delta(g)}{25}$	$\frac{(f)}{1023 I}$	$\frac{(j)}{138.89 t}$	$\Sigma_0^y(k)$	$\Sigma_0^y(l)$
y	z	Δz	$\Delta^2 z$	$1000000 \frac{\Delta^2 z}{\Delta y^2} \cdot \frac{t}{2}$	$\frac{EI}{1-\mu^2} \cdot \frac{\Delta^2 z}{\Delta y^2}$		$\frac{EI}{1-\mu^2} \cdot \frac{\Delta^3 z}{\Delta y^2}$	$\frac{EI}{1-\mu^2} \cdot \frac{\Delta^4 z}{\Delta y^2}$	$\frac{EI}{1-\mu^2} \cdot \frac{\Delta^4 z}{\Delta y^4}$	$1000000 \frac{\Delta^2 z}{\Delta y^2} \cdot \frac{t}{2}$	$\Delta^2 z$	$\Sigma_0^y \Delta^2 z = \Delta z$	$\Sigma_0^{60} \Delta z = z$
Elevation feet	Total deflection inches	Observed deflection in 60 in. = 60s in./60 in.	60 times difference in slope = 60Δs ratio	Strain 1000000ε in. per in.	Moment = M lb. in./in. or lb. ft/ft. Observed	Moment From Curve	5 times Shear = 5V lb./ft.	25 times Load = 25w, lb./ft.²	Load = w lb/ft.²	Strain 1000000 ε in./in.	60Δs ratio	60s in./60in.	Total deflection inches
60	-.3685					0				0			-.3771
		+.0066					-355					+.0069	
55	-.3751		+.0017	-6	+2130	+355		+1780	+71	-1	+.0003		-.3840
		+.0052					-2135					+.0066	
50	-.3803		-.0010	+3	-1065	+2490		+1065	+43	-7	+.0021		-.3906
		+.0062					-3200					+.0045	
45	-.3865		+.0039	-13	+4620	+5690		+3900	+156	-16	+.0048		-.3951
		+.0023					-7100					-.0003	
40	-.3888		+.0111	-37	+13150	+12790		+610	+24	-36	+.0108		-.3948
		-.0088					-7710					-.0111	
35	-.3800		+.0197	-66	+23450	+20500		-2630	-105	-57	+.0171		-.3837
		-.0285					-5080					-.0282	
30	-.3515		+.0223	-74	+26300	+25580		-7260	-291	-72	+.0216		-.3555
		-.0508					+2180					-.0498	
25	-.3007		+.0165	-60	+24600	+23400		-10570	-423	-57	+.0156		-.3057
		-.0673					+12750					-.0654	
20	-.2334		+.0045	-21	+12400	+10650		-29400	-1175	-18	+.0039		-.2403
		-.0718					+42150					-.0714	
15	-.1616		-.0052	+29	-28540	-31500		-46750	-1870	+32	-.0057		-.1689
		-.0666					+88900					-.0657	
10	-.0950		-.0154	+113	-194200	-120400		-45600	-1825	+70	-.0095		-.1032
		-.0512					+134500					-.0562	
5	-.0438		-.0074	+71	-211000	-254900		-42700	-1707	+88	-.0092		-.0470
		-.0438					+177200					-.0470	
0	0					-432100				+87			0

integration of the load curve must give exactly the same moment curve as that from which the load curve was obtained by differentiation.

The shear, moment, and deflection curves, shown as solid lines in Figs. 126 to 129, are, therefore, successive integral curves obtained from the corresponding load curves; and the substantial agreement of the deflection curve, shown as a solid line, with the dotted curve drawn through the plotted points satisfactorily checks the correctness of the mechanical operations in determining the loads from the observed deflections. The slope curves have been omitted from these diagrams because they have little direct bearing on the results. In all cases the solid lines represent the curves obtained by integration of the load curve. The plotted points for deflections were obtained by integration of the observed slopes. For strain and moment curves the plotted points shown as open circles were obtained by differentiation of the slope without modification of the observed values, and the points shown as solid circles (for the center line only) were obtained from the strain-gauge data. The plotted points for the load and shear curves represent values obtained by differentiation of the modified moment curve.

As already explained, the moment curve was modified by removing the inaccuracies. It will be seen that at and above the 15-ft. elevation this modification is very slight, while below this it is considerable. At these lower elevations the influence from the foundation is so great that it makes the results very doubtful. Among these foundation influences may be mentioned the separation of the dam from the foundation on the up-stream side and the consequent extreme pressure on the toe, together with the yielding of the bed-rock. At this place the telemeter and the strain-gauge measurements were so few that they do not give much information (at the bottom there were no strain measurements at all), but at these elevations the strain curve obtained from the modified moment curve gives a better agreement with the strains measured with the telemeter and strain-gauge than with those obtained from the clinometer (Fig. 126).

With so much room for selection of a moment curve to fit the points below the 15-ft. elevation, it is necessary to use some basis for arriving at the correct moment curve other than strains in the vertical direction. Since the total load at any elevation must be the sum of the loads carried by the vertical and the horizontal elements, and since, as shown in Chapters *K* and *L*, the load carried by the horizontal elements has been determined fairly satisfactorily, the remaining load has been assumed to be carried by the vertical elements. At the very bottom on the vertical center line all the load is assumed to be carried by the vertical element, since the horizontal element cannot deflect radially, as would be necessary if it were to carry load, either by bending or by direct compression (arch action).

The strains and moments obtained from the strain-gauge and telemeter data for the vertical center line of the dam are also shown in Fig. 126. Telemeter stations on both the up-stream and the down-stream faces indicate that there was little direct strain in a vertical direction in the dam; therefore, the vertical strains must be due principally to pure bending and this justifies the use of the strain-gauge data for computation of the bending

moments. The agreement between the bending moments obtained from the strain gauges and those obtained from the telemeter is very good and adds confidence in the correctness of the modified moment curve determined from the clinometer data.

62.—Specimen Determination of Vertical Bending Load.—In Section 54, the equation of the load on the rectangular element of the dam has been expressed in terms of the fourth derivative of the deflection, z. In order to determine the loads from this equation, a tabular method of differentiating the deflections has been used. Table 24 gives the average deflections found in Tests 11, 12, and 13 on the vertical center line of the dam for a head of 60 ft. and the various steps used in reducing the deflections to loads.

The vertical element is considered as a beam, and use is made of the fact that successive derivatives of the deflection, z, are equal to S, $\frac{M}{E\,I}$, $\frac{V}{E\,I}$, and $\frac{w}{E\,I}$, respectively, in which:

S = slope.
M = moment.
ε = strain.
E = modulus of elasticity.
I = moment of inertia of the section.
V = total shear per foot width for section considered.
w = load per unit of length at the point considered.

For the solution to be exact, deflections would have to be observed at points a differential distance, $d\,y$, apart. This, of course, was not done and could not be done. Therefore, instead of the limiting value, $\frac{d\,z}{d\,y}$, of the change of deflection per unit of distance along y, it is necessary to use the finite increments, $\Delta\,z$ and $\Delta\,y$, and the ratios, $\frac{\Delta\,z}{\Delta\,y}$, $\frac{\Delta^2\,z}{\Delta\,y^2}$, $\frac{\Delta^3\,z}{\Delta\,y^3}$, and $\frac{\Delta^4\,z}{\Delta\,y^4}$, are used instead of the corresponding derivatives. If the deflection curve were made up of straight lines from point to point, the increment ratios, $\frac{\Delta\,z}{\Delta\,y}$, etc., would give exact values; the degree of approximation of the results may be judged by the variation of straight lines through measured deflections from smooth curves fitted to the same points. Therefore, the following notation will be used:

z = deflection at any point
$\Delta\,z$ = measured deflection in any 60-in. gauge length
$\Delta^2\,z = \Delta\,(\Delta\,z)$ = difference between any two successive values of $\Delta\,z$
$\Delta^3\,z = \Delta\,(\Delta\,\Delta\,z)$ = difference between any two successive values of $\Delta^2\,z$
$\Delta^4\,z = \Delta\,(\Delta\,\Delta\,\Delta\,z)$ = difference between any two successive values of $\Delta^3\,z$
$\Delta\,y$ = gauge length = 60 in.
$\Delta\,y^2, \Delta\,y^3, \Delta y^4$ = 3 600, 216 000, and 12 960 000, respectively.

The first line in Table 24 gives captions for the columns. The second line uses symbols, factors, and column captions of Line (1) to indicate the operations performed. For example, Σ_0^y (*b*) indicates that to obtain the deflections of Column (*a*) for any elevation, *y*, the observed deflections for each 60 in., given in Column (*b*), were added cumulatively, beginning at the bottom and ending at Elevation *y*. The symbol, $\Delta(b)$, in Column (*c*) indicates that to obtain the values in that column, differences were taken between the values given in Column (*c*). To obtain the values of moment given in Column (*e*), the values from Column (*c*) were multiplied by 1 023 *I*. To obtain the values given in Column (*f*), a smooth curve was drawn to fit the values given in Column (*e*), thus eliminating inaccuracies and taking account of the fact that near the bottom the load must be determined by subtracting from the total load the load carried by the horizontal element. The moments so indicated were tabulated in Column (*f*) and were used for succeeding operations, as indicated. To test the correctness of the moments shown by the smooth curve and tabulated in Column (*f*) the deflections corresponding to these moments were determined by the operations indicated in Columns (*j*), (*k*), (*l*), and (*m*).

In Fig. 126, the deflections, strains, and moments given in Columns (*a*), (*d*), and (*e*), are shown as plus signs and dotted lines. The shears (shown as a solid line) are one-fifth the values given in Column (*g*). The loads, moments, strains, and deflections shown as open circles and solid lines are the values given in Columns (*i*), (*f*), (*j*), and (*m*).

63.—Results.—In Figs. 126 to 129, inclusive, the loads carried by vertical bending are shown for all portions of the dam on which the data gave information. The deflections, strains, moments, and shears were used in determining the loads, and these also are given.

Positive loads are plotted to the left and negative loads to the right. The significance of the signs is the same as in Section 60, that is, a region of positive load is one in which the resultant of the bending loads on the vertical element causes an up-stream pressure; and a region of negative load is one in which the resultant gives a down-stream pressure.

In all cases the bending load on the vertical element was positive in the upper part of the dam; that is, the upper portion of the vertical element, instead of carrying load, was resting on the arch element, which at any elevation in the upper part of the dam was, therefore, carrying more than the total water pressure applied at that elevation. For nearly all sections shown in Figs. 126 to 129, the load on the vertical elements changed from positive to negative somewhere between the 40 and 35-ft. elevations and was negative from there to the bottom. As shown, the region of positive load on the upper part of the vertical elements extended from the center line to at least 30 ft. from the center line, and possibly farther. The load carried by vertical bending at any given elevation was approximately the same, regardless of the location, up to a distance of at least 30 ft. from the vertical center line. No corresponding statement can be made for the region more than 30 ft. from the center line, because for the only clinometer line within this area, not enough points were available for determining the load distribution.

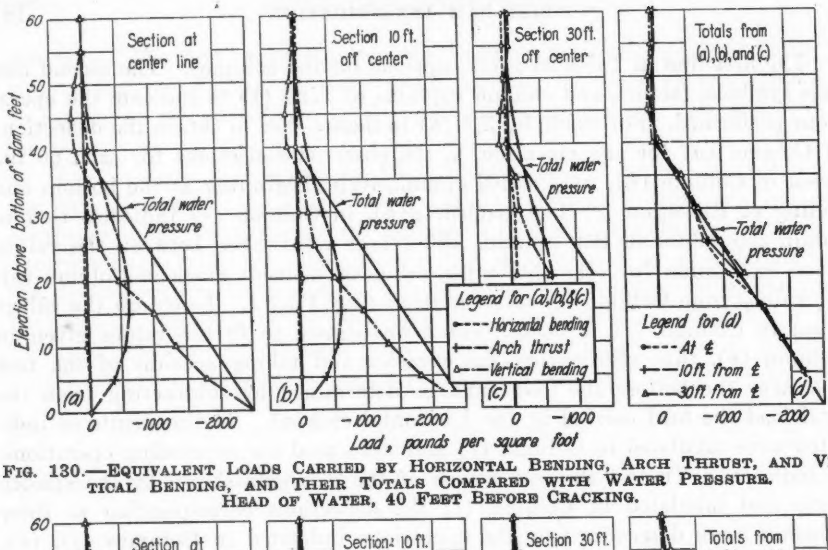


FIG. 130.—EQUIVALENT LOADS CARRIED BY HORIZONTAL BENDING, ARCH THRUST, AND VERTICAL BENDING, AND THEIR TOTALS COMPARED WITH WATER PRESSURE. HEAD OF WATER, 40 FEET BEFORE CRACKING.

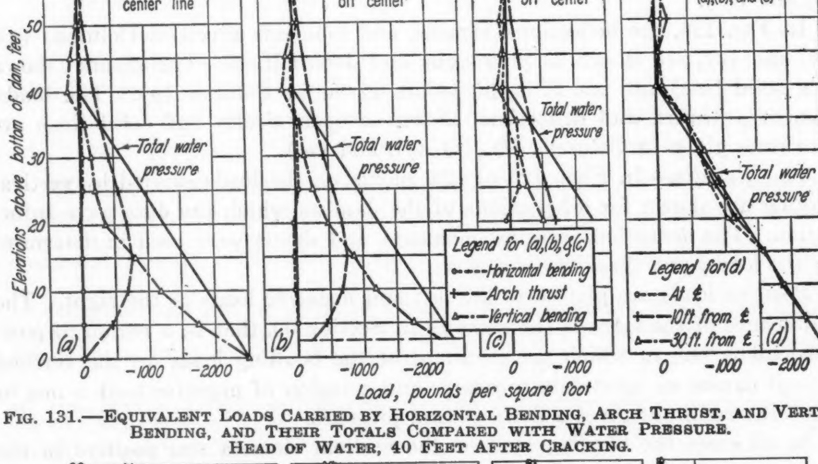


FIG. 131.—EQUIVALENT LOADS CARRIED BY HORIZONTAL BENDING, ARCH THRUST, AND VERTICAL BENDING, AND THEIR TOTALS COMPARED WITH WATER PRESSURE. HEAD OF WATER, 40 FEET AFTER CRACKING.

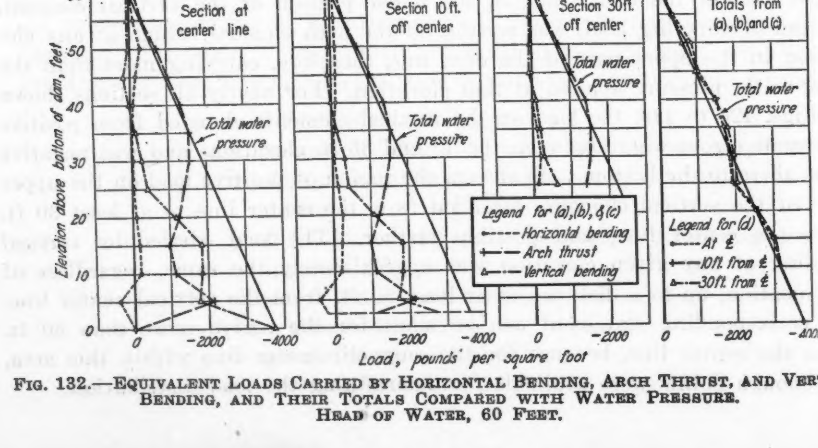


FIG. 132.—EQUIVALENT LOADS CARRIED BY HORIZONTAL BENDING, ARCH THRUST, AND VERTICAL BENDING, AND THEIR TOTALS COMPARED WITH WATER PRESSURE. HEAD OF WATER, 60 FEET.

The portion of the total load carried by direct compression and by bending in the vertical and horizontal directions may be seen in Figs. 130 to 133. This subject is discussed in Chapter *N*.

In the computation of the bending moment from the measured strains, the assumption of conservation of plane section was made (see Section 47). The relation between the vertical strains on the up-stream and the down-stream faces of the dam near the bottom as measured with the telemeters, indicates, however, that a plane section did not remain plane. (See Fig. 134 and Section 59). The indication is that either there was considerable direct vertical compression on a horizontal section near the bottom of the dam, or that there was a horizontal crack on the up-stream face at an elevation of about 10 ft. The presence of a resultant direct vertical pressure on the section would account for the excess of the strain on the down-stream face over that on the up-stream face, as shown in Fig. 135. However, it would not account for the departure from all semblance of conservation of plane section (Fig. 135). If a crack occurred it would be most likely to be at the construction joint, which was located at the 9.5-ft. elevation. The telemeters, the strains of which are plotted in Fig. 135, are located at the 10-ft. elevation, so that the strain shown for the up-stream side would be smaller if the crack were present than if it were not; also, sections near it would not be likely to remain plane, and thus the crack would explain both the bending of the section from a plane and the excess of the observed strain on the down-stream face over that on the up-stream face.

Further indication that there was a crack at the 10-ft. elevation on the up-stream face is seen in the excessive strain found from the clinometer data for the 60-ft. head, as shown in Fig. 126, both for the vertical section at the center line and for a section 10 ft. away from the center. If a crack occurred at the construction joints at the 30 and 35-ft. elevations under a strain of only about 0.000075 in. per in., it seems reasonable that there should be one here, where the strain was more than 0.000100 in. per in. However, no crack was discovered. Although searches were made for cracks on the up-stream face at the time of the test, this was a particularly difficult location for making an examination, both because of the narrow quarters and because of the constant flow of water. It would have been impossible to make an examination while the load was on and with the load not on, it is doubtful if such a crack could have been found.

N.—Assembly of Results

64.—Loads for 40-Foot Head Before Cracking.—The curves in Fig. 130 show the amount and distribution of the load, or equivalent load, on a vertical section of the dam at the center line and on sections 10 ft. and 30 ft. from the center. For the latter two distances the results represent the average for symmetrical sections on opposite sides of the center line. These values are the average results for Tests 6 and 7, which were made before the vertical crack at the center line in the upper part of the dam occurred. Fig. 130 is a compilation of loads carried by (1) direct thrust, as shown in Fig. 121; (2) vertical bending, as shown in Fig. 129; and (3) horizontal bending, as shown in Fig. 125. Since the direct thrust has been taken as constant

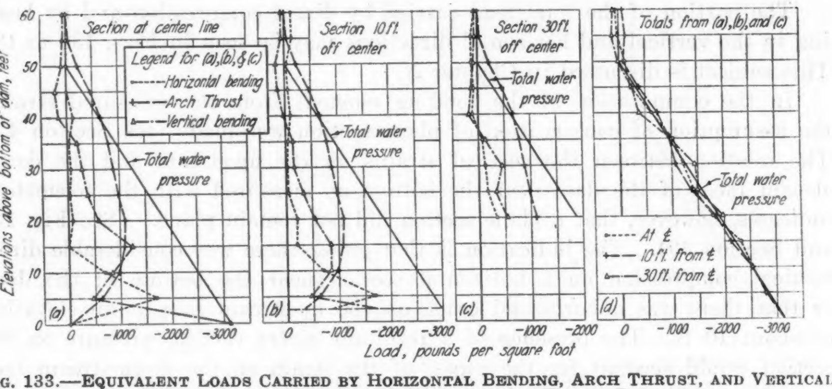


FIG. 133.—EQUIVALENT LOADS CARRIED BY HORIZONTAL BENDING, ARCH THRUST, AND VERTICAL BENDING, AND THEIR TOTALS COMPARED WITH WATER PRESSURE. HEAD OF WATER, 50 FEET.

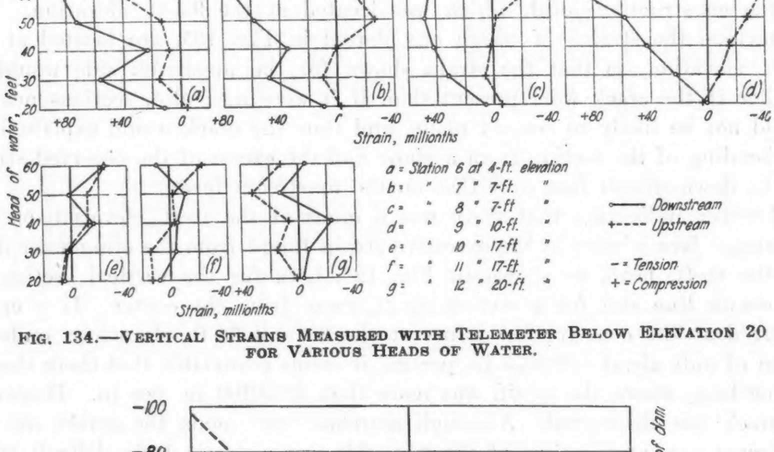


FIG. 134.—VERTICAL STRAINS MEASURED WITH TELEMETER BELOW ELEVATION 20 FOR VARIOUS HEADS OF WATER.

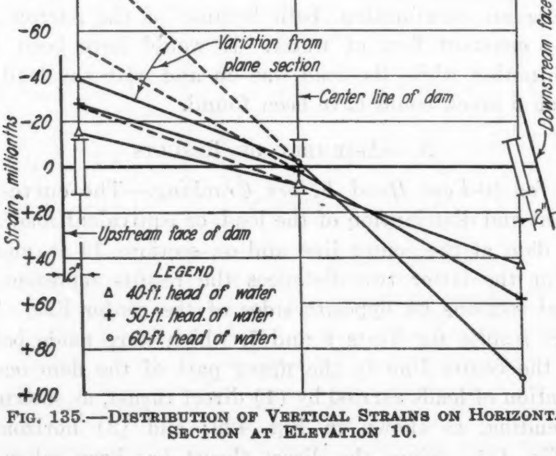


FIG. 135.—DISTRIBUTION OF VERTICAL STRAINS ON HORIZONTAL SECTION AT ELEVATION 10.

throughout the length of any arch rib (see Section 57), the equivalent loads carried by direct thrust as found for all three sections shown in Fig. 130 are equal, and this justifies using the values from Fig. 121 for all three sections. The equivalent loads carried by vertical bending on each of the three sections of Fig. 130 are taken from the values at the right in Fig. 129.

The load carried by horizontal bending was small for all the sections shown in Fig. 130 (*a*), (*b*), and (*c*), but there is some indication that it was greater 30 ft. away from the center than at the center. Fig. 125 shows more clearly the variation (with the distance from the center line) in the load carried by horizontal bending. The principal part of the load was carried by vertical bending and direct thrust. It is pointed out in Section 63 that for all three sections the bending load on the vertical element was positive in the upper part of the dam. Below an elevation of about 35 ft., the vertical bending loads were negative and increased in value toward the bottom as far as they were determined. The loads carried by direct thrust were everywhere negative and increased in value from approximately zero at the top to a maximum at an elevation between 15 and 20 ft. Smooth curves drawn through the points for equivalent load carried by direct thrust and vertical bending, respectively, indicate that at the bottom of the vertical center-line section practically all the load was carried by vertical bending and none by direct horizontal thrust. No determination of the load at the bottom could be made; hence this statement refers merely to the load distribution above the bottom. The load distribution for the sections away from the center had a tendency to follow lines similar to those described for the center section.

The sums of the loads carried by direct thrust, vertical bending, and horizontal bending, are shown in Fig. 130 (*d*) for sections at the center line and at 10 and 30 ft. from it. The agreement between the sums of the individual loads and the total water pressure is very good. For the sections at the center and 10 ft. away there was, however, a slight deficiency in the total load at the 25-ft. elevation. No satisfactory explanation for this deficiency has been found. Although there was no water pressure above the 40-ft. level, Fig. 130 indicates that there were loads above that point. These loads represent the reactions of horizontal upon vertical elements, and of vertical upon horizontal elements; therefore, they are not external loads so far as the entire dam is concerned. Fig. 130 (*d*), which gives the sum of the load carried by different agencies, shows that the sum of the negative loads above the 40-ft. level was very nearly equal to the sum of the positive loads and that the resultant was zero, as it should be since the water pressure was zero.

65.—Loads for 40-Foot Head After Cracking.—In raising the water a vertical crack occurred in the upper part of the dam on its center line when the head was between 47 and 50 ft. The water was then lowered to an elevation of 40 ft. and readings were taken. (See Section 40.) These observations afford a basis for determining the distribution of load after the crack formed. The loads so determined are shown in Fig. 131. The individual items of load are taken from Fig. 120, for load carried by direct thrust; Fig.

125, for load carried by horizontal bending; and Fig. 128, for load carried by vertical bending.

The characteristics of Fig. 131 are much the same as those of Fig. 130. The effect of the formation of the crack on the load distribution was unexpectedly small. The indication is that more load was carried by direct thrust and less by vertical bending after the crack occurred than before. While these results are based on only one test and should be taken merely as indications, the shifting of load in this manner need not be considered as impossible, nor even as improbable. The crack extended down from the top about 10 ft. on the down-stream face and 5 ft. on the up-stream face. The effect of this crack probably would be to cause shifting of the load from the horizontal to the vertical elements, but with the opening of the horizontal crack between the bottom of the dam and the foundation, during the previous loading to a head of 50 ft. (see Fig. 76), the reduction of the fixity of the vertical elements would cause a shift in load from the vertical to the horizontal elements. Fig. 131 indicates that this shifting more than balanced the shifting from the horizontal to the vertical elements.

The agreement of total loads in Fig. 131 (*d*) with the total water pressure is very good. The deficiency of load at the 25-ft. elevation is not so marked as that shown in Fig. 130, although it is still present. The agreement at the top is not as good as that found for the 40-ft. head before the cracking. Greater errors might well be expected in Fig. 131 than in Fig. 130, since the former represents results of only one test, whereas the latter represents the average results for two tests.

66.—Loads for 50-Foot Head.—The distribution of loads for a head of 50 ft. of water is shown in Fig. 132, the arrangement of which is the same as that of Figs. 130 and 131.

The shift of load from the vertical to the horizontal elements, noted in Section 65 and Fig. 131, is accentuated in Fig. 132 with the increase of the head to 50 ft. This shifting of load, although present, is not very marked above the 20-ft. level, where it consists in transferring from vertical bending to direct thrust. Below the 20-ft. elevation the shifting is mainly from vertical bending to horizontal bending.

In all previous cases the equivalent load carried by horizontal bending has been so small as to be almost negligible. With the 50-ft. head of water the load carried by horizontal bending became large for the first time, and near the bottom of the dam it was very important. With a head of 40 ft. it was noted (Section 64), that at the bottom the entire load appeared to be carried by vertical bending and none by horizontal bending, or direct thrust. With a head of 50 ft. (see Fig. 132) an extension of the load curves toward the bottom of the dam would indicate that all the load was carried by horizontal bending and none by vertical bending or direct thrust. However, for the reason given in Section 61, the entire load at the bottom has been assumed to be carried by vertical bending. This is purely an assumption, but whatever value is assumed for load at the bottom, the results for points higher up on the dam will not be affected.

The agreement of the total loads in Fig. 132 (*d*) with the total water pressure is as good as that shown in Fig. 130 and 131 (Sections 64 and 65).

Again there is a deficiency in the total load which is a little more marked at the 25-ft. level than at other elevations.

67.—Loads for 60-Foot Head.—The distribution of loads for a head of water of 60 ft. is shown in Fig. 133, the arrangement of which is the same as that for Figs. 130 to 132. For the 50-ft. head (Section 66) it was pointed out that smooth curves drawn through the load points indicated that at the bottom of the dam all the load was carried by horizontal bending, whereas for the 40-ft. head (Section 64) it was all carried by vertical bending. Under the 60-ft. head another change in load distribution evidently had taken place and a smooth curve drawn through the load points in Fig. 133 indicates that all or most of the load at the bottom was carried by direct thrust, that is, by arch action.

With the increase of head from 50 to 60 ft. the proportionate load carried by horizontal bending evidently grew much larger since the horizontal tension on the down-stream face near the bottom of the dam increased rapidly until a vertical crack occurred under a tensile stress of about 500 lb. per sq. in. with the reservoir standing full. With the formation of this crack the tensile stress in its vicinity must have disappeared, or nearly so, and correspondingly the ability of the section to resist horizontal bending moment must have been greatly decreased. The effect of the formation of this crack may be seen in Fig. 133 in the almost complete disappearance of the load carried near the bottom by horizontal bending. In fact, the horizontal bending load at the 5-ft. elevation appears to have been negative.

With the decrease in the proportionate load carried by horizontal bending, there was a marked increase in the load carried by direct compression near the bottom. However, for the reasons given in previous sections, the vertical element was assumed to carry all the load at the bottom.

For this case the sums of the loads carried by horizontal bending, arch thrust, and vertical bending, agree well with the total water pressure on the dam at any point in its height. However, the slight deficiency at Elevation 25 is still present.

68.—Comparison of Observed with Computed Stresses in Horizontal Elements.—In order to bring out the relation between the results of the tests and the results obtained by some of the methods used in designing arch dams, Fig. 136 has been prepared. Horizontal elements at the 30 and the 50-ft. elevations, respectively, were chosen as representing typical elements of the arch dam, and the stresses on the down-stream face obtained from the measured strains by a modulus of elasticity of 3 600 000 lb. per sq. in., and a Poisson's ratio of 0.15, are shown, including both total stresses and bending stresses.

For the 30-ft. elevation a continuous row of strain-gauge stations was available which gave the total strains at 10-in. intervals throughout the entire length of the arch. For this study the average curves shown in Fig. 114 for the total and the bending strains, respectively, were used. For the 50-ft. elevations total strains at symmetrical stations on opposite sides of the center line were averaged, and a curve drawn to correspond. The bending strains for the 50-ft. elevation were obtained from the clinometer data by

the process described in Section 58. Both curves for the 50-ft. elevation are shown in the upper part of Fig. 122. For the comparison with "measured" arch stresses, stresses were computed on the assumptions, first, that the arch ribs were fixed in direction and position at the ends; second, that they were hinged but fixed in position at the ends; and, third, that they were free at the ends, both as to direction and position.

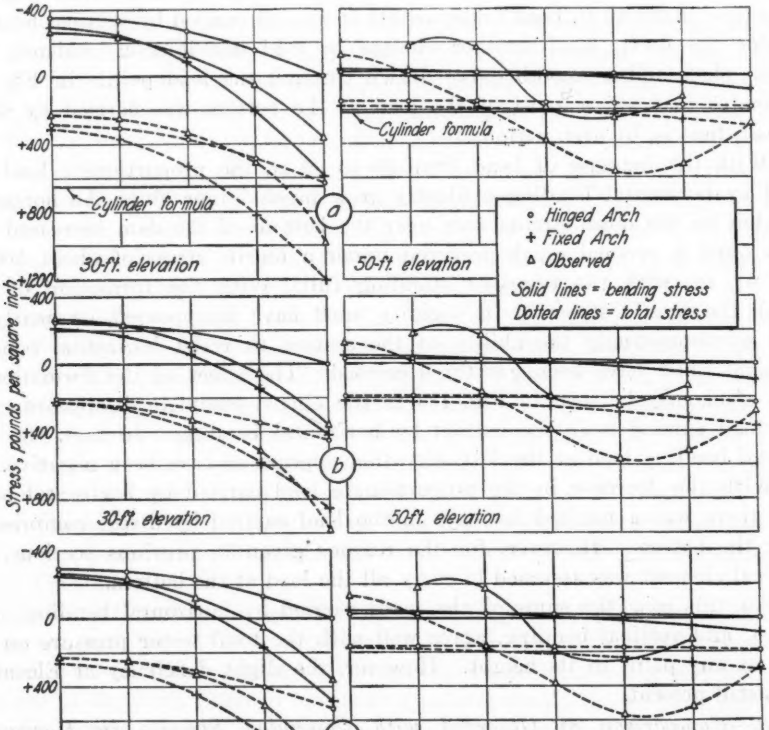


FIG. 136.—RELATION OF RESULTS OF TESTS AND RESULTS OF METHODS USED IN DESIGNING ARCH DAMS.

A condition corresponding to the third assumption exists when a complete cylindrical section is subject to uniform radial pressure. The results for this condition are obtained by the application of the "cylinder" formula to a circular arc. For the first two conditions the application of the analysis of an elastic circular arch is necessary. For this study the analysis according to Cain* was used. Uniform radial loads were considered in all three cases. In Fig. 136 (a) full water pressure was assumed. The load used in Fig. 136 (b)

* "The Circular Arch Under Normal Loads," by William Cain, M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. LXXXV (1922), p. 233.

was that determined in the design of the dam* to have been carried by the horizontal elements. In Fig. 136 (*c*) a load of the intensity found from the test to have been carried by the horizontal element at the center of its span was assumed as applied throughout the length of the element. This load was very nearly equal to the average load carried by the horizontal element. All stresses in the following comparison apply to the down-stream face of the dam.

Since with conditions under which the cylinder formula was applied in this study, no bending can occur, the total stress found is a direct compression, and the compression is necessarily constant from end to end of the arch. The value of this compression is shown in Fig. 136 (*a*) for the full load, by straight horizontal lines. At the 30-ft. elevation the stress is 650 lb. per sq. in. and is seen to be larger than the observed total stress for all points except for a few feet close to the abutment. This stress is about 180% in excess of the observed stress at the center line, while at the abutment it is about 35% less than the observed stress. At the 50-ft. elevation the stress by the cylinder formula is 217 lb. per sq. in. compression, whereas the observed stress was a tension of about 125 lb. per sq. in. at the crown, and a compression of about 290 lb. per sq. in. at the abutment. About 20 ft. from the abutment the observed stress was about 575 lb. per sq. in. compression, or about 165% greater than that given by the cylinder formula. It is quite obvious that the cylinder formula is entirely inadequate to represent conditions found in an elastic arch of this kind. For the second and third loading conditions, Fig. 136 (*b*) and (*c*), a comparison of the stresses is not shown.

For the 50-ft. elevation the total stresses computed for the fixed or the hinged arch are much the same as those computed by the cylinder formula, and the agreement with the test results is, therefore, little or no better than that just discussed for the cylinder formula. Much the same comment regarding the agreement with observed stresses at the 50-ft. elevation may be made with respect to the stresses computed for the 40-ft. and the 60-ft. elevations and for the loads used in Fig. 136 (*b*) and (*c*), since, as shown in Fig. 124, the form of curve of observed strain is much the same for the 40, 50, and 60-ft. elevations. Furthermore, it would not be possible with any uniform radial loading to produce the reversal of curvature shown in the strain curves for these elevations. Therefore, for the portion of the dam at and above the 40-ft. elevation, the assumption of an elastic arch with uniform radial loading did not give stresses sufficiently close to those found from the test to warrant this assumption as a basis for design. That is, even when cantilever action was taken into account, assuming the load carried by all vertical elements at a given elevation as equal, the remaining load on the arch elements did not give computed stresses in the arches even reasonably close to the observed stresses for this portion of the dam. If in design the variation in the loads carried at a given elevation by the different vertical elements could be correctly determined and taken into account, an examination of the arch ribs under the influence of the resulting variable radial load should be expected to show better agreement between computed and observed stresses.

* Described in Part II, Section 3, p. 13.

The conditions for the 30-ft. elevation are quite different. Here for the full water load the total stresses (sum of bending and direct stresses on the down-stream face) as computed for either the fixed or the hinged arch (see Fig. 136 (*a*)), varied from the center to the abutment in much the same manner as the observed stresses. For the fixed arch the computed stresses are 40, 60, and 20% greater than the observed stresses at the center, quarter-point, and abutment, respectively, and at no place was the observed stress the greater of the two. For the hinged arch the computed stress was nearly equal to the observed stress at the center and at the quarter-point, while at the abutment it was about 35% less than the observed stress.

The bending stresses computed for full load on the fixed arch at the 30-ft. elevation are not very different from the observed bending stresses, but in general are slightly greater. Those for the hinged arch are much higher in tension than the observed stresses.

Fig. 136 (*b*) shows that the computed total stresses for the fixed arch under the load determined in the design of the dam agree fairly well with those found in the test. At the abutment the computed stress was about 15% less than the observed stress. At all other points the computed stress is greater than the observed stress. For the hinged arch the variation between computed and observed stresses is greater than for the fixed arch and the computed stresses are generally the smaller. The differences between the computed stresses for the fixed arch and the observed stresses are slightly larger for the bending than for the total stresses. For the bending stresses the differences are about the same as for the total stresses. Thus, as judged by this loading, the true conditions were more nearly those of a "fixed" than of a "hinged" arch.

The loads considered in preparing Fig. 136 (*c*) gave computed total stresses which for both the hinged and the fixed arch agree with observed stresses better than those in Fig. 136 (*a*), but not quite so well as those in Fig. 136 (*b*).

The observed stresses for the 30-ft. elevation lie for the most part between the stresses computed for the fixed arch and those for the hinged arch, for each of the three loadings considered in Figs. 136 (*a*), (*b*), and (*c*).

69.—Comparison of Observed Deflections, Moments, and Loads on Central Vertical Element with Those Used in Design of Test Dam.—In Fig. 126 is shown at the left the observed deflection of the central vertical element of the dam, and the deflection computed in the design.*

In the middle and right-hand diagrams are shown the moments and loads deduced from the measured deflections, and for comparison with them, the moments and loads used in the design of the dam. In the design the dam was assumed to be fixed at the bottom, that is, a tangent to the elastic curve at the bottom was assumed to remain vertical throughout the loading, although the deflection curve shows that this condition did not exist. The modulus of elasticity used in the design was only 2 000 000 lb. per sq. in., while that found in the test and used in reducing the observed deflections to moments was 3 600 000 lb. per sq. in. This difference in modulus of elasticity should be

* Described in Part II, Section 3, p. 13.

given consideration in comparing the deflections. The agreement of the observed with the computed deflection is good only at the 30-ft. elevation. However, the agreement or disagreement in the amount of deflection is of no significance, as it is the curvature of the deflection curve in combination with the modulus of elasticity that determines the moments. The difference of curvature of the two deflection curves is offset by the difference of the corresponding moduli of elasticity. The computed and the observed moment curves agree fairly well.

The agreement of the loads obtained from the observed deflections with those corresponding to the design deflections is fairly good. The only place where the disagreement is important is close to the bottom, where there is a marked irregularity in the curve of loads deduced from the test. Fig. 128 shows that this irregularity was not present in the test to the 40-ft head. The change in the manner of carrying the load is discussed in Sections 66 and 67. If as good agreement with the design loads were found for the vertical elements away from the center as is shown in Fig. 126, a better agreement between the observed and the computed arch stresses should be expected than that pointed out in Section 68.

In the design only the central element was examined, and the load found for it was assumed to apply to all the vertical elements. Assuming that the equivalent load carried by direct thrust in any horizontal element is constant, then for a given elevation the variation in the load carried by the vertical elements must be equal (but opposite in sign) to the variation in equivalent bending load along the length of a horizontal element. Therefore, the load curves in Fig. 124 may be used for judging the variation in load carried by the vertical elements at different distances from the center line. The variation is sufficiently marked to show that the difference between the observed and the designed stresses in the arch elements at the 50-ft. elevation should not be unexpected.

70.—Summary of Results.—The dimensions of the dam as constructed conformed closely to those shown in the design. This applies to the curvature and thickness of the dam, and to the slope and regularity of the abutment lines.

The concrete showed a satisfactory degree of uniformity and strength. The 28-day strengths averaged about 10% greater than the designed strength of 1 800 lb. per sq. in. The average strength at 3 months was about 2 800 lb. per sq. in., and the uniformity appears somewhat better at 3 months than at 28 days.

During the hardening period the strains computed from the temperature, using an average coefficient of expansion for Elevations 30 and 40, followed the same general trend as that of the measured vertical strains. For the 50-ft. elevation the strains computed from the temperature changes followed more nearly the measured horizontal strains. For all three elevations there was reasonable correspondence between the measured horizontal strains and the strains computed from the deflections by an approximate formula.

Later, during the curing period, it was found that the relation between temperature and strain in the dam was about as should be expected if the

coefficient of expansion was 0.0000077 in. per in. per degree centigrade, the average value determined in the Berkeley tests. This value appears unusually low, but seems to be confirmed throughout the experimental work on the test dam.

When the dam dried out, this was generally accompanied by a temperature rise, and a large part of the resulting strains may be accounted for by the temperature change alone, without assuming any strain due primarily to drying out.

The test results show a reasonable agreement between observed deflections and the deflection computed by the Cain formula from the corresponding temperature changes. The agreement between observed and computed deflections is best if, in the computations, it is assumed that near the bottom of the dam the horizontal elements were fixed and that near the top they were hinged.

To a large extent the stresses caused by temperature changes probably were bending stresses. For temperature changes of about 8 and 3.5° cent., at the top and bottom, respectively, the highest apparent bending stress was about 100 lb. per sq. in. The actual stress probably was somewhat smaller than this.

The first evidence of cracking was a separation from the abutments on the up-stream face about 12 ft. above the bottom of the dam. The separation occurred within a few days after the pouring of the concrete at that elevation. Before the beginning of the load-testing program in July, the dam on the up-stream side was cracked loose from the abutments most of the way from the top down to about the 12-ft. elevation.

On the down-stream face the cracking away from the abutments extended down about 10 ft., prior to the period from June 24 to July 2, during which time the dam was allowed to dry. During this latter period the cracking extended downward to about Elevation 30. There is no evidence that it extended farther at a later date.

The cracks that occurred previous to the load tests were between the dam and the bed-rock. None occurred within the dam itself. A crack between the dam and the foundation on the up-stream face at the bottom occurred under a head of water of between 30 and 40 ft. This crack increased in width to about 0.05 in. at the maximum head of 60 ft. Under a head of water of about 50 ft. a vertical crack appeared on the vertical center line of the dam extending downward from the top to an elevation of about 49 ft. on the down-stream face. It has since extended downward to about Elevation 40. Although this crack extended entirely through the thickness of the dam, it was narrower on the up-stream than on the down-stream face. Under a head of 60 ft. a vertical crack occurred near the bottom of the dam under a tensile stress of about 500 lb. per sq. in. This crack finally extended to a height of about 13 ft. It is not known how far into the interior of the dam it went. Extremely fine horizontal cracks in the construction joints at Elevations 30 and 35 were discovered when the dam had been under a head of water of 60 ft. for about 2 days.

Two additional fine cracks were found after the dam had been through several storms and had been under a load of débris for more than six

months. Starting at a point on the abutments on opposite sides of the center of the dam, at an elevation of about 8 ft., these cracks incline slightly toward the center line.

With a head of less than 50 ft. of water, there was no leakage through the dam, although there was a small amount around its ends. With a head of 50 ft., or more, there was a slight leakage through the dam and increased leakage at the abutments. The total was estimated at about 6 gal. per min., of which only a very small proportion came through the dam.

The temperature changes between the no-load and the load readings in all tests were very small. It is to this fact that a considerable part of the success in interpreting the data must be attributed.

The measured movement of bed-rock increased consistently with the increase of head. For a head of 60 ft. the measured radial movement at the bottom was about 0.0012 in. At elevations of 17 and 33 ft. the movements along the long chord were about 0.037 and 0.032 in., respectively. At the top of the dam the net change in chord length was approximately zero. Where it was possible to compare these movements with results computed by the use of Vogt's formula, a fair agreement was found.

The regularity and smoothness of the deflection curves are evidence of the accuracy of the measured deflections. The accuracy was sufficient to determine strains, moments, shears, and loads by successive differentiation of the deflection curves. The maximum deflection occurred near the mid-height of the dam for all loadings. With the 60-ft. head the maximum deflection was about 0.39 in. The upper horizontal element of the dam deflected up stream within a distance at either end equal to a little less than one-third the length of the element. At some places an up-stream deflection extended as far down as 20 ft. below the top of the dam. The region covered by the up-stream deflection did not change materially with variation in the head of water.

Tilting of the dam from its original vertical position at the base took place. Accompanying the tilting there appear to have been strains which were not distributed in accordance with the law of conservation of plane section. This caused serious difficulties in the effort to interpret the deflections and strains into bending moments.

The strains measured directly with various instruments agreed well with each other and with those found by differentiation of the measured deflections.

For the most part the directions of the principal strains were approximately vertical and horizontal. This indicates that the division of the dam into imaginary vertical and horizontal elements for purposes of analysis not only serves convenience but also indicates that the stresses found in this manner will generally be close to the principal stresses.

Below the 20-ft. elevation the larger of the principal strains near the foundation was in general nearly perpendicular to the abutment line. At the interior points it took approximately a horizontal direction. The largest strain found corresponds to a stress of about 1 100 lb. per sq. in.

The dam underwent severe floods which caused a flow of water at least 3 ft. deep on its crest and which deposited débris against the up-stream face to a maximum depth of about 45 ft. There was no evidence that the dam

was near failure at any time due to these floods, but there was a slight increase in deflection, and an additional fine crack occurred near the bottom.

There was considerable variation in the values for the modulus of elasticity found for different specimens and by different methods. The average value of 3 600 000 lb. per sq. in. was used in interpretation of the test data. A check on the reasonableness of this as a working modulus is found in the fact that the total loads at various heights computed from the test data with the use of this modulus agree very well with the total water pressures applied.

The average value of Poisson's ratio was found to be about 0.15 and this was used in the interpretation of the test data. Because of the smallness of this value the effect of Poisson's ratio on the results generally has not been very great, although in some instances its neglect would cause an error of 18 to 20% in the stress computed from the observed strain.

The strain due to arch thrust was found to be nearly constant along the arch ring at the 30-ft. elevation. At other elevations the variations from uniformity along the arch ring were not consistent enough to warrant any other assumption than that of uniformity of direct thrust along the arches. The strains due to the bending of horizontal elements were quite large. Evidence of this is found in the formation of cracks in the dam on the center line near the top and near the bottom. At the one-quarter points near the top of the dam the tensile strains were so great as to indicate the imminence of cracks. The point of zero bending moment was about 25 ft. from the center line.

Points of maximum tensile strain in the vertical elements occurred on the down-stream face of the dam at approximately the 30-ft. elevation, both for the center-line section and for the sections away from the center line and for all heads of water. The largest tensile strain found was about 0.000075 in. per in. This probably is sufficient to account for the cracks that developed along the construction joints after the dam had been under a 60-ft. head of water for 2 days. Except for the presence of the construction joints, a crack probably would not have formed under this strain.

There are indications that under the 60-ft. head a horizontal crack occurred on the up-stream face of the dam at the 10-ft. elevation. However, no crack at this location actually was found.

The effect of torsion on the load distribution was small enough to be neglected, as far as any indications from the test results are concerned. Except near the bottom of the dam under a head of water of 50 ft., the equivalent load carried by horizontal bending was very small everywhere. However, on account of the great length and slenderness of the upper horizontal elements, the strains due to horizontal bending loads were important and even resulted in a vertical crack near the top. The greater part of the pressure was carried to the abutments by direct thrust, that is, by horizontal arch action. In fact, in the upper 20 ft. of the dam, more than the total load was carried in this way. Consequently, the load carried by vertical bending in this region was principally an up-stream pressure. The point at which no load was carried by the vertical elements generally lay at an elevation of about 37 ft. Below this elevation an important part of the total load was carried to the

FIG. 137.—VIEW OF STEVENSON CREEK TEST DAM, DECEMBER 4, 1926, WITH WATER FROM THE GREAT STORM OF LATE NOVEMBER STILL FLOWING OVER THE CREST.

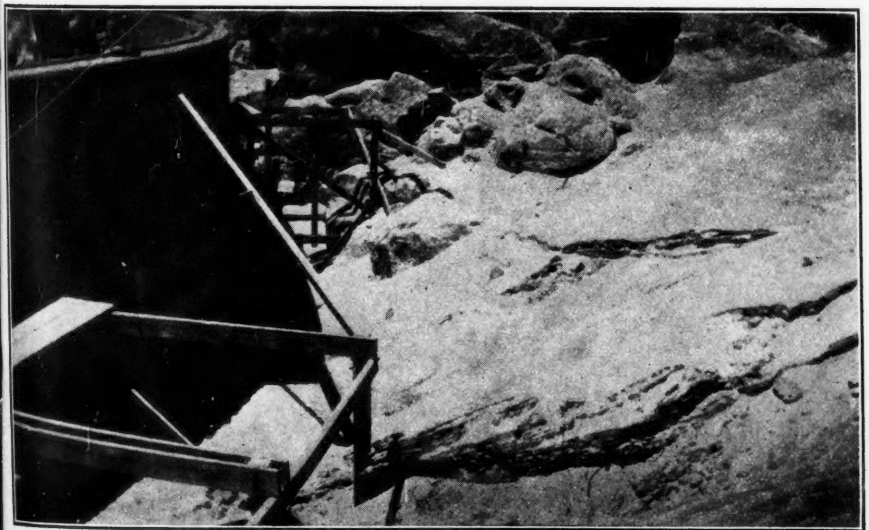


FIG. 138.—SAND AND ROCK DÉBRIS BACK OF STEVENSON CREEK TEST DAM, JULY 14, 1927, DEPOSITED BY FLOODS OF NOVEMBER AND DECEMBER, 1926.

abutn
to th
was, i
Tl
condi
Fe
tion
suffici
as a
comp
with
result
than
Fe
obtai
good.
71
days
flowe
about
quan
or mo
early
leavi

abutments by vertical bending. The sum of the loads found to be carried to the abutments by horizontal bending, direct thrust, and vertical bending was, in all cases, very nearly equal to the total water pressure.

The "cylinder formula" was found to be inadequate for representing the conditions existing in an elastic arch of this type.

For the portion of the dam at and above the 40-ft. elevation, the assumption of an elastic arch with uniform radial loading did not give stresses sufficiently close to those found from the test to warrant such an assumption as a basis for design. For the 30-ft. elevation the agreement of the stresses computed on the assumption of uniform radial load on the arch elements with those found from the test was reasonably good. At this elevation the results based on an assumed fixed arch agreed with the observed results better than those based on an assumed hinged arch.

For the central vertical element the agreement of the moments and loads obtained from the test with those used in design was found to be reasonably good.

71.—Present State of the Dam.—Conditions at Stevenson Creek Dam a few days after the great flood of late November, 1926, are shown by Fig. 137. Water flowed over the dam all that winter, with depths on the crest varying from about 3 ft. during the flood to a fraction of an inch at later dates. A large quantity of fine and coarse rock débris, including fragments probably 1 cu. yd. or more in volume, was washed into the reservoir. During the late spring and early summer, the water drained out through the obstructed under-sluice, leaving the reservoir in the condition shown by Fig. 138.

The
stresse
mate k
ture is
for wh
the st
For th
straine
tions c
the ax

Con
from t
section
to arc
tary a
due to
cantile
ring i
from c

It
cured
entire
place
tent c
to cha

Th
stance
filled
reserv
when
canno
Betwe
chang
at an
50° F
moist
the re
satura
ring

*
of Cal

PART V.—PHYSICAL PROPERTIES OF CONCRETE

By Raymond E. Davis,* M. Am. Soc. C. E.

1.—Preliminary Statement

The translation of field measurements of deflection and deformation into stresses, where actions are as complicated as in an arch dam, requires an intimate knowledge of the physical properties of the concrete of which the structure is composed. Unlike the metals, concrete is a non-homogeneous material for which the elastic properties vary with age and with other factors, and the stress-strain relation, even for low stresses, is not that of a straight line. For the simple case of pure compression, such as might occur in an unrestrained element of the arch near the top of the dam under constant conditions of temperature and moisture content of the concrete, it is required that the axial stress-strain relation be known.

Considering the relation between stress and deformation at points remote from the top of the dam, the problem becomes more complicated. A cross-section of any element of the arch ring is subjected to direct compression due to arch action and to bending due to horizontal beam action. Any elementary area in the plane of the horizontal is subjected to direct compression due to the weight above, and to flexural stresses due to vertical beam or cantilever action. Under these conditions the lateral strains, or those occurring in a plane at right angles to the direction of stress, cannot be omitted from consideration, and Poisson's ratio becomes an important factor.

It is well known that when concrete is cured in air it contracts and when cured in water it expands, and that for a long period, probably during its entire life, the volume changes with variations in moisture, expansion taking place as the moisture content increases and contraction as the moisture content decreases. Also, like other structural materials, concrete is subjected to changes in length due to variations in temperature.

These factors may be confined within very narrow limits in certain instances, as, for example, in an experimental dam where the reservoir may be filled and emptied quickly, where the dam may be kept saturated when the reservoir is empty, and where observations of strains may be taken at night when temperature fluctuations are small. On the other hand, these conditions cannot be controlled in dams subjected to the normal conditions of service. Between periods of reservoir full and reservoir empty there is likely to be a change in temperature of 100° Fahr. in certain portions of many dams and at any given time the temperatures within the dam might have a range of 50° Fahr. When the reservoir remains empty for a considerable period, the moisture content is likely to be small and the arch ring will contract; when the reservoir is full, the concrete near the up-stream face of the dam will be saturated and that near the down-stream face may be nearly so, and the arch ring will be elongated due to moisture absorption.

* Cons. Structural Engr.; Prof. of Civ. Eng., in Chg., Materials Testing Laboratory, Univ. of California, Calif.

Concrete is plastic, that is, under continued stress, its deformations tend to increase with the passage of time, coming to rest when plastic equilibrium is reached. Thus, in a dam subjected to continuous water pressure over a period of time, strains would increase in magnitude, and accompanying this continued flow a redistribution of stresses might be expected, the higher stresses perhaps being in a measure relieved.

These and other considerations led to laboratory tests to determine the physical properties of concrete which would have a bearing on the interpretation of data collected in the field. Since the fall of 1924 tests have been continuously in progress in the Materials Testing Laboratory of the University of California, under the direction of the writer. In September, 1926, tests were begun on concrete identical in composition with that entering the Test Dam. For most of the tests the time element is a factor; the tests are still in progress and are likely to be continued for some time in the future.

2.—Tests Completed or in Progress

The principal tests in progress are those to determine:

(1) Modulus of elasticity and Poisson's ratio, as influenced by mix, aggregate, age, and other factors.

(2) Volumetric changes due to causes other than variation in temperature, as influenced by mix, gradation and character of aggregate, age, humidity, etc.

(3) Thermal coefficient of expansion and the influence of temperature, moisture, age, character of aggregate, mix, etc.

(4) Flow under constant sustained compressive stress, and the effect of mix, gradation of aggregate, age, stress, moisture conditions, etc., upon flow.

(5) Permeability, rate of penetration, rate of percolation, and the influence of age and time upon these quantities at various pressures.

3.—Temperature and Humidity Control

Early in the investigation it was deemed necessary to secure close control of temperature and moisture conditions. To this end the specimens have been for the most part stored and the tests have been carried out in insulated rooms, now four in number, specially constructed and electrically heated, with thermostat controls, so that any desired temperature above normal outside temperature may be maintained. One room is equipped with water sprays and the water is electrically heated to room temperature before being sprayed into the air. The second room has a vaporizing device and a hydrostat by which the humidity may be regulated to any desired percentage above 50% and which so far has provided a constant humidity of 70 per cent. The third room (Fig. 139) is equipped with a hydrostat and a dehydrator which automatically maintain the humidity at any desired percentage below that outside and which thus far have maintained a relative humidity of 50 per cent. The fourth room, in addition to being electrically heated, is provided with refrigeration coils so that any temperature between 10° and 135° Fahr. may be maintained.

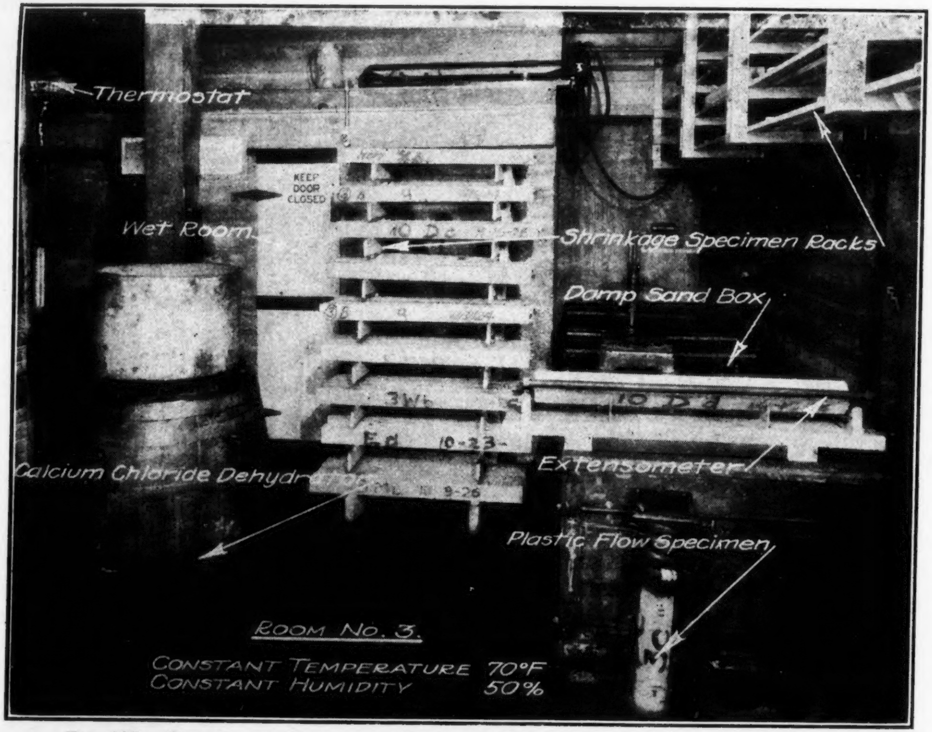


FIG. 139.—ONE OF THE INSULATED ROOMS FOR THE CONCRETE TESTS AT THE UNIVERSITY OF CALIFORNIA.

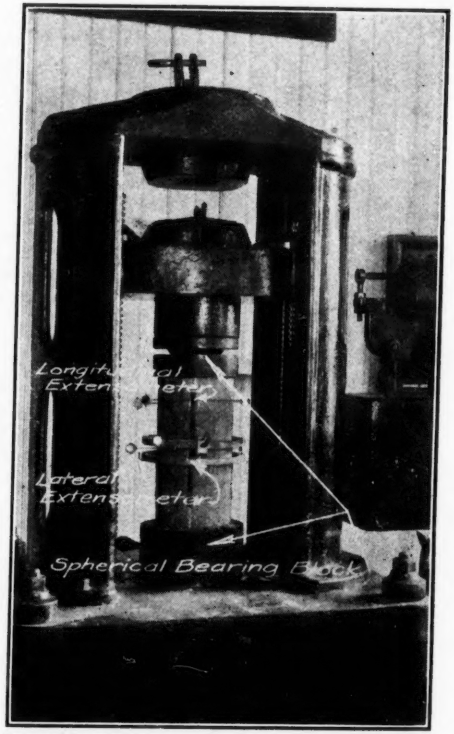


FIG. 140.—MODULUS OF ELASTICITY AND POISSON'S RATIO OF CONCRETE: A CYLINDER WITH EXTENSOMETERS ATTACHED.

4.—Modulus of Elasticity and Poisson's Ratio—1924 Series

These tests are being made to determine the effect of richness of mix and of age. Mixes are from 1:3.5 to 1:6, the aggregate being trap rock and beach sand. The series is designed to include ages up to 3½ years. The specimens are 6 by 12-in. cylinders, and loads are applied in compression. The cylinders are stored in damp sand. Both axial and transverse strains are measured with mirror extensometers, observations being made to 0.000001 in. When the series is completed in April, 1928, more than two hundred cylinders will have been tested. (See Fig. 140.)

TABLE 25.—Secant Modulus of Elasticity* and Poisson's Ratio, 1924 Series.

Specimen No.	Mix.	Age, in days.	Average ultimate strength, in pounds per square inch.	Average secant modulus at 1 000 lb. per sq. in.	Average Poisson's ratio at 1 000 lb. per sq. in.
1a- 6a	1:3.5	42	3 760	3 580 000	0.17
19a-24a		73	4 450	3 800 000	0.15
31a-36a		95	4 400	4 210 000	0.19
7b-12b		105	4 790	4 117 000	0.18
31b-36b		225	4 980	4 263 000	0.20
49b-54b		316	5 347	4 840 000	0.23
1c- 6c		465	6 100	4 890 000	0.19
19c-24c		576	5 150	4 650 000	0.17
37c-42c		683	6 515	5 200 000	0.20
13a-18a	1:4.5	40	3 620	3 730 000	0.17
37a-42a		73	4 050	4 180 000	0.18
49a-54a		95	3 810	4 480 000	0.18
1b- 6b		100	3 800	4 447 000	0.19
19b-24b		220	5 260	4 817 000	0.21
37b-42b		313	4 855	4 860 000	0.20
7c-12c		465	5 040	5 100 000	0.19
25c-30c		576	5 640	4 830 000	0.18
43c-48c		683	4 520	5 210 000	0.20
7a-12a	1:6	42	2 980	3 450 000	0.18
25a-30a		73	3 020	3 680 000	0.18
43a-48a		95	3 240	3 810 000	0.16
13b-18b		100	3 020	3 552 000	0.18
25b-30b		220	3 125	4 048 000	0.20
43b-48b		312	3 403	3 905 000	0.20
13c-18c		465	3 650	4 630 000	0.20
31c-36c		576	3 430	4 080 000	0.18
49c-54c		683	3 320	4 560 000	0.20

* The secant modulus of elasticity is obtained by dividing the unit stress (pounds per square inch) by the corresponding unit deformation (inch per inch). It is represented by the slope of the line connecting the origin with the proper point on the stress-deformation curve.

The result so far (Table 25) may be summarized as follows:

(1) For concrete in compression both axial and transverse stress-strain diagrams are curved lines; but for a given mixture, the older the concrete, the more nearly do the curves approach straight lines.

(2) The indications are that, other things being equal, an increase in the cement ratio is accompanied by an increase in the modulus of elasticity until a certain maximum is reached beyond which

if the cement ratio is still further increased, there is a reduction in the value of the modulus.

(3) The modulus of elasticity, like the strength, increases with age, but after a year the increase is very slow. This increase with age is more marked for rich concretes than for lean ones. At the age of 2 years the modulus may be 50% higher than at 1 month.

(4) In a general way Poisson's ratio varies with the modulus of elasticity as demonstrated by Fig. 141, which shows for each cylinder the relation between the secant modulus of elasticity and Poisson's ratio at 1 000 lb. per sq. in. compressive stress. Hence, in general, it increases with the age of the concrete and with the cement ratio until the maximum is reached, beyond which an increase in the cement ratio results in a decrease in Poisson's ratio. In this particular, however, the trend is not so marked as in the modulus of elasticity.

(5) Poisson's ratio varies with the stress, being, in general, somewhat greater for low unit stresses than for high ones; but, usually, for stresses of more than 200 lb. per sq. in. the ratio does not increase more than 10%, and for a given concrete with a stress greater than 1 000 lb. per sq. in. the ratio seems to be fairly constant.

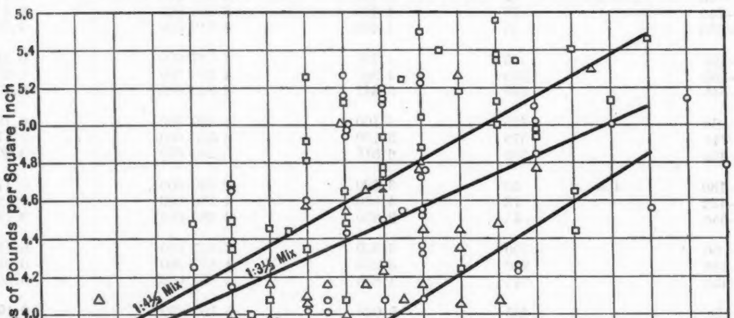


FIG. 141.—DISTRIBUTION DIAGRAM SHOWING SECANT MODULUS AND POISSON'S RATIO AT 1 000 POUNDS PER SQUARE INCH. SERIES, 1925–26–27.

5.—MODULUS OF ELASTICITY AND POISSON'S RATIO, STEVENSON CREEK DAM CONCRETE

For these tests fifty-four 6 by 12-in. cylinders were cast, the aggregates, cement, cement ratio, and water ratio being the same as for the concrete in Stevenson Creek Dam. In general, the specimens were stored in damp sand at a constant temperature of 70° Fahr. Tests have already been made on speci-

mens at the ages of 28 days, 3 months, and 6 months. Additional tests will be made at 1 year and at greater ages. At each age six specimens are tested, three being loaded to the ultimate and the remainder to a unit compressive stress of 1 000 lb. per sq. in. and then again stored in damp sand to be tested at the next later age. Table 26 summarizes the results on specimens up to 6 months of age.

6.—Volumetric Changes in Concrete

The specimens for these tests are 3 by 3 by 40-in. bars, in the ends of which are embedded cylindrical rustless steel gauge plugs or contact posts. Changes in length of these bars are measured with a special extensometer (Fig. 142), observations being made to 0.0001 in. Observations are made periodically to determine not only changes in length, but also changes in weight. The specimens have been stored in the special rooms previously mentioned, so that temperature and moisture conditions have been maintained as desired. Observations, which are being carried out on about 200 bars, have extended over periods ranging from 6 months to 3 years.

7.—Volumetric Changes in Crushed Granite Concrete

These tests are being made upon bars of the same quality of concrete as in Stevenson Creek Dam. Some specimens have been stored continuously in water, the remainder, in damp sand for 28 days; and thereafter some continuously in air and others alternately in air and in water, the time between alternations varying for the several groups. The tests extend over a period of 6 months. Following are some of the results:

(1) Specimens stored in water increase in both length and weight. They increase in length at a nearly uniform rate for the first 2 or 3 months and at the end of 6 months appear nearly to have reached a state of volumetric equilibrium.

(2) Specimens stored in air at 50% relative humidity and 70° Fahr., after having been water-soaked, both shrink and lose weight, the rates of shrinkage and dehydration becoming less with the passage of time.

(3) At the end of 6 months the specimens in air shrink ¾ in. per 100 ft., which is about five times as great as the expansion during the same period when cured under water.

(4) Under conditions of alternate air and water storage there is no evidence that the expansion accompanying immersion is materially different from the contraction accompanying the preceding period of air storage in 10 days, or less, but for periods of air storage of greater duration it appears that the total expansion is less than the total contraction, leaving a residual shrinkage.

8.—Effect of "Fines" on the Shrinkage of Crushed Granite Concrete

These tests are being made upon the same materials as those in Stevenson Creek Dam, but not quite in the same proportions. For one series the fine aggregate (passing a ¼-in. sieve) as received contained 26% of granite dust passing the No. 100 sieve and 9% passing the No. 200 sieve. For a second series the fine aggregate as received contained 25% of dust passing the No. 100 sieve and 16% passing the No. 200 sieve. The ratio of fine to coarse aggregate was 2:3. For the first series the mix was 1 cement to 4 mixed aggregate;

TABLE 26.—SECANT MODULUS OF ELASTICITY AND POISSON'S RATIO.

Group.	Number of test specimens.	Storage before testing.	Percentage of moisture.	Age at loading.	Times previously loaded at 1 000 lb. per sq. in.	Modulus of Elasticity at:			Poisson's Ratio at:			Ultimate strength, in pounds per square inch.
						100 lb. per sq. in.	600 lb. per sq. in.	1 000 lb. per sq. in.	100 lb. per sq. in.	600 lb. per sq. in.	1 000 lb. per sq. in.	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
SAU	3	Damp sand........	7.19	28 days............	None	3 500 000	2 100 000	1 500 000	0.250	0.139	0.146	1 480
SAR	3	" "	....	" "	"	2 600 000	2 000 000	1 500 000	0.195	0.129	0.187	.....
SAR	3	" "	....	3 months..........	One at 28 days......	3 500 000	2 700 000	2 400 000	0.157	0.122	0.125	.....
SBU	3	" "	6.37	" "	None	4 500 000	3 300 000	2 800 000	0.281	0.187	0.174	2 600
SBR	3	" "	....	" "	"	2 900 000	2 400 000	2 200 000	0.148	0.128	0.140	.....
WBU	6	Water for 12 days.	7.62	" "	"	3 800 000	2 800 000	2 300 000	0.200	0.160	0.156	2 160
DBU	6	Air at 50% humidity for 16 days...	5.72	" "	"	4 000 000	2 800 000	2 400 000	0.204	0.130	0.124	2 470
SAR	3	Damp sand........	....	6 months..........	At 28 days and 3 months..........	6 100 000	4 100 000	3 400 000	0.400	0.246	0.216	.....
SBR	3	" "	....	" "	At 3 months.......	3 800 000	3 100 000	2 800 000	0.220	0.174	0.178	.....
SCU	3	" "	6.24	" "	None..............	4 000 000	3 300 000	3 000 000	0.252	0.187	0.182	3 200
SCR	3	" "	....	" "	"	4 200 000	2 700 000	2 100 000	0.270	0.154	0.130	.....

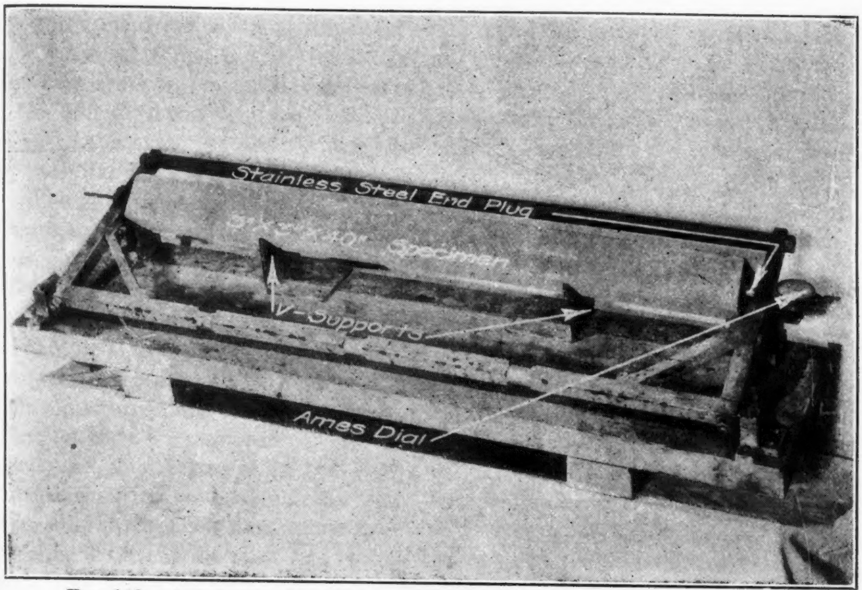


FIG. 142.—APPARATUS FOR DETERMINING VOLUMETRIC CHANGES IN CONCRETE.

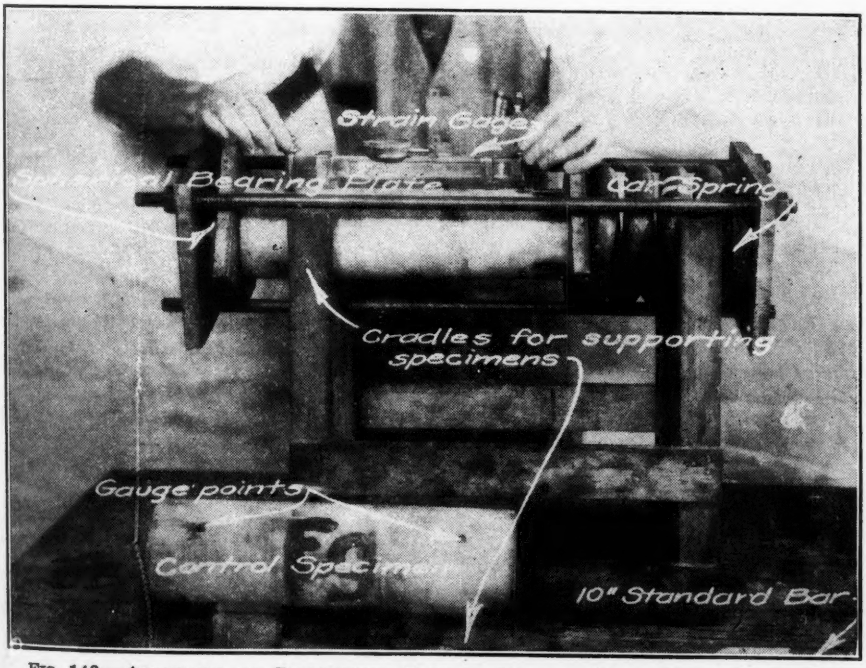


FIG. 143.—APPARATUS FOR DETERMINING FLOW OF CONCRETE UNDER CONTINUED STRESS.

for th
mix v
fine a
a No.
sieve
in the

Th
length
kept c
of 4 c
then c
to dry
tempe
being
Final
cent.
final
Fig. 1
specin
ence
indep
unusu

Th

9.—V

T
scree
The
each
ness
unde
for 2
dried
and
cure

for the second series, the consistency was kept constant (slump, 3 in.), the mix varying from 1:4.5 to 1:5.2. In one group of tests of each series the fine aggregate was used as received; in a second group all material passing a No. 100 sieve was removed; and in a third group, "fines" passing a No. 100 sieve to the extent of 6% by weight of the fine aggregate were incorporated in the mix.

The specimens used were 3 in. square and 40 in. long, and the change of length due to shrinkage was measured from end to end. The specimens were kept covered by wet burlap at a temperature of about 12.8° cent. for a period of 4 days after casting. During this time no shrinkage was found. From then on, the specimens were subjected to a higher temperature and allowed to dry for a certain length of time. They were then returned to the constant-temperature room and brought to the initial temperature of 12.8° cent. before being measured and weighed to determine the shrinkage and loss of weight. Finally, they were oven-dried to a constant weight at a temperature of 40.5° cent. and then returned to the initial temperature of 12.8° cent. before the final measuring and weighing. This period of oven-drying is indicated in Fig. 10, which gives results up to about 3 weeks. Although in general the specimens having the most fine material showed the least shrinkage, the difference was slight, and in both series the shrinkage appeared to be practically independent of the quantity of fine material. In neither was the shrinkage unusually high.

The results based on observations extending over 18 months follow:

(1) The percentage of "fines" (granitic material passing a No. 100 sieve) has no appreciable effect on the magnitude of the shrinkage when the concrete is dried, nor on the expansion when the concrete is water-soaked for a long period.

(2) Specimens having the higher cement ratios, in general, show the larger shrinkages when oven-dried at an early age and the smaller net expansion when this is followed by a prolonged period of water-soaking.

(3) When, after prolonged immersion, the specimens are again oven-dried they are found to be both longer and heavier than they were at the termination of the early oven-drying period.

(4) The extreme change in length is about ¾ in. per 100 ft., or about one-half that for a similar concrete for which the aggregate is river gravel.

9.—Volumetric Changes in Gravel Concrete as Affected by Variations in Cement Ratio, Gradation of Aggregate, and Moisture Conditions

These tests are being made on bars of gravel concrete, the gravel being screened and recombined so as to give four distinctly different gradations. The tests are divided into four series differing in gradation of aggregate, and each series is divided into four groups differing from one another in richness of mix, which varies from 1:2 to 1:6. Neat cement bars have also been under observation. At the date of this report the tests have been in progress for 34 months, during which time the specimens have been repeatedly oven-dried and water-soaked, and have been stored in air at constant temperature and regulated humidity for various periods. Tests were begun on specimens cured in water for about 28 days. Following are some of the results:

(1) When specimens of a given gradation and mix are thoroughly dried and then stored in an atmosphere of constant humidity and constant temperature, they expand and absorb moisture rapidly at first and then at a gradually decreasing rate until finally a state of volumetric and weight equilibrium is reached. Their rate of increase in length and in weight varies with the degree of humidity; and the magnitude of their expansion and absorption when equilibrium is reached is a function of the humidity.

(2) When concrete originally cured for a normal length of time in the condition of saturation is subjected to successive stages of thorough drying and long periods of air storage, it takes on a permanent shrinkage, which is not recovered when it is again saturated, although it returns practically to its original water-soaked weight. It appears, therefore, that over long periods changes in length do not run quite parallel with changes in weight.

(3) Furthermore, when concrete has passed through the stages mentioned and, after the final water-soaking, is stored in air for a long period and then thoroughly dried, its weight is greater than the original dry weight but its length may be greater or less than the original dry length, depending on the richness of the mix and the gradation of the aggregate. For aggregates of low surface modulus, the length is likely to be greater than the original dry length, and for aggregates of high surface modulus it is likely to be less; for rich mixes, greater, and for lean mixes, less.

(4) When dry concretes containing aggregates of a given gradation and varying as to richness of mix are stored in air at a given humidity, the richer the mix the greater the expansion and the greater the absorption.

(5) The maximum shrinkage from the original water-soaked length is greater for rich mixes than for lean ones and is also likely to be greater where the aggregates are of high surface modulus* than where they are of low surface modulus, which, in effect, means that those concretes for which the individual voids in the aggregate are small and numerous are, for the same richness

TABLE 27.

Series.	Fineness modulus.*	Surface modulus.†	Shrinkage in Percentage.			
			1:2 mix.	1:3 mix.	1:4.5 mix.	1:6 mix.
1	4.83	4.74	0.122	0.115	0.107	0.106
2	6.17	3.73	0.135	0.111	0.103	0.101
3	5.04	7.05	0.155	0.134	0.125	0.105
4	4.93	5.90	0.144	0.140	0.097	0.082

* The fineness modulus of an aggregate is obtained by determining the percentages coarser than each of the following sieves: 100-mesh, 48-mesh, 28-mesh, 14-mesh, 8-mesh, 4-mesh, ⅜-in., ¾-in. and 1½-in. These percentages are added together and their sum divided by 100. The result is known as the fineness modulus. It is low (2 to 4) for fine aggregate and high (7 to 9) for coarse aggregate. The number of meshes are per inch of length.

† The surface modulus closely represents the surface area of an aggregate. If p_1 represents the percentage passing the 100-mesh sieve, p_2 that between the 100-mesh and 48-mesh sieves, p_3 that between the 48-mesh and 28-mesh sieves, etc, using the intervals between the 28-mesh, 14-mesh, 8-mesh, 4-mesh, ⅜-in., ¾-in., and 1½-in. sieves, then s, the surface modulus, is as follows:

$$s = p_1 + \tfrac{1}{2} p_2 + \tfrac{1}{4} p_3 + \tfrac{1}{8} p_4 + \tfrac{1}{16} p_5 + \tfrac{1}{32} p_6 \text{ etc.}$$

The number of meshes are per inch of length.

of mix, likely to undergo greater volumetric changes than are those in which the individual voids are large. Table 27 gives the maximum observed shrinkages.

10.—Flow Under Continued Stress

These tests are being made on concrete cylinders, the constant compressive stress being maintained by means of car springs (Fig. 143). In 1925 a series of tests was begun on gravel concretes to determine the effect of richness of mix, gradation of aggregate, and moisture conditions on the flow under a constant compressive stress of 640 lb. per sq. in. The cylinders were 6 in. in diameter by 24 in. long and deformations have been measured with a strain-gauge over a length of 20 in.

In order to determine the effect of moisture conditions on flow, the tests were divided into two series, the specimens of one series being kept constantly wet by spraying at a temperature of 70° Fahr. and those of the other series being kept at 70° Fahr. and 70% relative humidity. To determine the effect of richness of mix and of the gradation of aggregate, each series was divided into four groups: One of a rich mix and high fineness modulus; a second of a lean mix and high fineness modulus; a third of a rich mix and low fineness modulus; and a fourth of a lean mix and low fineness modulus. Specimens were loaded at the age of 7 months, those of the wet series having been cured continuously in water and those of the dry series having been cured 2 months in water and 5 months in air. The crushing strength at the age of 1 year was slightly greater for dry concretes than for the corresponding wet ones. Following is a summary of results:

(1) Under a constant compressive stress concrete tends to flow as a plastic material, the rate of flow being most rapid immediately after the application of load and gradually decreasing with time until finally it ceases. The magnitude of this plastic deformation may be several times the deformation that takes place as the load is applied, which, for convenience, may be called the instantaneous deformation.

(2) Other things being equal, a concrete in the saturated state flows materially less than one in the dry state and reaches a state of plastic equilibrium in a comparatively short time.

(3) Other things remaining equal, the leaner the mix, the larger the plastic deformation.

(4) Other things remaining equal, the lower the water-cement ratio, the less the plastic deformation.

(5) The magnitude of the flow after 5 months of constant compressive stress of 640 lb. per sq. in. varies from 0.002% for the wet specimens of high fineness modulus and rich (1:4) mix to 0.044% for the dry specimens of low fineness modulus and lean (1:7) mix.

11.—Flow of Crushed Granite Concrete

These tests are being made on cylinders 4 in. in diameter by 14 in. long of the same materials as the concrete of Stevenson Creek Dam. The tests have been in progress for approximately one year. A constant compressive stress is maintained by car springs. Seventy specimens are under observation, deformations being measured periodically by a strain-gauge of the

type used at Stevenson Creek. Part of the specimens are kept constantly submerged in water at 70° Fahr. and the remainder are stored in air at 70° Fahr. and 70% relative humidity. The specimens were loaded at ages varying from 2 days to 3 months, and the magnitude of the constant compressive stress varies, among the several specimens, from 200 to 1 200 lb. per sq. in.

Table 28 gives results after periods of loading varying from 267 to 321 days. The values in Column (7) are net plastic deformations, in inches per inch, occurring after the application of load, the effect of changes in length due to causes other than flow having been eliminated by observing similar unloaded specimens stored under the same conditions. Column (8) gives values of the deformations which occurred immediately upon application of the load. At this writing it appears that, except for the wet specimens loaded to 300 lb. per sq. in. at the age of 7 days, flow is still taking place although continuing at a greatly decreased rate.

The general behavior of the specimens throughout the test period is illustrated by Fig. 144, in which for both wet and dry groups (designated *W* and *D*) loaded at 28 days, there are given diagrams for stresses of 300, 600, and 900 lb. per sq. in. (marked 3, 6, and 9, respectively).

Examining the results of the tests, the following relations appear to be established:

(1) The older the concrete at the time of loading, the less rapid the rate of flow, other conditions remaining constant.

(2) Other things remaining equal, the flow under continued compressive stress varies with the magnitude of the stress, but this variation is not uniform, being more rapid at the higher stresses.

(3) After a considerable period the flow under given conditions is materially less for water-soaked specimens than for those maintained in air, although in some instances the rate of flow is more rapid for wet specimens than for dry ones during the period immediately following the application of load.

(4) Six months or so after the load has been applied the specimens in water exhibit a flow which is, in general, less than the instantaneous deformation which accompanies the application of load; but the specimens in air show a flow materially greater than the strain accompanying the application of load.

(5) Other things remaining equal, the flow, in a general way, varies with the instantaneous deformation and hence varies inversely as the modulus of elasticity of the concrete.

12.—Permeability Tests

Tests to determine the permeability of concrete of the same materials as those in the Stevenson Creek Dam are in progress. The specimens are disks 18 in. in diameter and 4 in. thick. During the test they are supported on an unyielding metal frame and water pressure is applied to the opposite face. The water which passes through the central part of the disk is collected by a knife-edged funnel, 6 in. in diameter, which bears against the specimen. Thus far, only preliminary tests have been run. For the principal tests distilled water will be used and that which percolates through the specimens will be subjected to chemical analysis. Specimens will be tested at various ages, and the effect of continued percolation will be studied.

TABLE 28.—PLASTIC FLOW TESTS.

Group. (1)	Curing conditions. (2)	Storage while loaded. (3)	Age, at time of loading. (4)	Unit stress, in pounds per square inch. (5)	Days loaded. (6)	Unit plastic flow, inches per inch. (7)	Unit initial elastic deformation. (8)	Total unit deformation, Columns (7) and (8). (9)	Ratio of Column (7) to Column (8). (10)
WA 2	Water	Water	2 days	200	275	0.00017	0.00032	0.00048	0.53
WB 3	"	"	7 "	300	288	0.00015	0.00018	0.00032	0.83
WB 6	"	"	7 "	600	288	0.00055	0.00082	0.00134	0.67
WC 3	Damp sand	"	28 "	300	317	0.00014	0.00010	0.00022	1.40
WC 6	" "	"	28 "	600	317	0.00028	0.00044	0.00071	0.63
WC 9	" "	"	28 "	900	317	0.00050	0.00074	0.00122	0.68
DC 3	" "	Air at 70% humidity	28 "	300	321	0.00027	0.00009	0.00034	3.00
DC 6	" "	" " " "	28 "	600	321	0.00053	0.00028	0.00078	1.89
DC 9	" "	" " " "	28 "	900	321	0.00088	0.00062	0.00144	1.42
WD 6	" "	Water	3 months	600	267	0.00014	0.00021	0.00033	0.67
WD 9	" "	"	3 "	900	267	0.00025	0.00033	0.00054	0.76
WD 12	" "	"	3 "	1 200	267	0.00035	0.00051	0.00082	0.69
DD 6	" "	Air at 70% humidity	3 "	600	270	0.00042	0.00019	0.00053	2.20
DD 9	" "	" " " "	3 "	900	270	0.00063	0.00035	0.00088	1.80
DD 12	" "	" " " "	3 "	1 200	270	0.00091	0.00050	0.00125	1.82

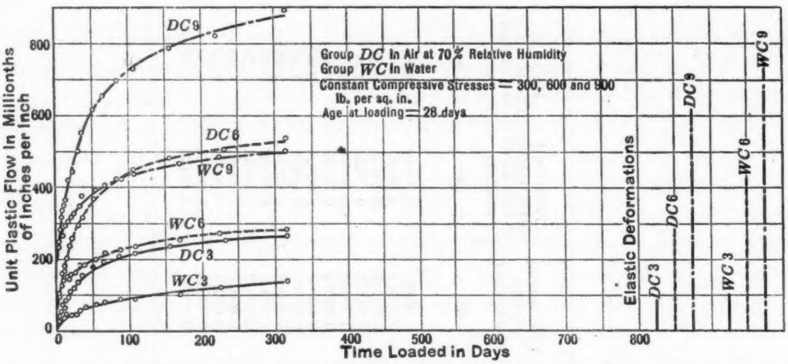


FIG. 144.—TESTS ON PLASTIC FLOW OF CONCRETE ACCOMPANYING CONSTANT SUSTAINED LOAD.

13.—COEFFICIENT OF THERMAL EXPANSION

These tests are in progress on 3 by 3 by 40-in. bars, of materials identical with those used in Stevenson Creek Dam, the specimens being similar to those used in studying volumetric changes except that each is provided with embedded thermo-couples to enable the determination of its true temperature. Part of the specimens are stored in water and the remainder in air. During each test, observations of changes in length are made at various temperatures between 39° and 125° Fahr., the tests being carried out in one of the insulated rooms. Temperatures are controlled within 1° Fahr. Determinations of changes in length are made with a compensating extensometer of 40-in. gauge length especially designed for these tests, and the compensating extensometer is itself checked by use of a compensating standard bar. Table 29 gives the average coefficients of thermal expansion per 1° Fahr. for both wet and dry specimens at ages of 7 weeks and 4 months.

TABLE 29.

Condition.	COEFFICIENT OF THERMAL EXPANSION.	
	At age of 7 weeks.	At age of 4 months.
Wet	0.00000447	0.00000410
Dry	0.00000379	0.00000390

The dry specimens consistently show a lower coefficient than the wet ones, but so far there is nothing to indicate that either the moisture condition or the age has any marked effect on the value of the coefficient. Furthermore, no marked variation between coefficients at high and low temperatures has been observed, although it appears that the coefficient is slightly lower at the low temperatures. It is significant that the observed coefficients are only about two-thirds of the value commonly assumed, although these low values seem logical in view of the coefficients of thermal expansion of granite, which have been determined elsewhere. It is proposed to determine what, if any, is the effect of repeated extremes of temperature on the thermal expansion.

PART VI.—TESTS ON MODELS AT BOULDER, COLORADO

By J. L. Savage* and Ivan E. Houk,† Members, Am. Soc. C. E.

Comprehensive experiments on the Test Dam have shown certain facts about a comparatively thin arch dam built in a narrow, symmetrical, V-shaped canyon like Stevenson Creek. There are many problems on which further experimental data are desired: What modifications must be applied in order to adapt the Stevenson Creek data to the design of arch dams for other shapes of canyon, such as a narrow gorge with vertical walls; or a comparatively wide gorge with vertical sides; or a comparatively wide gorge with sloping sides; or a decidedly non-symmetrical gorge; or a gorge containing pronounced irregularities in profile? What would be the effect of varying the cross-section of the dam? What would be the action of a constant-angle arch dam built in the same gorge? What would be the effect of using non-concentric arcs on the up-stream and down-stream sides?

The ideal, an elaborate system of measurements, such as was made at Stevenson Creek, on several full-sized dams of different design, built in canyons having different shapes of cross-section, is not practicable because of excessive cost. It is believed that a fairly satisfactory solution can be obtained through careful study of small scale models in conjunction with suitable mathematical treatment, the comprehensive Stevenson Creek data, and such experimental observations as it is practicable to secure on other full-sized dams.

One of the first questions to be decided by the Sub-Committee on Models was whether the tests should be carried out at a field site, where natural rock abutments were available, or in a concrete pit to be built in an existing testing laboratory. A site of either type could be obtained at Boulder, Colo., about thirty-five miles from Denver. In view of desirable co-operation, as well as the other advantages of a laboratory, it was decided to adopt the latter. Plans for a concrete pit, with massive, heavily reinforced side-walls and floor, to be built below the floor of the basement of the Engineering Testing Laboratory at the University of Colorado, were prepared; and arrangements for its construction by University forces at actual cost, were made. The pit is ready for use. It has a maximum depth of 5 ft., a maximum inside length of 18 ft., and a maximum inside width of 12 ft. Fig. 145 shows the plan, with a longitudinal and a transverse section. The octagonal, stepped design will permit the testing of models of practically any shape, symmetrical or non-symmetrical, and of practically any size not exceeding the maximum inside dimensions of the pit.

Careful study has been given to the possibility of using different materials in the models—hard rubber, celluloid, plasticine, gypsum, cast-iron blocks, and others; and also to different methods of loading the models—by springs,

* Chf. Designing Engr., U. S. Bureau of Reclamation, Denver, Colo.
† Research Engr., U. S. Bureau of Reclamation, Denver, Colo.

by mercury, and by water pressure transmitted through enclosed horizontal lengths of rubber hose laid along the up-stream face of the model. The conclusions reached were that the first models should be built of concrete, that they should be tested under triangular loads obtained by holding a thick film of mercury against the up-stream face of the model, and that experiments with other materials and other methods of loading should be deferred until the results of the first tests were available.

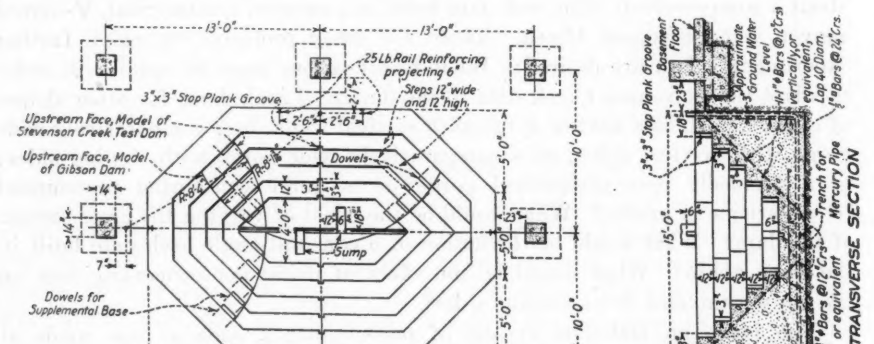


FIG. 145.—PLAN AND SECTIONS OF CONCRETE TESTING PIT, ENGINEERING TESTING LABORATORY, UNIVERSITY OF COLORADO.

The first experiment will be an investigation of a one-twelfth scale model of the Stevenson Creek Test Dam. It will be attempted to reproduce the actual conditions, as nearly as possible, in moisture, temperature, anchorage, and materials. In order that the concrete in the model and test specimens may be as closely as possible identical with that in the actual dam, some fine aggregate from Stevenson Creek has been sent to Boulder. The decision to build a model of the Stevenson Creek Dam for the first experiments was based on the fact that complete data on deflections, strains, bending, movement of abutments, temperature, effects, etc., are available. It is expected that comparison of the experimental data obtained on the model with that obtained on the actual dam will determine the relations between a small scale concrete model loaded with mercury and a full sized dam loaded with water. These relations can then be used in interpreting the results of subsequent model tests. They will also be of value in deciding whether or not it is advisable to conduct experiments on models built of other materials.

Loads on the model will be obtained by forcing mercury into a rubber bag held against the up-stream face of the model. The rubber bag will be held in place by a ¼-in. sheet steel form curved to the same radius as the up-stream face. Heavy timber bracing will hold the form in position, and air pressure or water pressure will be used to force the mercury from the supply tank into the rubber bag. Since mercury is 13.6 times as dense as water, a dam 5 ft. high, loaded with mercury, will be subjected to the same pressure at the bottom as a 68-ft. dam loaded with water.

Very careful measurements will be made of the movements of the model when the load is applied. Deflections of the down-stream face will be observed at approximately forty points, by means of dials reading directly to 0.0001 in. Invar steel rods, the lengths of which do not change appreciably with the usual variations in temperature, will transmit the movements of the down-stream face to the measuring dials. The movement of the model at the up-stream edge of the base will be measured at the crown section where the formation of a crack is anticipated. Movements of the abutments will be observed at three elevations corresponding to the elevations at which similar measurements were made at the Stevenson Creek Dam. Strains at the down-stream face of the model will be measured with the Tuckerman optical strain-gauge (Part VII). Investigations are being made to determine the feasibility of using this instrument in measuring other deformations of the model, such as changes in length of mid-ordinates in a 10-in. chord length, angular rotation of different elements of the model, deflections near the base and abutments where the movements are almost infinitesimal, etc.

Test specimens of the mortar of which the model is built will be preserved for determining its ultimate compressive strength and modulus of elasticity. These will be 3 by 6-in. cylinders for determining the compressive strength, 3 by 10-in. cylinders for determining the modulus of elasticity, and a test beam for use in checking the modulus of elasticity shown by the cylinders. Measurements of Poisson's ratio are not considered necessary in view of the similarity between the mortar in the model and the concrete in the Stevenson Creek Dam. Elaborate investigations of Poisson's ratio for the concrete used in the Stevenson Creek Dam were made by Raymond E. Davis, M. Am. Soc. C. E., at the University of California (Part V).

When the experiments on the Stevenson Creek model are completed, it is proposed to investigate the action of a model of the Gibson Dam now being built by the U. S. Bureau of Reclamation on the Sun River Irrigation Project of Montana. Since the valley at the Gibson site has a relatively wide cross-section, definitely different from the narrow V-shaped section at the Stevenson Creek site, it is expected that a different load distribution between cantilever and arch elements will be found. Computations made in designing the Gibson Dam indicate that approximately three-fourths of the load will be carried by gravity (cantilever) action, whereas, in the Stevenson Creek Dam, the greater part of the load was carried by arch action. Tentative plans for a concrete

model of the Gibson Dam are shown in Fig. 146, although in the end some other material may be selected. The arrangement of testing equipment shown is similar to that planned for the model of the Stevenson Creek Dam.

It is anticipated that the experience gained in testing models of Stevenson Creek and Gibson Dams, will point out what further experimental work is

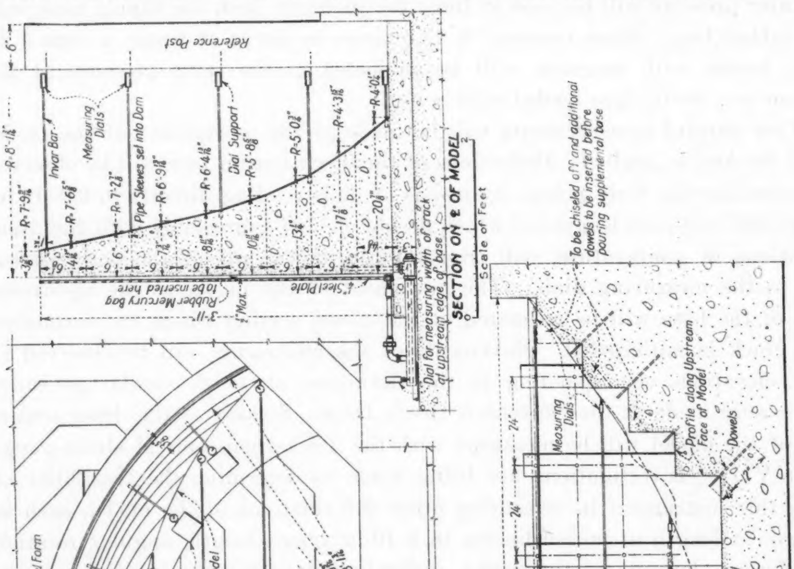


FIG. 146.—PLAN, ELEVATION, AND SECTION, GIBSON DAM.

needed. Doubtless, it will be found desirable to build and test models of other dams, including the 675-ft. Boulder Canyon Dam proposed for the control of the Colorado River. It probably will be desirable to build models of other materials than concrete, especially for comparatively high dams.

PART VII.—TEST OF A CELLULOID MODEL OF THE STEVENSON CREEK DAM

By George E. Beggs,[*] M. Am. Soc. C. E.

1.—Historical Note

In April, 1926, Fred A. Noetzli, M. Am. Soc. C. E., Secretary of the Committee on Arch Dam Investigation, inquired as to the possibility of using a model to check the data obtained from the test of the Stevenson Creek Dam. The writer, having carried out numerous experiments on elastic models of structures,[†] suggested a celluloid model with a reservoir of mercury. Celluloid was suggested because it is many times more flexible than concrete, and mercury was chosen because it is much heavier than water. Although the model proposed would be only one-fortieth size, its deflections and strains would be readily measurable.

2.—Construction of the Model

The celluloid model (Fig. 148) is 18 in. high and 0.6 in. thick for the upper 9 in., increasing to more than 2 in. at the bottom. Four sheets of celluloid, each 0.6 in. thick and of various heights, after having been heated in boiling water for 2 hours and clamped until cold in heavy zinc-lined wooden forms of the proper curvature, were welded together by injecting celluloid cement under pressure between them through small drilled holes, this operation driving out all air and insuring a continuous celluloid filled joint. The cemented sheets were then seasoned in ovens provided by the Celluloid Company, Newark, N. J. The final profile on the down-stream face was accurately formed to a metal template by hand-carving done by a pattern-maker.

The celluloid sheets were made of sufficient length for the dam to be anchored into the reinforced concrete abutments not less than 4 in. To form a reservoir for the mercury, an additional curved sheet of celluloid, 0.3 in. thick, was placed ¼ in. up stream from the dam. From its bottom, a heavy rubber tube led to the bottom of an iron container holding about 100 lb. of mercury and hung from a chain hoist. Raising or lowering the container caused the mercury to fill or empty the reservoir. The time for filling and emptying and for making any observation did not exceed 5 min., and, therefore, errors due to variation in temperature were eliminated.

3.—Control Test Samples

Sample bars of celluloid, 0.6 in. thick, from the same sheets from which the model was made, were put through identical processes of heat treatment and seasoning. From deflection and strain measurements on simple beams made from the control test bars, it appeared that the coefficient of elasticity

---

[*] Assoc. Prof., Civ. Eng., Princeton Univ., Princeton, N. J.

[†] *Proceedings*, Am. Concrete Inst., 1922 and 1923; *Transactions*, Am. Soc. C. E., Vol. 88 (1925), pp. 1208–1230; *Engineering News-Record*, April 29, 1927; *Beton und Eisen*, November 5, 1923.

of the celluloid remained practically constant during several months, with an average value of 263 750 lb. per sq. in. Poisson's ratio for this material through the range of strain in the dam was found by experiment at the National Bureau of Standards to be 0.42.

4.—Formulas for Predicting Deflections and Strains of Concrete Dam from Observations on Celluloid Model

The formulas for translating the deflections and strains of the celluloid model into corresponding values for the full-sized concrete dam are as follows:

$$\text{Predicted Deflection of Concrete Dam} = \text{Deflection of Model} \times \left(\frac{n^2}{q\,z} = 8.6\right) \qquad (30)$$

$$\text{Predicted Horizontal Unit Strain for Concrete Dam} = \text{Unit Horizontal Strain in Model} \times \left(\frac{n}{q\,z} = 0.216\right) \qquad (31)$$

$$\begin{aligned}\text{Predicted Vertical Unit Strain for Concrete Dam} &= [\text{Unit Vertical Strain in Model} - (\mu_2 - \mu_1)\,(\text{Unit Horizontal Compressive Strain at Middle Surface of Model})] \times \frac{n}{q\,z} \\ &= [\text{Unit Vertical Strain in Model} - (0.27)\,(\text{Unit Horizontal Compressive Strain at Middle Surface})] \times 0.216 \qquad (32)\end{aligned}$$

in which,

n = ratio of size of arch dam to its model.

q = specific gravity of mercury used in testing model, 13.57.

z = ratio of moduli of elasticity of concrete and celluloid $= \frac{3\,600\,000}{263\,750} = 13.7$.

μ_1 = Poisson's ratio for concrete = 0.15.

μ_2 = Poisson's ratio for celluloid = 0.42.

Equations (30), (31), and (32) are readily derived for the special case where Poisson's ratios for the concrete dam and its model are equal. Inasmuch as Poisson's ratios for celluloid and concrete were found to differ, rational modifications of the several formulas were made. Poisson's ratio can be expressed as a function of the coefficients of elasticity in shear and direct stress. Inasmuch as shearing deflection or torsional deflection probably had little to do with the radial deflections of the concrete dam or its model, it appeared reasonable to neglect the Poisson's ratio factor in translating the deflections of the model into those for the full-sized concrete dam. From Equation (30), the multiplier for model deflections is as small as 8.6, so that deflection readings to the precision of 0.001 in. on the celluloid model would give predicted deflections on the full-sized concrete dam to 0.01 in. The experimental difficulties of predicting arch dam deflections from a celluloid model of this scale are, therefore, slight.

The formula for predicting horizontal strains in the concrete dam from model observations also omits the effect of Poisson's ratio. This seems logical, for in neither the model nor the dam is there any considerable vertical direct stress at the surface midway between up-stream and down-stream faces. A

study of the many vertical strains measured on down-stream and up-stream faces of the celluloid arch dam confirmed this. Consequently, the simple multiplier, $\frac{n}{q\,z} = 0.216$ (Equation (31)), serves for the translation of horizontal strains.

It is notable that the unit strains on the celluloid model are about five times larger than the corresponding unit strains on the large concrete dam; it follows, therefore, that a 2-in. gauge length on the celluloid model has nearly the same total strain as a 10-in. gauge length on the concrete dam. Hence, the prediction of strains from observations on a celluloid model does not present great difficulty.

The formula for predicting vertical strains in a concrete dam from model observations logically includes an allowance for the difference in Poisson's ratio for celluloid and concrete. A study of experimental data both for the celluloid model and for the concrete arch dam indicated for any elevation the presence of a constant horizontal thrust at the mid-surface between up-stream and down-stream faces. This horizontal thrust, by its Poisson's ratio effect, would increase the vertical strains more for the model than for the dam, for Poisson's ratio is 0.42 for celluloid and 0.15 for concrete.

The unit vertical strains measured on the celluloid model were accordingly relatively too great for direct translation into vertical strains for the concrete dam. For a model of concrete, the measured unit vertical strains would have included 0.15 of the strains due to unit horizontal thrust; but in the celluloid model, these vertical unit strains contained 0.42 of the strains from unit horizontal thrust. Therefore, the measured unit vertical strains in the model should be reduced by 0.42 — 0.15, or 0.27 times the corresponding measured strains at mid-surface from the horizontal thrust in the celluloid model before multiplying by the $\left(\frac{n}{q\,z}\right)$ factor to find the corresponding predicted vertical unit strains in the full-sized concrete dam (Equations (32)).

5.—Measurement of Deflections

The apparatus for measuring deflections of the model is shown in Fig. 148. The deflections of points on the down-stream face were communicated through wooden rods to Ames dials reading to 0.001 in. and mounted on a central vertical axis. Inasmuch as the reservoir could be emptied more rapidly than filled, deflection readings were recorded for movement of the dam caused by emptying. The celluloid control beam from which the coefficient of elasticity was measured, was also read from the loaded to the unloaded condition. In each case, 1 min. was allowed to elapse after removing the load before the no-load readings were taken. Every deflection and strain recorded is an average of not less than four observations.

The radial deflections of the celluloid dam are recorded in Table 30. Deflections were read at stations on the down-stream face every 3 in. horizontally and every 1½ in. vertically, corresponding to 10-ft. and 5-ft. intervals on the concrete dam. In Table 31 are recorded the predicted deflections for

TABLE 30.—DEFLECTIONS OF THE CELLULOID MODEL.

(To Obtain Deflections of Stevenson Creek Dam Multiply the Values by the Proper Value of $\frac{n^2}{q\,z}$, in Which, n = Scale of Actual Structure to Scale of Model = 40; q = Specific Gravity of Mercury = 13.57; and z = Ratio of E (Concrete) to E (Celluloid).)

Elevation, in inches.	18 in. left.	15 in. left.	12 in. left.	9 in. left.	6 in. left.	3 in. left.	Center.	3 in. right.	6 in. right.	9 in. right.	12 in. right.	15 in. right.	18 in. right.
18	-0.00195‡	−0.00755‡	−0.0091‡	−0.0037‡	+0.00927§	+0.02313§	+0.0307*	+0.0268*	+0.01403*	+0.00023*	−0.0085*	−0.00787*	−0.0029*
16½	−0.0005‡	−0.0045‡	−0.00515‡	+0.00073‡	0.0127‡	0.02583‡	0.03255†	0.03016†	0.0177*	0.00468*	−0.00397*	−0.00467*	−0.00105*
15	....	−0.0017‡	−0.0014‡	0.0051‡	0.01687§	0.02963§	0.03548*	0.03255*	0.0207*	0.00818*	+0.00007*	−0.00153*	0*
13½	....	+0.00015‡	+0.0015‡	0.008‡	0.0198‡	0.03273‡	0.0391†	0.03612†	0.02397*	0.0111*	0.00273*	+0.00017*	....
12	....	....	0.0033‡	0.011‡	0.02307§	0.03587§	0.04192*	0.03835*	0.0267*	0.01335*	0.00407*	0.00073*	....
10½	....	....	0.0031‡	0.0114‡	0.02363‡	0.0358‡	0.04225†	0.0391†	0.02637*	0.0125*	0.00363*	....	....
9	....	....	....	0.01105‡	0.02283§	0.03328§	0.03927*	0.03542*	0.0248*	0.0112*	0.002*	....	....
7½	....	....	....	0.0073‡	0.01767‡	0.02753‡	0.0324†	0.02854†	0.0183*	0.00703*	....	....	....
6	....	....	....	....	0.01207§	0.01913§	0.02395*	0.02098*	0.01233*	0.00281*	....	....	....
4½	....	....	....	....	0.00678‡	0.01227‡	0.01533†	0.0129†	0.007*	....	....	....	....
3	....	....	....	....	....	0.00637§	0.009*	0.0071*	0.00273*	....	....	....	....
1½	....	....	....	....	....	0.003§	0.00462*	0.00375*	....	....	....	....	....

* Observations taken May 31 E (celluloid) = 261 000.
† Observations taken June 1 E (celluloid) = 261 000.
‡ Observations taken June 2 E (celluloid) = 264 000.
§ Observations taken June 3 E (celluloid) = 269 000.
E (concrete) = constant = 3 600 000.

TABLE 31.—DEFLECTIONS OF STEVENSON CREEK DAM CALCULATED FROM THOSE OF THE CELLULOID MODEL.

Elevation, in feet.	60 ft. left.	50 ft. left.	40 ft. left.	30 ft. left.	20 ft. left.	10 ft. left.	Center.	10 ft. right.	20 ft. right.	30 ft. right.	40 ft. right.	50 ft. right.	60 ft. right.
60	−0.017	−0.065	−0.079	−0.032	+0.082	+0.204	+0.262	+0.229	+0.120	+0.002	−0.073	−0.067	−0.025
55	−0.004	−0.039	−0.045	+0.006	0.110	0.223	0.278	0.258	0.151	0.040	−0.084	−0.040	−0.009
50	....	−0.015	−0.012	0.044	0.149	0.261	0.303	0.278	0.177	0.070	+0.001	−0.013	0
45	....	+0.001	+0.013	0.069	0.171	0.283	0.334	0.308	0.205	0.095	0.023	+0.001	....
40	....	....	0.029	0.095	0.203	0.316	0.358	0.328	0.228	0.114	0.035	0.006	....
35	....	....	0.027	0.099	0.204	0.310	0.361	0.334	0.225	0.107	0.031	....	....
30	....	....	....	0.096	0.201	0.293	0.335	0.303	0.208	0.096	0.017	....	....
25	....	....	....	0.063	0.153	0.238	0.277	0.244	0.156	0.060	....	....	....
20	....	....	....	....	0.106	0.169	0.205	0.179	0.105	0.024	....	....	....
15	....	....	....	....	0.058	0.106	0.131	0.110	0.060	....	....	....	....
10	....	....	....	....	....	0.056	0.077	0.061	0.023	....	....	....	....
5	....	....	....	....	....	0.026	0.040	0.032	....	....	....	....	....

the Stevenson Creek Dam as obtained by multiplying the deflections in Table 30 by the $\left(\frac{n^2}{q\,z}\right)$ factor (Equation (30)).

In Fig. 147 is shown a comparison of these predicted deflections with those actually measured on the Stevenson Creek Dam. The agreement is noteworthy in both value and sign. Where measured deflections are not shown, they were too close to the predicted deflections to be plotted separately. The slightly greater deflection of the concrete dam at the 60-ft. elevation near the center line was probably due to the vertical crack which weakened the dam and allowed it to deflect more than was natural for a continuous structure. Considering that the predicted deflections differ nowhere as much as 0.1 in. from the measured deflections, it appears that the experimental method using a celluloid model is fundamentally practical.

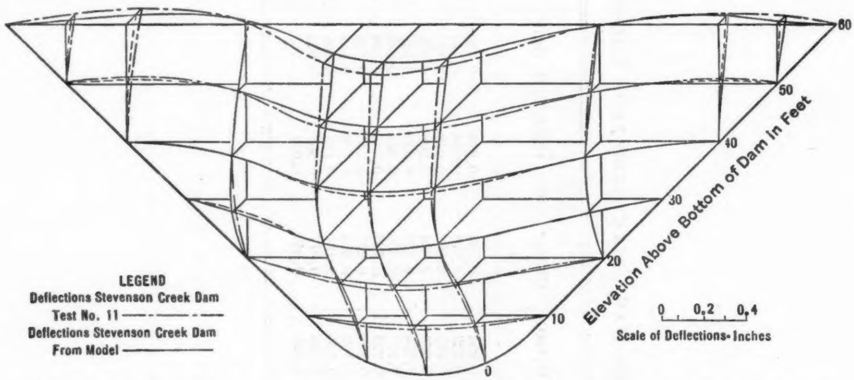


FIG. 147.—COMPARISON OF DEFLECTIONS OF STEVENSON CREEK DAM FROM TEST NO. 11, WITH THOSE COMPUTED FROM DEFLECTIONS OF A CELLULOID MODEL.

6.—MEASUREMENT OF SURFACE STRAINS ON DOWN-STREAM FACE

The strains on the down-stream face of the model caused by emptying the mercury were measured with the Tuckerman optical strain-gauge* (Fig. 149). It consists of two parts: The auto-collimator supported on the stand, and the 2-in. strain-gauge attached to the down-stream side of the model. Briefly, a beam of light carrying the image of a scale travels from the auto-collimator to the strain-gauge and is reflected back from the strain-gauge on to a readable scale in the auto-collimator. The light ray enters the strain-gauge through a special roof prism, is reflected to a stellite lozenge, which rotates with varying strain, and from this lozenge is reflected back to the readable scale. Readings to 0.000001 in. per in. on a 2--in. gauge length could be made with confidence.

The apparatus is remarkably well adapted to measuring small strains, and is commendably free of many common instrumental errors. The readings are independent of the position of the auto-collimator. Shaking the stand on which the auto-collimator is supported, does not affect the reading. The gauge itself weighed but a few ounces, and was readily supported against the vertical

* *Engineering* (Lond.), Vol. 116, No. 3007, pp. 222–223, August 17, 1923; *Proceedings*, Am. Soc. for Testing Materials, Vol. 23, Pt. 2, pp. 602–610, 1923.

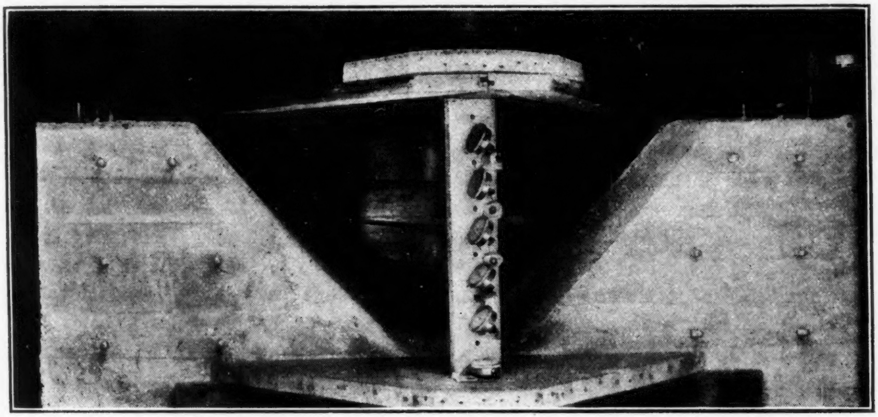


FIG. 148.—CELLULOID MODEL OF ARCH DAM WITH DEFLECTION APPARATUS. NOTE SIZE OF MAN AT BASE OF DAM.

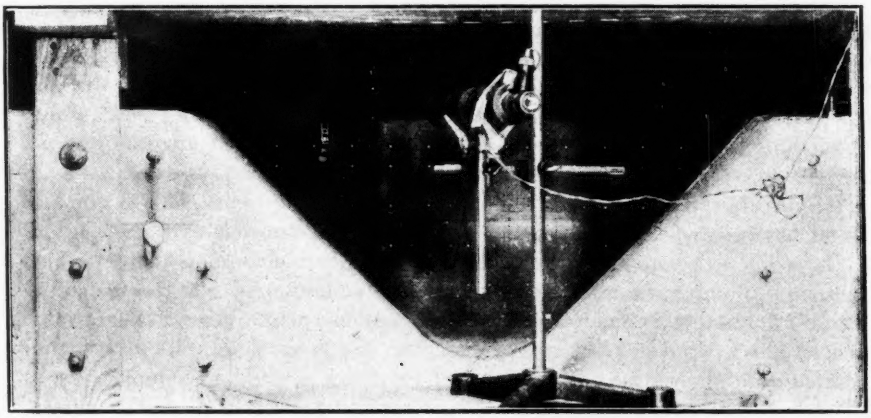


FIG. 149.—TUCKERMAN'S OPTICAL STRAIN-GAUGE APPARATUS FOR MEASURING DOWN-STREAM UNIT STRAINS OF CELLULOID DAMS.

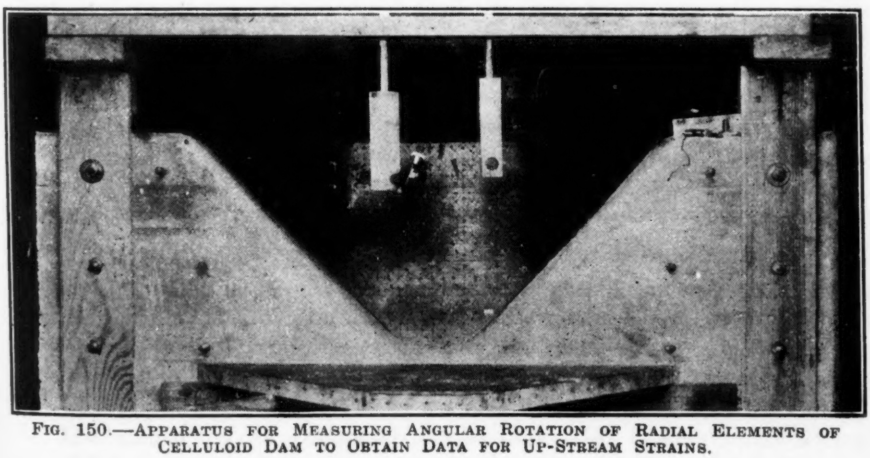


FIG. 150.—APPARATUS FOR MEASURING ANGULAR ROTATION OF RADIAL ELEMENTS OF CELLULOID DAM TO OBTAIN DATA FOR UP-STREAM STRAINS.

surf
into
S
filled
The
unit
cate
has
that
maj
It f
dete
Sim
for

whe
find
in
the
in.
of
the
fiel

det
un
it
str
at
an
St
rec
tal
in
ele
ve
Th

st
zo
li
po

th
ir

surface of the celluloid with a rubber band attached to two small tacks driven into the celluloid on either side.

Strains in millionth inches per inch caused by emptying the completely filled reservoir, were read horizontally, vertically, and in two 45° directions. The stations at which strains were measured and the average of the observed unit strains on the down-stream face are fully shown in Fig. 151. To indicate in a general way the nature of strains on the down-stream face, Fig. 152 has been drawn showing to an exaggerated scale the deformed shapes of areas that were circular when the reservoir was empty. It is notable that the great majority of ellipses have their major axes vertical or horizontal—not inclined. It follows that maximum stress on the down-stream face may be found by determining vertical and horizontal stresses and disregarding inclined stresses. Similar observations were also made in connection with strain ellipses drawn for the up-stream face.

7.—Measurement of Surface Strains

Evidently, it was not possible to read a strain-gauge on the up-stream face when the reservoir was filled with mercury. It was therefore necessary to find the up-stream strains by indirect measurement. The apparatus is shown in Fig. 150. At the ends of each 2-in. gauge length for which the strain on the down-stream face had been measured, two steel knitting needles, about 15 in. long, were driven firmly into holes in the celluloid. On the outer ends of these needles corks were pressed. Through these corks at right angles to the needles, pins were pushed until their two points could be observed in the field of a filar micrometer microscope reading to 0.0001 in.

The separation or approach of these pin-points on emptying the mercury determined the angular movement of the needles. Knowing the down-stream unit strain, the thickness of the dam, and the angular change of the needles, it was a simple problem in geometry to calculate the corresponding up-stream strains. Then, by averaging the up-stream and down-stream strains, the strains at the middle surface were calculated. The results for the up-stream and middle surface strains are not recorded, as corresponding values for the Stevenson Creek Dam are available for comparison at only a few points. A reduction of the middle surface strains to stresses (in which account was taken of Poisson's ratio by the usual formulas) gave results showing that in the celluloid arch dam the horizontal thrust at the middle surface at any elevation is nearly constant. Also, these calculations indicated that there was very little vertical tension or compression at the middle surface of the model. These observations are in agreement with assumptions commonly made.

8.—Comparison of Predicted and Measured Surface Strains.

In Figs. 153 and 154 are comparisons of predicted and measured surface strains on the down-stream face of the Stevenson Creek Dam for horizontal and vertical strain, respectively. For example, for a point on the line marked —75, in Fig. 153, the horizontal compressive strain in inches per inch on the down-stream face of the dam is 0.000075.

The comparison for horizontal strains (Fig. 153) is highly satisfactory, the differences being logically explainable by the vertical cracks that formed in the concrete dam in the areas of horizontal tension. The agreement between

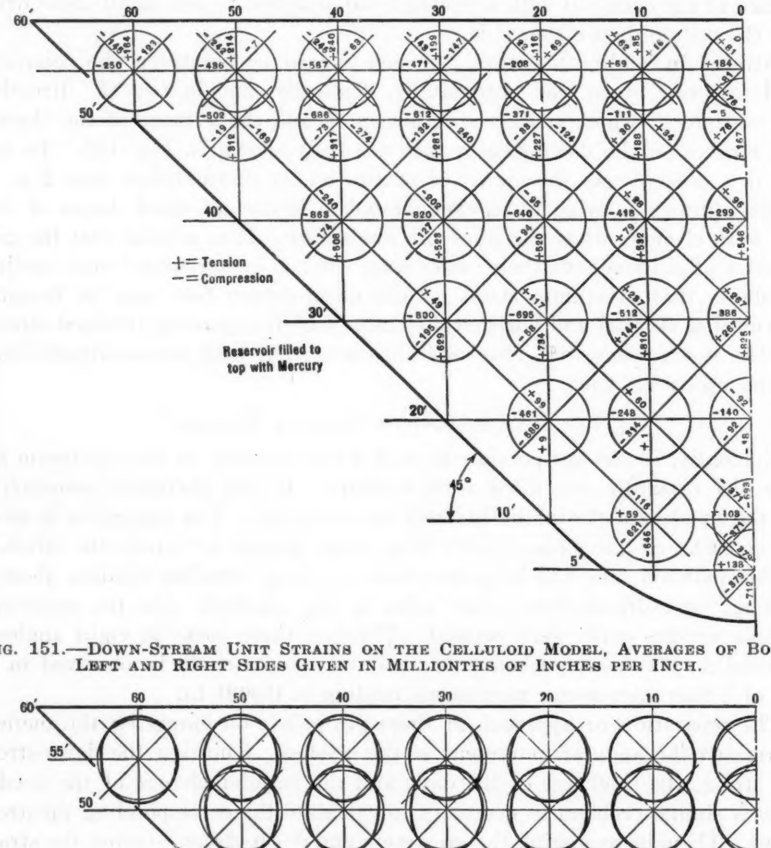


FIG. 151.—DOWN-STREAM UNIT STRAINS ON THE CELLULOID MODEL, AVERAGES OF BOTH LEFT AND RIGHT SIDES GIVEN IN MILLIONTHS OF INCHES PER INCH.

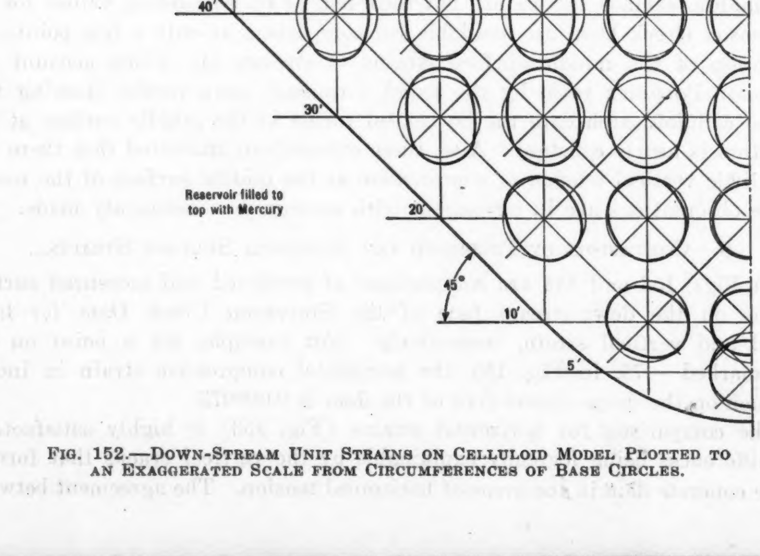


FIG 152.—DOWN-STREAM UNIT STRAINS ON CELLULOID MODEL PLOTTED TO AN EXAGGERATED SCALE FROM CIRCUMFERENCES OF BASE CIRCLES.

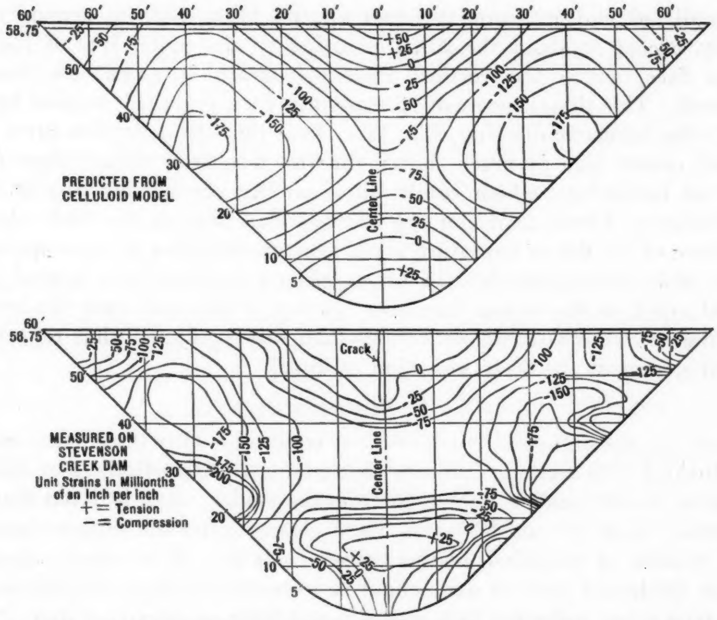


FIG. 153.—COMPARISON BETWEEN MEASURED AND PREDICTED VALUES FOR LINES OF EQUAL HORIZONTAL STRAIN, DOWN-STREAM FACE, FOR 60-FOOT HEAD.

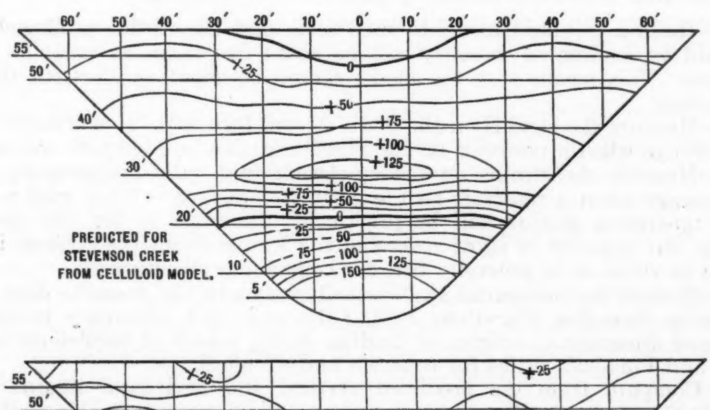


FIG. 154.—COMPARISON BETWEEN MEASURED AND PREDICTED VALUES FOR LINES OF EQUAL VERTICAL STRAINS ON DOWN-STREAM FACE, FOR 60-FOOT HEAD.

the predicted and measured vertical strains (Fig. 154) is generally satisfactory, except at about the 30-ft. elevation on the center line of the down-stream face, where the vertical measured strain is much less than that predicted. This divergence can be explained in a practical manner by reference to the isometric drawing, Fig. 147. Note that the deflection curve on the vertical center line is much flatter for the measured values than for the predicted deflections, which results in a smaller vertical bending strain for the Stevenson Creek Dam, on the down-stream face at the 30-ft elevation. The cause of the flatter curvature is the greater deflection of the concrete dam at the 60-ft. elevation; this, in turn, appears to have been caused by the vertical crack on the center line near the top, which weakened the arch and allowed greater down-stream deflections than were predicted, thus relieving the vertical strains at the 30-ft. elevation as noted.

9.—Summary

Tests on the full-sized concrete dam compared with those on a celluloid model loaded with mercury indicate that quantitative predictions of deflection and strain can be made for the dam from the model. It is believed that these predictions from the model agree more closely with the observations than would results of practical mathematical analysis. Even closer agreement between the model and the dam would be expected if the model had been cut where its tension indicated that cracks would form in the actual dam.

For the practical use of models to evaluate stresses in proposed or existing concrete arch dams, the following procedure is suggested:

1.—Build a celluloid model to such scale that the strains produced in the celluloid by loading of mercury will be about five times as great as for the concrete. This means that the model should be about one-fortieth the scale of the dam.

2.—Measure the strains on the down-stream face with Tuckerman's optical strain-gauge when a reservoir on the up-stream face is filled with mercury.

3.—Measure the strains on the up-stream face with Tuckerman's optical strain-gauge when a reservoir on the down-stream face is filled with mercury. These up-stream strains will be practically the same as for the up-stream loading, but opposite in sign; reversing the load reverses the stresses in sign, but not in value, as is generally true for continuous structures.*

4.—Predict the horizontal and vertical strains in the concrete dam by the conversion formulas, Equations (30), (31), and (32), allowance being made for linear dimensions, weights of loading fluids, values of coefficients of elasticity, and Poisson's ratios for concrete and celluloid.

5.—Compute from the predicted vertical and horizontal strains in the concrete the corresponding vertical and horizontal stresses, making allowance by the usual formula for Poisson's ratio for the concrete.

For skillful and patient assistance in making thousands of measurements and computations and for other valuable help, the writer is indebted to D. B. Sloan, Jun. Am. Soc. C. E., a graduate student at Princeton University. The University provided services and facilities. The Power Division of the American Society of Civil Engineers made a substantial contribution to the funds, and the Committee on Arch Dam Investigation contributed the remainder.

---

* This procedure would have practically reversed the load on the Stevenson Creek Dam model but not exactly. The approximation should be sufficiently accurate.

PART VIII.—THEORETICAL ANALYSIS OF THE STRUCTURAL ACTION OF THE STEVENSON CREEK ARCH DAM

By H. M. Westergaard,* M. Am. Soc. C. E.

Synopsis

The general method described herein applies to any nearly vertical, fairly thin, symmetrical concrete arch dam. The numerical computations refer to the Stevenson Creek Dam.

The analysis rests on the assumption that the dam acts mechanically as an elastic shell; and that this shell acts as if monolithic with the supporting rock, which is elastic.

Each small block extending through the thickness of the shell and bounded by two horizontal planes close together and two vertical planes close together (see Fig. 155) is in equilibrium under the influence of all the forces and couples acting upon it. The couples are the following: (1) The bending couples contained in a vertical plane and acting upon the horizontal sections; (2) the bending couples contained in a horizontal plane and acting upon the vertical sections; and (3) the twisting couples acting upon vertical and horizontal sections. The forces are the following: Transverse shears (in radial directions); horizontal thrusts (or arch thrusts); vertical thrusts; horizontal and vertical central shearing forces (perpendicular to the radius); and, finally, the weight of the block and the water pressure.

These forces and couples and the changes in temperature cause the block to deform. Plastic flow of the concrete and swelling and shrinkage due to variation of moisture, although important, are not considered in the present study. The blocks, which fit one another so as to form a continuous shell before deformation, must do so after deformation also; that is, the geometrical continuity is preserved.

From the two principles, that of equilibrium, and that of geometrical continuity, various general equations are derived, especially a differential equation for the deflections of the dam. A general method is given for solving this equation under the assumption that the arch thrust is constant at each elevation, that is, it varies with the elevation only. The inaccuracies introduced by this assumption are discussed in a general way, but are left otherwise for correction by other methods. The deformations of the bed-rock are taken into consideration by an approximate method, based on the idea of extending the shell some distance into the bed-rock to an imagined fixed abutment. Two different abutment lines are found: One to account for the rotation of the true abutment, the other to account for the effects of the thrusts.

The general equations reveal certain features of the influence of Poisson's ratio (the ratio of lateral contraction to longitudinal elongation). One may

* Prof. of Theoretical and Applied Mechanics, Univ. of Illinois, Urbana, Ill.

ask: What is the influence of a change of Poisson's ratio? This question presents itself in connection with the tests of models. It is desired to interpret the test of a celluloid model as applying to a concrete dam. On account of the difference in Poisson's ratio for the two materials similar loads on the dam and the model will not produce exactly similar distributions of the deflections. Nor can one expect that they will produce exactly similar distributions of the stresses. By introducing additional loads on the model one may, however, make it deflect similarly to the dam. The question arises: What are these additional loads? The equations show that except for discrepancies that can have only a small influence, it is sufficient to introduce additional loads at the crest of the model. These loads consist of horizontal radial forces and couples contained in radial planes. Their purpose is to bring about similarity in the conditions of deformations at the crests of the model and the dam, respectively. A feature of the general analytical solution suggests an experimental procedure for determining these additional loads. This suggestion of supplementary tests to correct for Poisson's ratio is included on account of the importance of investigations by models.

The general solution referred to allows a certain freedom in the choice of the type of functions in terms of which the numerical work is carried out. A judicious choice is necessary if a reasonable degree of accuracy is to be obtained without an excessive amount of work. The choice made first, while this investigation was in progress, led to a difficulty of "small differences". After lengthy computations results expected to be significant appeared as small differences between large numbers, and the accuracy was lost. It was necessary, then, to return to the general statement of the solution, select a new type of functions, and adopt a less ambitious plan for the numerical work. The discarded plan involved the numerical integration of two simultaneous ordinary linear differential equations of fourth order with variable coefficients. In the plan adopted these two equations are replaced by one. The solution of this equation does not represent the final answer, but it lends itself to supplementary computations of a much simpler kind. The solution corresponds to a distribution of the load on the dam close to the true distribution. The supplementary loads necessary to reproduce the true distribution can be computed readily, and their effects can be estimated by various means. These loads are of such a nature that if they are considered to act upon an independent arch of the dam, they will add practically nothing to the thrust in this arch; that is, they produce bending of the arch only. These supplementary loads are important close to the top of the dam. The up-stream deflections are accounted for by considering them.

A.—Introduction

1.—Purpose of the Present Theoretical Investigation and Its Relation to Other Analyses of Arch Dams.—Papers published during recent years have

contributed much to the understanding of the structural action of arch dams.* Using these investigations an engineer with judgment may design an arch dam rationally. On account of the complexity of the whole structural action of an arch dam, it is expedient in each investigation to concentrate the attention on particular phases of the problem. It is a profitable feature of this series of papers that the emphasis varies. Messrs. Smith, Noetzli, and Howell and Jaquith consider the dam to consist of horizontal arches and vertical cantilevers, and emphasize rightly the joint action of these arches and cantilevers. Messrs. Cain and Jakobsen, on the other hand, have contributed especially to the theory of the arches; the thick arches, which may be described as sharply curved beams, have distributions of stress different from those of ordinary arches or beams. Professor Cain's discussions of the different papers are particularly valuable in that they represent a balanced judgment of the different emphases.

It is hoped that the present investigation may take its place among the others as one that throws light on certain phases of the problem. *The new features taken into consideration herein are the influence of Poisson's ratio on the flexure, and the influence of the twisting moments.* Both influences may be described by referring to the arches and cantilevers. Due to Poisson's ratio a change of curvature of an arch at a particular point is accompanied by a tendency of the intersecting cantilever to change its curvature at the same point, and *vice versa*. Twisting moments are produced when two adjacent arches or two adjacent cantilevers at a certain point rotate in different amounts. The twisting moment resists this relative rotation of two parallel elements. One may say also that the twisting moment removes bending moment from one cantilever or arch, and puts it into the adjacent one.

Twisting moments play a part in the flexure of slabs. A rectangular slab supported on four sides deflects and is stressed differently from a grate-like structure of the same shape, consisting of two systems of crossing beams, tied together at the points of intersection. The torsional resistance provides stiffness and strength in the slab, and it must have some influence in the dam. When the influences of Poisson's ratio and of the twisting moments are considered, the simple picture no more exists in which independent arches and cantilevers deflect equally at each point in common, when the load is divided properly between arches and cantilevers. The structure, therefore, is treated herein as an elastic shell, rather than as a system of intersecting arches and cantilevers.

* Especially the following papers, published with discussions in *Transactions*, Am. Soc. C. E.: "Arched Dams," by B. A. Smith, M. Am. Soc. C. E., Vol. LXXXIII (1919–20) p. 2027; "Gravity and Arch Action in Curved Dams," by Fred A. Noetzli, M. Am. Soc. C. E., Vol. LXXXIV (1921), p. 1; "The Relation Between Deflections and Stresses in Arch Dams," by Fred A. Noetzli, M. Am. Soc. C. E., Vol. LXXXV (1922), p. 234; "The Circular Arch Under Normal Loads," by William Cain, M. Am. Soc. C. E., Vol. LXXXV (1922), p. 233; "Stresses in Thick Arches of Dams," by B. F. Jakobsen, M. Am. Soc. C. E., Vol. 90 (June, 1927), p. 475; and the following very recent paper: "Analysis of Arch Dams by the Trial Load Method," by C. H. Howell, M. Am. Soc. C. E., and the late A. C. Jaquith, Esq., *Proceedings*, Am. Soc. C. E., January, 1928, Papers and Discussions, p. 61. See, also, the recent book on dams where, in addition to a treatment of the subject, further references are given: "Die Staumauern," by N. Kelen, Berlin (Jul. Springer), 1926; and the book on an allied subject: "Berechnung von Behältern," by Th. Pöschl, Second Edition, Berlin (Jul. Springer), 1926 (with a chapter by Charles Terzaghi, M. Am. Soc. C. E.).

In view of the thinness of the Stevenson Creek Dam it appeared admissible, in this study, not to consider any thick-arch action. The assumptions are maintained that a straight line drawn through the dam perpendicular to the middle surface before deformation remains straight and perpendicular to the middle surface after deformation, and that the stresses are distributed through the thickness of the dam according to straight-line diagrams.

The method described is too elaborate to be used directly in design. The present stage of the work, however, is that of research, and here one may justify elaborate computations if they lead somewhere. While the ultimate aim of this work is design, the immediate aim is to account for the structural behavior of the Stevenson Creek Dam.

B.—General Theory

2.—Notation.—A number of the symbols about to be introduced appear in Fig. 155.

Co-Ordinates and Dimensions

A cylindrical surface containing the center line of the top of the dam is used for reference. In the case of the Stevenson Creek Dam this cylinder is the middle surface of the upper 30 ft.

r = radius of this cylinder of reference, equal to 99 ft. in the case of the Stevenson Creek Dam.

x = horizontal distance measured on this cylinder from the vertical plane of symmetry of the dam. The co-ordinate, x, identifies the radial plane in which any point is located.

y = vertical co-ordinate of any point, measured downward from the horizontal plane of the top of the dam. The two co-ordinates, x and y, identify the horizontal radius upon which any point is located.

r_y = horizontal radius of the middle surface; r_y is a function of y only, and does not depart greatly from r. In the Stevenson Creek Dam, $r_y = r =$ 99 ft. within the upper 30 ft. of the dam.

$r' = \dfrac{d\,r_y}{d\,y}$; $r'' = \dfrac{d^2\,r_y}{d\,y^2}$.

s = x-co-ordinate of the imagined fixed abutment introduced in the computations for the purpose of taking into account the rotation of the true abutment due to the bending moments. This imagined abutment lies a small distance inside the bed-rock.

l = distance to be added to s for the purpose of taking into account the deformations of the bed-rock due to the arch thrust. The over-all length of arch, assumed for this purpose, becomes $2\,(s + l)$.

t = thickness of the dam (2 ft. on the upper 30 ft. of the Stevenson Creek Dam).

λ = unit of distance used in the computations (in order to avoid awkwardly large or small numbers). The value chosen in the numerical work is $\lambda = 10$ ft.

Deflections

z = component of the deflection of the middle surface in the direction of the horizontal radius; this deflection is considered positive down stream.

Properties of the Material and Measures of Stiffness of the Dam

E = modulus of elasticity of the concrete; in the numerical work E is assumed to be 3 600 000 lb. per sq. in.

μ = Poisson's ratio of lateral contraction to longitudinal elongation for the concrete; numerical values assumed, $\mu = 0.15$.

ε_T = coefficient of temperature expansion for the concrete.

$$N = \frac{E\,t^3}{12\,(1-\mu^2)} = \text{measure of stiffness of the dam in flexure.}$$

$$N' = \frac{d\,N}{d\,y},\ N'' = \frac{d^2\,N}{d\,y^2}.$$

$$K = \frac{2\,N'}{N},\ k = \frac{N''}{N}.$$

Loads and Change of Temperature

w = water pressure per unit of area of the cylinder with radius, r.

w_c = weight of the concrete per unit of volume.

T = uniform drop of temperature.

Internal Forces and Couples

In discussing internal forces and couples reference is made to "the section, $d\,x$", and "the section, $d\,y$". These sections extend through the thickness of the dam. The section, $d\,x$, is horizontal and is bounded by two radii with the x-co-ordinates, x and $x + d\,x$. The section, $d\,y$, is a part of a vertical radial plane, and lies between the depths, y and $y + d\,y$. These sections appear as faces of the block in Fig. 155. The forces and couples shown in Fig. 155 are resultants of internal normal stresses and shears. The symbols which follow represent the values per unit of length of the dimension, $d\,x$ or $d\,y$, respectively. The forces may be stated in pounds per inch, the couples in inch-pounds per inch, or, in pounds $\left(\frac{\text{inch-pounds}}{\text{inches}} = \text{pounds}\right)$.

P_x = horizontal thrust (arch thrust), acting upon the section, $d\,y$, per unit of length of the dimension, $d\,y$.

P_y = vertical thrust, acting upon the section, $d\,x$, per unit of length of the dimension, $d\,x$.

P_{xy} = vertical central shear, acting upon the section, $d\,y$. P_y is considered positive downward when acting upon the part of increasing values of x; that is, the part of the material having larger values of x than the cross-section.

P_{yx} = horizontal central shear, acting upon the section, $d\,x$, along a tangent to the middle surface. When acting upon the part below the section, P_{yx} is considered positive in the direction of increasing x.

Q_x = transverse shear (in the radial direction), acting upon the section, $d\,y$, positive up stream when acting upon the part of increasing values of x.

Q_y = transverse shear (in the radial direction), acting upon the section, $d\,x$, positive up stream when acting upon the part below the section.

M_x = bending moment acting upon the section, $d\,y$, positive when it tends to produce compression in the up-stream surface.

M_y = bending moment acting upon the section, $d\,x$, positive when it tends to produce compression in the up-stream surface.

M_{xy} and M_{yx} = twisting moments acting upon the sections, $d\,y$ and $d\,x$, respectively. On account of the theorem of equality of components of shearing stress in two sections perpendicular to each other, the two values are nearly equal when, as assumed here, r_y, the radius of the middle surface, differs only by a small amount from r, the radius of the cylindrical surface along which the dimension, $d\,x$, is measured. The twisting moment is considered positive when the two twisting moments together tend to produce compression in the up-stream surface in a diagonal direction of increasing values of x and y.

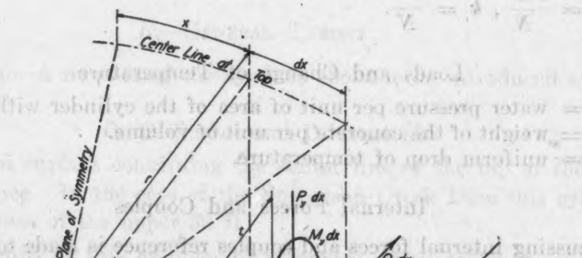


FIG. 155.—FORCES AND COUPLES ACTING UPON AN ELEMENT OF THE DAM.

Stresses

σ_x = normal horizontal unit stress in the direction of the increment, $d\,x$, positive as tension.

σ_y = normal vertical unit stress, positive as tension.

τ_{xy} = shearing unit stress in the same two sections, in the directions of the increments, $d\,x$ and $d\,y$, respectively. This shearing

stress is considered positive when it tends to produce elongation in a diagonal direction of increasing values of x and y.

Strains and Detrusions

ε_x and ε_y = unit elongations in the directions of $d\,x$ and $d\,y$, respectively.
γ_{xy} = detrusion corresponding to the shearing stress, τ_{xy}, positive when τ_{xy} is positive.

*3.—Equations of Equilibrium.**—The block shown in Fig. 155 is bounded by radial planes with x-co-ordinates, x and $x + d\,x$, respectively; by two horizontal planes at the depths, y and $y + d\,y$, respectively; and by the up-stream and down-stream surfaces of the dam. The two radial planes will be referred to as the back and the front, and the two horizontal planes as the top and the bottom, respectively. The dimensions, $d\,x$ and $d\,y$, are, of course, to be interpreted as infinitesimal distances. The forces and couples shown in the diagram (and defined in Section 2, under the heading, "Notation") hold the block in equilibrium.

Consider, first, the components of forces in the direction of the radius through the center of the block. The transverse shear, Q_x, acts on the back of the block over the height, $d\,y$. The total value is $Q_x\,d\,y$. In passing from the back to the front, Q_x increases at the rate, $\frac{\delta\,Q_x}{\delta\,x}$†, over the distance, $d\,x$. The resulting increase is $\frac{\delta\,Q_x}{\delta\,x}\,d\,x$, and the total transverse shear on the front has the value stated in Fig. 155. Ignoring differences which are infinitesimal of an order higher than the second, one finds a down-stream force resulting from the transverse shears on the back and the front equal to $\frac{\delta\,Q_x}{\delta\,x}\,d\,x\,d\,y$. Likewise, the transverse shears on the top and the bottom result in a down-stream force equal to $\frac{\delta\,Q_y}{\delta\,y}\,d\,x\,d\,y$. The water pressure is $w\,d\,x\,d\,y$. The only forces that remain to be considered are the arch thrusts ($P_x\,d\,y$ on the back). The lines of action of the two thrusts make an angle, $\frac{d\,x}{r}$. By drawing a force triangle for the thrusts on the front and the back, one finds, ignoring again the quantities which are infinitesimal of an order higher than the second, a resulting up-stream force equal to $P_x\,d\,y \times \frac{d\,x}{r}$.

Summing up these forces, and dividing by the common factor, $d\,x\,d\,y$, one finds the equation of equilibrium of radial forces,

$$\frac{\delta\,Q_x}{\delta\,x} + \frac{\delta\,Q_y}{\delta\,y} = -\,w + \frac{P_x}{r} \quad \text{.....................} (33)$$

* The equations about to be derived may well be compared with the corresponding ones applying to a slab. See, for example, the book by A. Nádai, "Die elastischen Platten," Berlin (Jul. Springer), 1925; or the paper by W. A. Slater, M. Am. Soc. C. E., and the writer, "Moments and Stresses in Slabs," *Proceedings*, Am. Concrete Inst., v. 17, 1921, p. 415 (or, National Research Council, Reprint and Circular Series, No. 32).

† The symbol, δ, is used in this Report instead of ∂, which is preferred by mathematicians and is indicated in the diagrams.

Next, take moments with respect to a vertical axis through the center of the block. The bending moments acting upon the back and the front have the difference, $\frac{\delta M_x}{\delta x} d x d y$. The twisting moments acting on the top and the bottom contribute their difference, $\frac{\delta M_{yx}}{\delta y} d x d y$. The transverse shears on the back and the front contribute (with the term containing $\frac{\delta Q_x}{\delta x}$ omitted because it is infinitesimal of third order) the moment, $Q_x \, d y \times d x$, in the opposite direction. The horizontal distance between the central shears on the top and bottom is $- r' \, d y$ (a positive distance in Fig. 155 since r' is negative). They contribute the moment, $- P_{yx} \, d x \times r' \, d y$. Summing up, and dividing by the common factor, $d x \, d y$, one finds the equation of equilibrium of moments about a vertical axis,

$$\frac{\delta M_x}{\delta x} + \frac{\delta M_{yx}}{\delta y} = Q_x - P_{yx} \, r' \quad \ldots\ldots\ldots\ldots (34)$$

Likewise, by taking moments with respect to an axis drawn through the center of the block in the direction of the increment, $d x$, one obtains the equation,

$$\frac{\delta M_y}{\delta y} + \frac{\delta M_{xy}}{\delta x} = Q_y - P_y \, r' \quad \ldots\ldots\ldots\ldots (35)$$

By summing up the components of forces in the direction of the increment, $d x$, and in the vertical direction, respectively, one finds the two equations of equilibrium,

$$\frac{\delta P_x}{\delta x} + \frac{\delta P_{yx}}{\delta y} + \frac{Q_x}{r} = 0 \quad \ldots\ldots\ldots\ldots (36)$$

$$\frac{\delta P_y}{\delta y} + \frac{\delta P_{xy}}{\delta x} = w_c \, t \quad \ldots\ldots\ldots\ldots (37)$$

In considering, finally, the moments with respect to a radius drawn through the center of the block, it is observed that the twisting moments on the back and the front contribute an amount proportional to the angle, $\frac{d x}{r}$, between the back and the front. By laying off the two couples as vectors, one finds this amount to be $M_{xy} \, d y \times \frac{d x}{r}$. Thus, one obtains the equation of equilibrium,

$$P_{yx} - P_{xy} = \frac{M_{xy}}{r} \quad \ldots\ldots\ldots\ldots (38)$$

Equations, (33) to (38), inclusive, are necessary and sufficient for the equilibrium of the block.

If the two shears and the twisting moment occurring in Equation (38) are about equally significant in producing shearing stresses, it may be observed that since r is large compared with t, the quantity on the right side of the equation will be very small compared with each of the two terms on the left

side. Consequently, if the central shears are significant at all, one may replace Equation (38) with sufficient accuracy by the simpler relation,

$$P_{xy} = P_{yx} \qquad (39)$$

For the reason stated when M_{xy} was defined (under "Notation"), one may write, furthermore, with sufficient accuracy,

$$M_{xy} = M_{yx} \qquad (40)$$

By differentiating Equation (34) with respect to x and Equation (35) with respect to y, and combining with Equation (33) so as to eliminate Q_x and Q_y, making use of Equations (37), (39), and (40), one finds the following relation between the loads, the arch thrust, the bending moments, and the twisting moments:

$$\frac{\delta^2 M_x}{\delta x^2} + 2\frac{\delta^2 M_{xy}}{\delta x \, \delta y} + \frac{\delta^2 M_y}{\delta y^2} = -w + \frac{P_x}{r} - P_y r'' - w_c t r' \qquad (41)$$

If the rate of change of the arch thrust, $\frac{\delta P_x}{\delta x}$, occurring in Equation (36) is important at all, the term, $\frac{Q_x}{r}$, can be only small compared with the other two terms. By ignoring the term, $\frac{Q_x}{r}$, and admitting Equation (39), the two Equations (36) and (37) are replaced by the following pair, which is simpler:

$$\frac{\delta P_x}{\delta x} + \frac{\delta P_{xy}}{\delta y} = 0 ; \quad \frac{\delta P_y}{\delta y} + \frac{\delta P_{xy}}{\delta x} = w_c \, t \qquad (42)$$

These equations are of the form of those which led to the introduction of Airy's stress function.* In terms of this method the two Equations (42) become equivalent to the statement that a function, F, exists, called a stress function, by means of which the three quantities, P_x, P_y, and P_{xy}, can be expressed as follows:

$$P_x = \frac{\delta^2 F}{\delta y^2} ; \quad P_y = \frac{\delta^2 F}{\delta x^2} + w_c \int_0^y t \, d\, y ; \quad P_{xy} = -\frac{\delta^2 F}{\delta x \, \delta y} \qquad (43)$$

The equations derived thus far are equations of equilibrium. They are independent of the properties of elasticity of the material.

4.—Relations Between the Internal Forces and Couples, the Stresses, and the Deformations.—In the relations which follow it is advisable to interpret t as the thickness measured in a direction perpendicular to the middle surface, rather than in a horizontal direction. Since the middle surface of the dam investigated does not depart greatly from a vertical cylinder, the difference between the two interpretations is not important, and the choice may be left as a matter of judgment.

The stresses at the middle surface may be expressed as follows (see the explanations of the symbols under "Notation", Section 2):

$$\sigma_x = -\frac{P_x}{t} ; \quad \sigma_y = -\frac{P_y}{t} ; \quad \tau_{xy} = -\frac{P_{xy}}{t} \qquad (44)$$

* "Mathematical Theory of Elasticity," by A. E. H. Love, Third Edition, 1920, p. 86; "Drang und Zwang," by A. and L. Föppl, v. 1, Second Edition, 1924, p. 246.

Since no thick-arch action is considered, the stresses produced in the down-stream surface by the bending moments and twisting moments may be found by dividing the moments by the section modulus per unit of width, $\frac{t^2}{6}$. Thus, one finds the following resultant stresses in the surfaces, with the upper signs referring to the up-stream, and the lower to the down-stream, surface:

$$\sigma_x = -\frac{P_x}{t} \mp \frac{6\,M_x}{t^2};\ \sigma_y = -\frac{P_y}{t} \mp \frac{6\,M_y}{t^2};\ \tau_{xy} = -\frac{P_{xy}}{t} \mp \frac{6\,M_{xy}}{t^2} \quad (45)$$

In terms of the modulus of elasticity for tension and compression, E, and Poisson's ratio, μ, the modulus of elasticity in shear is $\frac{E}{2\,(1+\mu)}$. The strains and detrusions corresponding to the stresses stated in Equations (44) or (45), respectively, may be written:

$$\varepsilon_x = \frac{1}{E}\,(\sigma_x - \mu\,\sigma_y);\ \varepsilon_y = \frac{1}{E}\,(\sigma_y - \mu\,\sigma_x);\ \gamma_{xy} = \frac{2\,(1+\mu)}{E}\,\tau_{xy} \quad (46)$$

Solving these equations for the stresses, one obtains the relations which are useful in interpreting given deformations (for example, strains obtained experimentally) in terms of stresses:

$$\sigma_x = \frac{E}{1-\mu^2}\,(\varepsilon_x + \mu\,\varepsilon_y);\ \sigma_y = \frac{E}{1-\mu^2}\,(\varepsilon_y + \mu\,\varepsilon_x);\ \tau_{xy} = \frac{E}{2\,(1+\mu)}\gamma_{xy} \quad (47)$$

In the following statements the quantity, r, is used several times instead of r_y. The distinction between the two is unimportant at this place. If all points of the shell deflect the same amount, z, the curvature of the horizontal center line (horizontal curve on the middle surface) changes by the amount, $\pm\frac{z}{r^2}$. If the deflection remains zero at a particular point while the shell deforms, the curvature of the horizontal center line changes by the amount, $\pm\frac{\delta^2 z}{\delta x^2}$. In the general case, the changes of the curvature of the horizontal center line and of the vertical center line (curve on the middle surface in a vertical radial plane) may be stated as $-\frac{\delta^2 z}{\delta x^2} - \frac{z}{r^2}$ and $-\frac{\delta^2 z}{\delta y^2}$, respectively. The choice of negative signs in these expressions has the advantage that changes called positive will correspond to positive bending moments. The twist of the shell at a given point for horizontal and vertical directions is numerically equal to the rate at which the horizontal tangent rotates as one moves in a vertical direction, or the rate at which the tangent to the vertical center line rotates as one moves in a horizontal direction. This twist is stated as $-\frac{\delta^2 z}{\delta x\,\delta y}$.

The state of moments, $M_x \neq 0$, $M_y = 0$, at a point produces a change of curvature of the horizontal center line equal to $\frac{12}{E\,t^3}\,M_x$ and a change of curvature

of the vertical center line equal to $-\dfrac{12}{E\,t^3}\,\mu\,M_x$. The state of moments, $M_y \neq 0$, $M_x = 0$, produces corresponding changes, with the indices, x and y, and the corresponding directions interchanged. The twisting moment, M_{xy}, does not contribute to these curvatures. The arch thrust, P_x, by shortening the center line, contributes an amount $\pm \dfrac{P_x}{E\,t.r}$ to the curvature of the horizontal center line. This amount and a corresponding contribution from P_y, containing the factor, μ, are relatively unimportant, and can be ignored. An uneven temperature may curve the shell, but only uniform changes of temperature are considered here. Thus, in the general case considered, with M_x and M_y contributing to the curvatures, the change of curvature of the horizontal center line may be expressed as follows:

$$-\frac{\delta^2 z}{\delta x^2} - \frac{z}{r^2} = \frac{12}{E\,t^3}\,(M_x - \mu\,M_y) \quad\ldots\ldots\ldots\ldots\ldots (48)$$

and that of the vertical center line as follows:

$$-\frac{\delta^2 z}{\delta y^2} = \frac{12}{E\,t^3}\,(M_y - \mu\,M_x) \quad\ldots\ldots\ldots\ldots\ldots (49)$$

The twisting moment, M_{xy}, produces a twist, $-\dfrac{\delta^2 z}{\delta x\,\delta y}$. The amount is the same as if the shell were a flat plate of the same thickness. The state of moments, $M_x = M_y = 0$, $M_{xy} \neq 0$, may be described in terms of moments which refer to the inclined directions of dx_1 and dy_1 making angles of 45° and 135° with the direction of dx. This equivalent state of moments consists of the bending moments, $M_{x_1} = M_{xy}$ and $M_{y_1} = -M_{xy}$, and the twisting moment, $M_{x_1 y_1} = 0$. The corresponding curvatures of the flat plate (compare Equations (48) and (49)) are defined by the statement,

$$-\frac{\delta^2 z}{\delta x_1^{\,2}} = \frac{\delta^2 z}{\delta y_1^{\,2}} = \frac{12\,(1+\mu)}{E\,t^3}\,M_{xy}$$

With $x_1 = \dfrac{1}{\sqrt{2}}(x+y)$, $y_1 = \dfrac{1}{\sqrt{2}}(-x+y)$, one finds,

$$\frac{\delta z}{\delta x} = \frac{\delta z}{\delta x_1} \times \frac{\delta x_1}{\delta x} + \frac{\delta z}{\delta y_1} \times \frac{\delta y_1}{\delta x} = \frac{1}{\sqrt{2}}\left(\frac{\delta z}{\delta x_1} - \frac{\delta z}{\delta y_1}\right)$$

or,

$$\frac{\delta}{\delta x} = \frac{1}{\sqrt{2}}\left(\frac{\delta}{\delta x_1} - \frac{\delta}{\delta y_1}\right)$$

Likewise, one finds,

$$\frac{\delta}{\delta y} = \frac{1}{\sqrt{2}}\left(\frac{\delta}{\delta x_1} + \frac{\delta}{\delta y_1}\right)$$

Thus,

$$-\frac{\delta_z^{\,2}}{\delta x\,\delta y} = -\frac{1}{2}\left(\frac{\delta}{\delta x_1} - \frac{\delta}{\delta y_1}\right)\left(\frac{\delta}{\delta x_1} + \frac{\delta}{\delta y_1}\right) z = \frac{1}{2}\left(-\frac{\delta_z^{\,2}}{\delta x_1^{\,2}} + \frac{\delta^2 z}{\delta y_1^{\,2}}\right)$$

That is, in view of the foregoing equation containing M_{xy}, the twist of the plate, or the shell becomes,

$$-\frac{\delta^2 z}{\delta x \, \delta y} = \frac{12 (1 + \mu)}{E t^3} M_{xy} \quad \text{.................... (50)}$$

By solving Equations (48), (49), and (50) for the moments and introducing N, one obtains:

$$N = \frac{E t^3}{12 (1 - \mu^2)} \quad \text{.......................... (51)}$$

$$M_x = N \left(- \frac{\delta^2 z}{\delta x^2} - \frac{z}{r^2} - \mu \frac{\delta^2 z}{\delta y^2} \right) \quad \text{.............. (52)}$$

$$M_y = N \left(- \frac{\delta^2 z}{\delta y^2} - \mu \frac{\delta^2 z}{\delta x^2} - \frac{\mu}{r^2} z \right) \quad \text{.............. (53)}$$

$$M_{xy} = - N (1 - \mu) \frac{\delta^2 z}{\delta x \, \delta y} \quad \text{.................. (54)}$$

5.—*The Equation of Flexure and the Equation of the Central Forces.*—By substituting the values of the moments given in the last three equations in Equation (41), and by using the symbols, $K = \frac{2 N'}{N}$ and $k = \frac{N''}{N}$, in which,

$$N' = \frac{d N}{d y}; \; N'' = \frac{d^2 N}{d y^2} \quad \text{...................... (55)}$$

one obtains the following equation:*

$$\frac{\delta^4 z}{\delta x^4} + 2 \frac{\delta^4 z}{\delta x^2 \, \delta y^2} + \frac{\delta^4 z}{\delta y^4} + \frac{1}{r^2} \frac{\delta^2 z}{\delta x^2} + \frac{\mu}{r^2} \frac{\delta^2 z}{\delta y^2}$$
$$+ K \left(\frac{\delta^3 z}{\delta y^3} + \frac{\delta^3 z}{\delta x^2 \, \delta y} + \frac{\mu}{r^2} \frac{\delta z}{\delta y} \right) + k \left(\frac{\delta^2 z}{\delta y^2} + \mu \frac{\delta^2 z}{\delta x^2} + \frac{\mu}{r^2} z \right)$$
$$- \frac{1}{N} \left(w - \frac{P_x}{r} + P_y r'' + w_c t r' \right) = 0 \quad \text{.............. (56)}$$

Equation (56) is called the equation of flexure because of the prominent place occupied in it by the function, z, the deflection. Most of the numerical work is concerned with the solution of this equation.

Equation (56) contains, in addition to the function, z, also the two unknown functions, P_x and P_y, the thrusts. These two may be expressed by Equations (43) in terms of the one function, F, the stress function. The two functions, z and F, jointly, define the structural action of the dam. Since there are two such functions, an additional equation is needed.

It is possible to derive an additional differential equation for F and z, in which F occupies a prominent place similar to that of z in Equation (56). Since F defines the central forces, P_x; P_y, and P_{xy}, the new equation is called the equation of the central forces. This differential equation is not used in the numerical work which follows, but is replaced by an approximate equation. For this reason it will be derived here only in the relatively simple form applicable to parts of the dam where the thickness is constant, and the

* This equation corresponds to Lagrange's equation in the theory of slabs. See the book by Nádai, or the paper by W. A. Slater, M. Am. Soc. C. E., and the writer, previously referred to.

middle surface is a vertical cylinder (such as the upper 30 ft. of the Stevenson Creek Dam). There is no particular difficulty in deriving the equation in the general form, applicable to the general case of variable thickness and slightly variable r_y.

Let ξ = deflection of the point, x, y, of the middle surface in the direction of the increment, $d\,x$; and,

η = deflection of the same point in the direction of y.

Then the corresponding strains and detrusions in the middle surface may be expressed, as follows:

$$\varepsilon_x = \frac{\delta\,\xi}{\delta\,x} - \frac{z}{r}\,;\ \varepsilon_y = \frac{\delta\,\eta}{\delta\,y}\,;\ \gamma_{xy} = \frac{\delta\,\xi}{\delta\,y} + \frac{\delta\,\eta}{\delta\,x} \qquad (57)$$

By applying to these equations the differential operators, $\frac{\delta^2}{\delta\,y^2}, \frac{\delta^2}{\delta\,x^2}$, and $-\frac{\delta^2}{\delta\,x\,\delta\,y}$, respectively, and adding, one finds the following equation (called equation of compatibility, being a condition of geometrical continuity):*

$$\frac{\delta^2\,\varepsilon_x}{\delta\,y^2} + \frac{\delta^2\,\varepsilon_y}{\delta\,x^2} - \frac{\delta^2\,\gamma_{xy}}{\delta\,x\,\delta\,y} + \frac{1}{r}\,\frac{\delta^2\,z}{\delta\,y^2} = 0 \qquad (58)$$

By substituting the values, $\varepsilon_x = \frac{1}{E\,t}\,(-P_x + \mu\,P_y)$, $\varepsilon_y = \frac{1}{E\,t}\,(-P_y + \mu\,P_x)$, and $\gamma_{xy} = -\frac{2\,(1+\mu)}{E\,t}\,P_{xy}$, with P_x, P_y, and P_{xy} expressed in terms of the stress-function, F, by Equations (43), one finds the equation of the central forces (for constant, t and r_y):

$$\frac{\delta^4\,F}{\delta\,x^4} + 2\,\frac{\delta^4\,F}{\delta\,x^2\,\delta\,y^2} + \frac{\delta^4\,F}{\delta\,y^4} - \frac{E\,t}{r}\,\frac{\delta^2\,z}{\delta\,y^2} = 0 \qquad (59)$$

The problem of the thin arch dam is solved when the two simultaneous partial differential equations of the fourth order, Equations (56) and (59), are solved, with proper consideration of the boundary conditions. It is a difficult matter, however, to deal with two such simultaneous equations. At the present stage of the investigation it appears advisable to dispose of the second equation, the equation for F, by introducing simplifying assumptions concerning the central forces. It is possible to estimate the inaccuracies resulting from this procedure; they are most noticeable at the top of the dam.

The simplifying assumptions are:

$$P_x = P = \text{function of } y \text{ only} \qquad (60)$$

and,

$$P_y = w_c \int_0^y t\;d\,y;\ P_{xy} = 0 \qquad (61)$$

These assumptions are equivalent to the statement that F is a function of y only. It is assumed, furthermore, that P may be expressed in terms of the deformations as if ε_y were zero. One finds then from the second Equation

*Compare Equations (58) and (59) with the corresponding ones for $r = \infty$ (plane co-ordinates, x and y). See A. and L. Föppl, "Drang und Zwang," v. 1, Second Edition, 1924, pp. 53 and 248.

(46), for the purpose of this particular computation, $\sigma_y = \mu\ \sigma_x$, and then, from the first Equation (46), the unit shortening due to the thrust, $-\varepsilon_x = \frac{1-\mu^2}{E\ t} P$. In considering one-half the arch, this shortening is assumed to occur over the length, $s + l$, in which, l is a distance added to s in order to account for the deformations of the bed-rock. The left side of Equation (62) expresses the total shortening with the effect of a drop of temperature, T, included. The right side expresses the same total shortening of the curve in terms of the deflections. The equation is,

$$\frac{1-\mu^2}{E\ t} P\ (s + l) + \varepsilon_T\ T\ s = \frac{1}{r} \int_0^s z\ d\ x \qquad (62)$$

Equation (62), although not exact, will be used in place of Equation (59) in conjunction with the equation of flexure (Equation (56)). Equation (62) becomes nearly correct if P is interpreted as the average value of P_x at the particular elevation.

6.—Boundary Conditions at the Top of the Dam.—The following conditions apply at the top of the dam:

$$P_y = P_{xy} = M_y = 0 \qquad (63)$$

In addition, there is a condition referring to the transverse shears, Q_y, and the twisting moments, M_{xy}. The same boundary condition applies to a slab. The peculiar feature that the two quantities should lead to only one boundary condition was explained in 1867 by Thomson (later, Lord Kelvin) and Tait.*

Assume for the time being that transverse shears and twisting moments act at the top as shown in Fig. 156(*a*). The twisting couple, $M_{xy}\ d\ x$, acting upon Element No. 1, is to be thought of as the resultant of the shearing stresses parallel to the increment, $d\ x$. This couple is equivalent to the couple, shown in Fig. 156(*b*), which consists of two transverse, equal and opposite forces, M_{xy}, with the arm, $d\ x$. It is permissible to replace the original twisting couple by the equivalent couple because this replacement causes only minor local disturbances in the state of stress. The slightly increased twisting couple acting upon the adjacent element, No. 2, may be replaced in the same manner by two transverse forces, as shown in Fig. 156. The result is a surplus transverse force, $\frac{\delta\ M_{xy}}{\delta\ x} d\ x$, acting up stream along the boundary between the two elements, that is, a transverse force, $\frac{\delta\ M_{xy}}{\delta\ x}$, per unit of length. Consequently, the combination of transverse shears, Q_y, and twisting moments, M_{xy}, acting on the top as external forces, may be replaced by the distributed reaction, positive up stream,

$$R_y = Q_y + \frac{\delta\ M_{xy}}{\delta\ x} \qquad (64)$$

This relation is known as the theorem of Kelvin and Tait. The boundary condition is:

$$R_y = 0 \qquad (65)$$

* "Natural Philosophy," by Thomson and Tait, 1867, see Articles 645–648 in the later editions; or, "Die elastischen Platten", by A. Nádai, 1925, p. 34.

The requirements, $P_y = P_{xy} = 0$, have already been disposed of by Equations (61). The condition, $M_y = 0$, is transcribed into a condition for z by substituting for M_y the right side of Equation (53). R_y, likewise, may be expressed in terms of z by the use of Equations (35), (53), and (54). One finds:

$$R_y = \frac{\delta M_y}{\delta y} + 2\,\frac{\delta M_{xy}}{\delta x} \quad \text{.......................(66)}$$

$$R_y = -N\left(\frac{\delta^3 z}{\delta y^3} + (2 - \mu)\,\frac{\delta^3 z}{\delta x^2\,\delta y} + \frac{\mu}{r^2}\,\frac{\delta z}{\delta y}\right)$$
$$- N'\left(\frac{\delta^2 z}{\delta y^2} + \mu\,\frac{\delta^2 z}{\delta x^2} + \frac{\mu}{r^2}\,z\right) \quad \text{.................(67)}$$

In the Stevenson Creek Dam, N' is zero at the top.

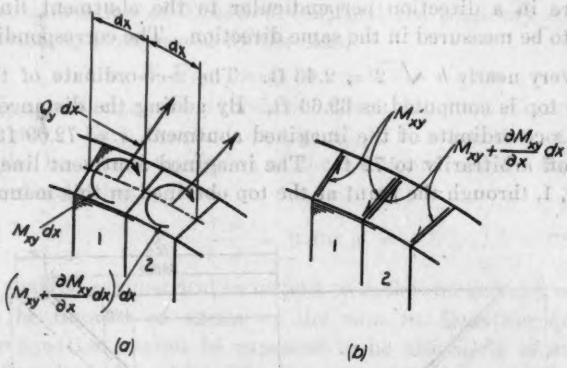


FIG. 156.—IMAGINED FORCES AND COUPLES AT TOP OF DAM.

7.—Deformations of Bed-Rock and the Boundary Conditions at the Abutments.—In stating the conditions which the solution of the equation of flexure must satisfy at the abutments, use is made of an investigation by Fredrik Vogt, dealing with deformations of bed-rock.* He gives formulas for average translations and rotations of the base of a rectangular block standing on deformable rock. He interprets the results by imagining this block to be extended into the rock, and to be supported on a new rigid foundation at the depth, h. He states the values of h necessary in order that particular motions of the original foundation may be reproduced in the imagined case at the same elevation.

The problem of the deformations of bed-rock is worthy of an intimate study. However, it is expedient to use now a quick approximate procedure for taking these deformations into consideration. The idea of the imagined fixed abutment is well-suited to this purpose. The procedure may be explained by referring to the Stevenson Creek Dam. Fig. 157 shows the true abutments and two imagined abutment lines.

* "Ueber die Berechmung der Fundamentdeformation", by Fredrik Vogt, Det Norske Videnskaps-Akademi, Oslo, Avhandlinger, Math.-Naturv. Klasse, 1925, No. 2. See, also, his discussion of the paper, referred to previously, by B. F. Jakobsen, M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., v. 90 (June, 1927), p. 554.

In using Vogt's formulas for the depth, h, the factor $\frac{m^2 - 1}{m^2}$, in which, $m = \frac{1}{\mu}$, will be replaced by unity. Furthermore, a factor equal to the ratio of the moduli of elasticity of the concrete and the bed-rock will be included. This ratio is assumed to be 1.8. For a long concrete wall of constant thickness, t, with a uniformly distributed couple tending to tip it over, Vogt's Formula (17a), which refers to the rotation of the base, gives, with modifications as stated,

$$h = \frac{3}{2\pi} \times 1.8\, t = 0.86\, t \qquad (68)$$

Applied to the upper part of the Stevenson Creek Dam, where the thickness is constant (2 ft.), this computation gives the distance, $h = 1.72$ ft., from the true abutment, A, to the imagined abutment, B. The principal bending moments are in a direction perpendicular to the abutment lines. The distance, h, is to be measured in the same direction. The corresponding horizontal distance is very nearly $h\sqrt{2} = 2.43$ ft. The x-co-ordinate of the true abutment at the top is computed as 69.66 ft. By adding the distance, 2.43 ft., one obtains the x-co-ordinate of the imagined abutment, $s = 72.09$ ft. This value is rounded off arbitrarily to 72 ft. The imagined abutment line, B, is drawn with a slope, 1, through the point at the top obtained in this manner.

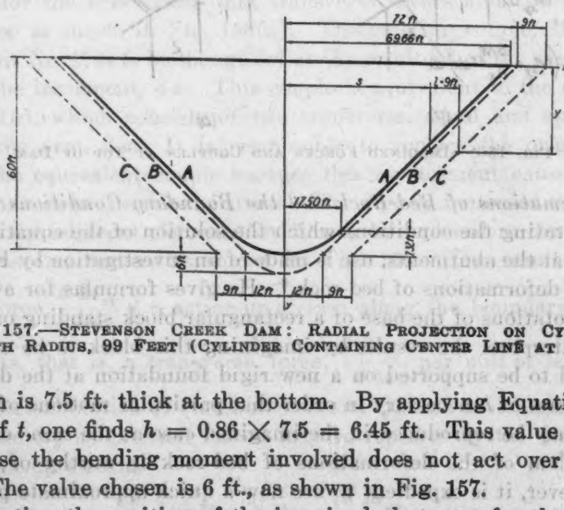


FIG. 157.—STEVENSON CREEK DAM: RADIAL PROJECTION ON CYLINDER WITH RADIUS, 99 FEET (CYLINDER CONTAINING CENTER LINE AT TOP).

The dam is 7.5 ft. thick at the bottom. By applying Equation (68) with this value of t, one finds $h = 0.86 \times 7.5 = 6.45$ ft. This value is a little too large because the bending moment involved does not act over a very large distance. The value chosen is 6 ft., as shown in Fig. 157.

In estimating the position of the imagined abutment for the arch thrusts, a block is considered with a base, 2 ft. by 60 ft. Vogt's Formula (9), which refers to forces perpendicular to the base, gives the distance to the imagined abutment, $h = 1.8\sqrt[3]{2^2 \times 60} = 11.2$ ft. By laying off this distance in a horizontal direction from the abutment, A, one obtains approximately the line, C, shown in Fig. 157. The horizontal distance from B to C is $l = 9$ ft. Since

the pressure considered is not perpendicular to the abutment, this procedure is in fact quite arbitrary. It may be improved upon by a combined use of Vogt's Formula (8), referring to a normal thrust, and Formula (23), referring to shearing forces. By this method a smaller value of the horizontal distance from A to C will be found (about 8 ft., or, possibly, about 9 ft. if the influence of the water pressure on the side of the canyon is included). In spite of its arbitrary character, the line C (Fig. 157), is used in the computations.

The boundary conditions applying to the abutments may now be stated as follows: There shall be no rotation at the line, B. There shall be no deflection at A or B; whether one specifies A or B, does not matter greatly, since the slope at B is zero. The deformations due to the thrusts shall be considered to extend to the line, C; at this line there shall be no displacement. This last condition is satisfied by use of Equation (62).

8.—General Method of Solving the Equation of Flexure.—The following general method is proposed for solving the equation of flexure (Equation (56)), with consideration of Equation (62) and the boundary conditions.

The deflection, z, is expressed by the sum,

$$z = \Sigma\, U_{(m)}\, \phi_{(m)};\ m = 1, 2, \ldots\ n \quad\ldots\ldots(69)$$

in which, $U_{(m)}$ and $\phi_{(m)}$ are functions. Each function, U (or $U_{(m)}$), is a function of y only. Each function, ϕ (or $\phi_{(m)}$), is a function of x and s, so chosen that the boundary conditions at the abutment will be satisfied. It is required accordingly that,

$$\phi = \frac{\delta\, \phi}{\delta\, x} = 0 \text{ for } x = s \quad\ldots\ldots(70)$$

Moreover, ϕ shall be symmetrical in respect to x, that is, $\phi\,(x, s) = \phi\,(-x, s)$.

With n, the number of terms in the sum in Equation (69), a finite number, this equation cannot be expected to be absolutely exact, even with well-chosen functions, ϕ. After choosing the functions, ϕ, the problem is to determine the functions, U, so that the expression for z becomes as accurate as possible.

A judicious choice of the functions, ϕ, is necessary if an acceptable solution is to be found with a small number of terms, $U\,\phi$, in Equation (69). In a numerical solution of this type the number, n, in fact, must be very small, for the dam not more than 2, if the amount of work is not to be excessive. The following two functions are proposed for the case, $n = 2$:

$$\phi_I = (1 - s^{-2}\, x^2)^2 \quad\ldots\ldots(71)$$

$$\phi_{II} = (1 - s^{-2}\, x^2)^2\, (1 - 11\, s^{-2}\, x^2) \quad\ldots\ldots(72)$$

These functions are represented graphically in Fig. 158. An inspection of the curves shows that by superposition of the two diagrams according to the formula,

$$z = U_I\, \phi_I + U_{II}\, \phi_{II} \quad\ldots\ldots(73)$$

with proper values assigned to U_I and U_{II}, one may reproduce quite well the diagrams of deflections obtained experimentally at the different elevations. The two functions are proposed with the idea that each shall express features of the diagram of deflections as different as possible from those expressed

by the other. This quality of the functions is indicated by the property of orthogonality, defined by the following relation, which is verified easily:

$$\int_0^s \phi_I \, \phi_{II} \, d\,x = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (74)$$

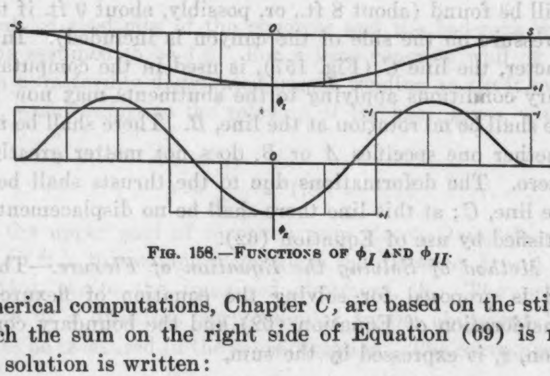


FIG. 158.—FUNCTIONS OF ϕ_I AND ϕ_{II}.

The numerical computations, Chapter *C*, are based on the still simpler solution in which the sum on the right side of Equation (69) is reduced to one term. This solution is written:

$$z = U\,\phi \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (75)$$

The function, ϕ, used herein is the same as ϕ_I in Equation (71). This solution does not show the up-stream deflections at the top of the dam, which are conspicuous in the tests, and it does not represent a final answer. Additional computations are needed, therefore, in connection with it. These computations will be based on the idea of supplementary loads, as described in the Synopsis, and they will furnish information about the features which are not accounted for directly by Equation (75).

The following statement of a general method of solution applies for any number of terms, n, and is not dependent on the particular choice of functions which is represented by Equations (71) and (72). A set of functions of x and s,

$$g_p = g_p\ (x, s) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (76)$$

is chosen, in which, $p = 1, 2 \ldots . n$. Assume for the time being that the solution, $z = \Sigma\, U\, \phi$, with n-terms in the sum, satisfies the equation of flexure exactly (as it may if the load is modified slightly). Let X denote the whole left side of the equation of flexure (Equation (56)), with $P\,x = P$ substituted from Equation (62). Then, one may write:

$$\int_0^s X \, g_p \, d\,x = 0,\ p = 1, 2, \ldots . n \ldots\ldots\ldots\ldots\ldots (77)$$

When the solution, $z = \Sigma\, U\, \phi$, is only approximately correct, one may assure a degree of approximation by specifying the n conditions represented by Equation (77), provided that the n functions, g_p, are well-chosen. By substituting $z = \Sigma\, U\, \phi$ in the expression for X (the left side of Equation (56) with $P_x = P$ taken from Equation (62)), and then carrying out the n integrations according to Equation (77), for $p = 1, 2 \ldots . n$, one obtains n simultaneous ordinary linear differential equations of fourth order for the n functions, U, which are functions of y only. These n ordinary differential

equations, with the one independent variable, y, replace the equation of flexure, which is a partial differential equation with the two independent variables, x and y.

The functions, g_1, g_2,....g_n, may be chosen in different ways. One possibility is to divide the distance from $x = 0$ to $x = s$ into n intervals, and to define the functions by the statement, $g_p = 1$, in interval number, p; otherwise, $g_p = 0$. The n equations (Equation (77)), then, will result in an average value of X equal to zero in each of the intervals. Another possible choice is $g_1 = 1$, $g_2 = x^2$, ... $g_p = x^{2(p-1)}$, ...; or, g_p may be a polynomial of the degree, $2(p - 1)$.

The following scheme, however, is recommended. Choose the functions, ϕ, so that they form a set of orthogonal functions $\left(\text{satisfying the conditions, } \int_0^s \phi_{(m)} \phi_{(p)}\, d\, x = 0, \text{ for all combinations of two different numbers, } m \text{ and } p\right)$. The set, of course, shall be one that lends itself to expressing the deflections at any elevation. Choose each function, g_p, proportional to the corresponding function, ϕ_p, that is, $g_p = c_p\, \phi_p$. It is advisable to determine the constant, c_p, so that $\int_0^s \phi_p\, g_p\, d\, x = 1$.

The advantage of choosing g_p proportional to ϕ_p may be observed by examining the case in which $n = 1$, and ϕ is the same as ϕ_I given in Equation (71) and shown in Fig. 1'8. In the integral, $\int_0^s X\, g\, d\, x$, for any particular elevation, all points at this elevation will receive consideration, but the emphasis will be on the middle portion, where g, ϕ, and the deflections are large. The parts near the abutments have already received consideration by choosing ϕ, so that the boundary conditions at the abutment will be satisfied. The scheme may be judged also from the point of view of the work of deformation. It is in agreement with the requirement that the work of deformation shall be as close to the correct value as possible.

The complete solution of the n ordinary linear differential equations of fourth order for the n-functions, U (functions of y), contains $4\,n$ integration constants, and is a linear function of these constants. Of the constants, $2n$ may be disposed of by considering the boundary conditions at the top, $M_y = R_y = 0$. These conditions are replaced by a requirement that M_y and R_y shall be as close to zero as possible. This requirement may be expressed by the $2n$ conditions,

$$\int_0^s M_y\, g_p\, d\, x = 0; \int_0^s R_y\, g_p\, d\, x = 0;\ p = 1, 2, \ldots\, n, \ldots\ldots \quad (78)$$

in which, M_y and R_y are interpreted as the right side of Equations (53) and (67), respectively, with the sum, $\Sigma\, U\, \phi$, substituted for z. By considering only the particular solutions which satisfy the conditions in Equations (78), there remain to be determined $2n$ integration constants. To determine them, there are available $2n$ conditions at the bottom of the dam. These

conditions may be stated in the manner of Equations (77) and (78), and they will take the form of $2n$ linear equations, which can be solved for the constants.

It may be necessary to introduce an additional intermediate boundary at any elevation at which a discontinuity occurs in s or N or their derivatives. Then new integration constants will be introduced. One set of constants will apply above the boundary, another below. In stating the corresponding additional conditions for the constants, one may use again the form of Equation (77), only replacing X by the expression for the difference in the values of z, or $\frac{\delta z}{\delta y}$, or M_y, or R_y above and below the boundary.

In order to use Equations (77) and (78), it is necessary to express in terms of $z = \Sigma\, U\, \phi$ first the derivatives of z and then M_y, R_y, and X.

The following notation is used:

U', U'', U''', U^{IV} = first, second, third, or fourth derivative, respectively, of U with respect to y;

ϕ', ϕ'', ϕ''', ϕ^{IV} = first, second, third, or fourth derivative, respectively, of ϕ with respect to x;

ϕ_m, ϕ_m', ϕ_m'' = m'th derivative of ϕ, ϕ', or ϕ'', respectively, with respect to s.

Examples of this scheme of notation are:

$$\phi'' = \frac{\delta^2 \phi}{\delta x^2};\ \phi_2 = \frac{\delta^2 \phi}{\delta s^2};\ \phi_2'' = \frac{\delta^4 \phi}{\delta x^2\, \delta s^2}$$

The derivatives of ϕ with respect to y are dependent on the derivatives of s with respect to y. Fig. 157 shows that in the case of the Stevenson Creek Dam, between $y = 0$ and $y = 60$ ft.,

$$s = 72 \text{ ft.} - y \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (79)$$

$$\frac{d\,s}{d\,y} = -1;\ \frac{d^2 s}{d\,y^2} \text{ and higher derivatives} = 0 \ldots\ldots\ldots\ldots (80)$$

The expressions in Equations (81) apply when Equations (80) are satisfied; that is, they apply to the Stevenson Creek Dam.

$$\left.\begin{aligned}
&z = \Sigma\, U\, \phi;\ \frac{\delta^2 z}{\delta x^2} = \Sigma\, U\, \phi'';\ \frac{\delta^4 z}{\delta x^2} = \Sigma\, U\, \phi^{IV} \\
&\frac{\delta z}{\delta y} = \Sigma\,(-U\,\phi_1 + U'\,\phi);\ \frac{\delta^3 z}{\delta x^2\,\delta y} = \Sigma\,(-U\,\phi_1'' + U'\,\phi'') \\
&\frac{\delta^2 z}{\delta y^2} = \Sigma\,(U\,\phi_2 - 2\,U'\,\phi_1 + U''\,\phi); \\
&\frac{\delta^4 z}{\delta x^2\,\delta y^2} = \Sigma\,(U\,\phi_2'' - 2\,U'\,\phi_1'' + U''\,\phi'') \\
&\frac{\delta^3 z}{\delta y^3} = \Sigma\,(-U\,\phi_3 + 3\,U'\,\phi_2 - 3\,U''\,\phi_1 + U'''\,\phi) \\
&\frac{\delta^4 z}{\delta y^4} = \Sigma\,(U\,\phi_4 - 4\,U'\,\phi_3 + 6\,U''\,\phi_2 - 4\,U'''\,\phi_1 + U^{IV}\,\phi)
\end{aligned}\right\} \ldots (81)$$

By substituting these values in Equation (53), and in Equation (67), with $N' = 0$, one finds the following two expressions (Equations (82) and (83)), which can be used in Equations (78):

$$-\frac{M_y}{N} = \Sigma \left[U \left(\phi_2 + \mu \, \phi'' + \frac{\mu}{r^2} \, \phi \right) - 2 \, U' \, \phi_1 + U'' \, \phi \right] \ldots\ldots (82)$$

$$-\frac{R_y}{N} = \Sigma \left[- U \left(\phi_3 + (2 - \mu) \, \phi_1'' + \frac{\mu}{r^2} \, \phi_1 \right) \right.$$
$$\left. + \, U' \left(3 \, \phi_2 + (2 - \mu) \, \phi'' + \frac{\mu}{r^2} \, \phi \right) - 3 \, U'' \, \phi_1 + U''' \, \phi \right] \ldots\ldots (83)$$

Equations (60) and (62) give,

$$\frac{P_x}{N \, r} = \frac{12 \, (1 - \mu^2) \, P}{E \, t^3 \, r} = \frac{12}{r \, (s + l) \, t^2} \left(\frac{1}{r} \, \Sigma \, U \int_o^s \phi \, d \, x - \varepsilon_T \, T \, s \right) \ldots (84)$$

The quantity, X, is the left side of Equation (56) with this expression substituted; X is to be used in Equation (77). One finds:

$$\left.\begin{aligned} X = \Sigma \Bigg[& U \left(\phi^{IV} + 2 \, \phi_2'' + \phi_4 + \frac{1}{r^2} \, \phi'' + \frac{\mu}{r^2} \, \phi_2 \right) \\ & - 4 \, U' \left(\phi_1'' + \phi_3 + \frac{\mu}{2 \, r^2} \, \phi_1 \right) \\ & + 2 \, U'' \left(\phi'' + 3 \, \phi_2 + \frac{\mu}{2 \, r^2} \, \phi \right) - 4 \, U''' \, \phi_1 + U^{IV} \, \phi \\ & - K \, U \left(\phi_1'' + \phi_3 + \frac{\mu}{r^2} \, \phi_1 \right) \\ & + K \, U' \left(\phi'' + 3 \, \phi_2 + \frac{\mu}{r^2} \, \phi \right) - 3 \, K \, U'' \, \phi_1 + K \, U''' \, \phi \\ & + k \, U \left(\phi_2 + \mu \, \phi'' + \frac{\mu}{r^2} \, \phi \right) - 2 \, k \, U' \, \phi_1 + k \, U'' \, \phi \Bigg] \\ & + \frac{12}{r \, (s + l) \, t^2} \left(\frac{1}{r} \, \Sigma \, U \int_o^s \phi \, d \, x - \varepsilon_T \, T \, s \right) \\ & - \frac{1}{N} \, (w + P_y \, r'' + w_c \, t \, r') \end{aligned}\right\} \ldots\ldots (85)$$

If the slope of the abutment were a constant, $\frac{\alpha}{1}$, instead of 1 $\left(\text{that is, } \frac{d\,s}{d\,y} = -\alpha\right)$, Equations (81), (82), (83), and (85) would be modified. The change to be made, however, is merely to multiply each derivative of the form, ϕ_m or ϕ_m'', by the factor, α^m.

9.—Suggestion Concerning Tests of Models: Proposed Supplementary Tests to Correct for a Difference in the Values of Poisson's Ratio.—For the time being, let the symbols which have been introduced refer to a celluloid model of the dam. Let the same symbols with bars over the letters refer to an imagined model, which is like the celluloid model in every respect, except that Poisson's ratio, $\bar{\mu}$, shall be equal to that of the dam. The second model would have the ideal quality that its deformations and stresses could be inter-

preted as applying to the dam, simply by multiplying them by constant ratios, which are determined easily.

Equations (51) and (55) show that,

$$\bar{N} = \frac{1-\mu^2}{1-\bar{\mu}^2} N;\ \bar{K} = K;\ \bar{k} = k \quad \ldots\ldots\ldots\ldots\ldots (86)$$

Equations (56), (60), and (62), with the terms containing w_c and T omitted, are assumed to govern the flexure of both models. If $\bar{z}$ denotes the deflections which would be found by loading the imagined model in the manner of the dam, then it will be observed that the solution,

$$z = \frac{1-\mu^2}{1-\bar{\mu}^2} z \quad \ldots\ldots\ldots\ldots\ldots (87)$$

will satisfy the equations referring to the celluloid model, except for discrepancies due to the relatively insignificant terms containing $\frac{\mu}{r^2}$, $k\,\mu$, or P_y. Ignoring these discrepancies, one may say that the solution satisfies the equation of flexure, as well as the boundary conditions at the abutments. To be produced physically, however, the solution requires at the top of the celluloid model both bending couples, M_y, and horizontal forces, R_y. The necessary values of these external couples and forces are defined by Equations (53) and (67), respectively.

If the values of these supplementary couples and forces at the top were known, it would not seem to be difficult to apply them in a test in conjunction with the liquid pressure. The benefit would be that the deformations produced by the combined load could be interpreted as proportional to those of the dam. It appears desirable, therefore, to indicate an experimental method of determining these supplementary loads.

The following new symbols are introduced:

$$\Delta\,\mu = \mu - \bar{\mu} \quad \ldots\ldots\ldots\ldots\ldots (88)$$

and,

$$Y = \frac{\delta^2 z}{\delta x^2} + \frac{z}{r^2};\ Z = -\frac{\delta^3 z}{\delta x^2\,\delta y} + \frac{1}{r^2}\frac{\delta z}{\delta y} \quad \ldots\ldots (89)$$

The quantity, Y, is numerically equal to the change of curvature in the direction of x.

By applying Equation (53) to the imagined model, and referring to Equation (87), one obtains the following statement of one of the conditions at the top:

$$\frac{\delta^2 z}{\delta y^2} + \bar{\mu}\frac{\delta^2 z}{\delta x^2} + \frac{\mu}{r^2} z = 0 \quad \ldots\ldots\ldots\ldots\ldots (90)$$

By applying Equation (53) to the actual model, and by using Equations (88), (89), and (90), this condition may be re-stated in the form,

$$M_y + N\,\Delta\,\mu\,Y = 0 \quad \ldots\ldots\ldots\ldots\ldots (91)$$

In a similar manner, by applying Equation (67), with $N' = 0$, one obtains the other condition at the top in the form.

$$R_y + N \varDelta \mu Z = 0 \qquad (92)$$

The desired load on the model may be obtained with any degree of approximation by combination of the $2n + 1$ individual loading schemes indicated in Table 32.

The following notation applies in connection with Table 32:

$m = 1, 2, \ldots n$; under practical conditions n must be a small number, perhaps 2;

$\phi_m = \phi_m(x)$ and $g_m = g_m(x)$ are functions of a type similar to that of $\phi_{(m)}$ and g_m in Section 8, except that the conditions in Equations (70) need not be satisfied; the functions, ϕ_m, shall lend themselves to expressing M_y and R_y by series of the type, $\Sigma\, c_m \phi_m$; a possible choice is $\phi_m = g_m = \cos \dfrac{(2m-1)\pi x}{2s}$.

$a_m, b_m, a'_m, b'_m =$ chosen constants; a possible arrangement, which simplifies some of the equations, yet may not be the most suitable for the testing, is $a_m = b'_m = 1$, $b_m = a'_m = 0$.

$A_m, A'_m =$ constants to be determined.

TABLE 32.—Loading Schemes for Model.

	Individual Loading Schemes.			Combined effect.
	W_0	W_m	W'_m	
(1)	(2)	(3)	(4)	(5)
Bending couple at top, M_y	0	$a_m \phi_m$	$a_m' \phi_m$	$\Sigma (A_m a_m + A_m' a_m') \phi_m$
Horizontal force at top, R_y	0	$b_m \phi_m$	$b_m' \phi_m$	$\Sigma (A_m b_m + A_m' b_m') \phi_m$
Liquid pressure	w	0	0	w
Y	Y_0	Y_m	Y_m'	$Y_0 + \Sigma (A_m Y_m + A_m' Y_m')$
Z	Z_0	Z_m	Z_m'	$Z_0 + \Sigma (A_m Y_m + A_m' Y_m')$

The load, W_m (Column (3)), consists of bending couples and horizontal forces at the top, distributed according to the law, $M_y = a_m \phi_m$, $R_y = b_m \phi_m$. The corresponding values, $Y = Y_m$ and $Z = Z_m$, defined by Equations (89), are to be measured throughout the length of the top. Similar remarks apply to Columns (2) and (4). After loading the model according to the $2n + 1$ individual schemes, each time measuring the values, Y and Z, one may express the combined effects, indicated in Column (5), as linear functions of the $2n$ unknown constants, A_m and A'_m. By applying to Equations (91) and (92) the scheme used in Equations (77) and (78), one obtains the $2n$ conditions,

$$\int_0^s (M_y + N \varDelta \mu Y)\, g_p\, d x = 0; \int_0^s (R_y + N \varDelta \mu Z)\, g_p\, d x = 0;$$

$$p = 1, 2, \ldots n \qquad (93)$$

With the expressions in Column (5) of Table 32 substituted, these conditions assume the form of $2n$ linear equations which may be solved for the constants,

A_m and A'_m. When these constants are known, the required values of M_y and R_y may be computed, and the combined load, which is to produce the desired deflections, z, may be applied in a final test.

The chief technical difficulty involved in this method, probably, is the measuring of the deformations, Y and Z, as defined by Equations (89).

C.—Numerical Computations Referring to the Stevenson Creek Dam

10.—Units, Dimensions, and Measures of Stiffness.—All computations following refer to the Stevenson Creek Dam. Unless stated specifically otherwise, distances, such as x, y, s, and t, will be measured in terms of the unit, $\lambda = 10$ ft. Deflections, z, are stated in inches. A slope, like $\frac{\delta z}{\delta y}$, accordingly, is stated in terms of the unit, in. λ^{-1}, and a change of curvature, like $-\frac{\delta^2 z}{\delta y^2}$, in terms of the unit, in. λ^{-2}.

Fig. 159 shows, in terms of the unit, λ, the positions of the same abutment lines, B and C, that were given in Fig. 157. The diagram shows also the division of the area into strips for the purpose of the numerical solution. Except close to the bottom, the height of each strip is one unit (that is, 10 ft.). The co-ordinates, $y = 3$ — and $3 +$, refer to the horizontal line, $y = 3$, indicating the points immediately above and below this line, respectively.

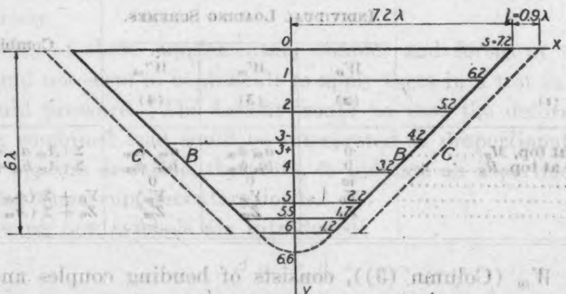


FIG. 159.—STEVENSON CREEK DAM: DIMENSIONS IN TERMS OF THE UNIT, $\lambda = 10$ FEET; DIVISION OF THE AREA INTO STRIPS.

Equation (79) assumes the form,

$$s = 7.2\,\lambda - y. \qquad \text{.......................... (94)}$$

The radius, r, is $9.9\,\lambda$. The radius of the vertical up-stream face is $10\,\lambda$.

The thickness, t, measured perpendicular to the middle surface, will be stated as follows:

$$\left.\begin{array}{ll} \text{For } 0 < y < 3: & \\ & t = t_0 = 0.2\,\lambda \\ \text{For } 3 < y < 6: & \\ & \dfrac{t}{t_0} = 1 + 0.297\,(y\,\lambda^{-1} - 3)^2 \end{array}\right\} \qquad \text{.................. (95)}$$

the latter formula applying with a satisfactory degree of approximation.

With $E = 3\,600\,000$ lb. per sq. in. and $\mu = 0.15$, one finds (see Equation (51)):

For $0 < y < 3$:

$$N = N_0 = \frac{E\,t_0^3}{12\,(1 - \mu^2)} = 294\,630\ \frac{\text{lb.}\ \lambda^2}{\text{in.}}$$

For $3 < y < 6$;

$$N = N_0 \left(\frac{t}{t_0}\right)^3 \qquad\qquad (96)$$

One finds, furthermore, according to Equations (55):

Above $y = 3$:

$$K = k = 0$$

Below $y = 3$:

$$\left.\begin{aligned} K\,s &= \frac{3.564\,(y - 3)\,s}{1 + 0.297\,(y - 3)^2} \\ k\,s^2 &= \frac{1.782\,(1 + 1.485\,(y - 3)^2)\,s^2}{(1 + 0.297\,(y - 3)^2)^2} \end{aligned}\right\} \qquad (97)$$

Table 33 contains numerical values according to these formulas.

TABLE 33.—DIMENSIONS AND MEASURES OF STIFFNESS.

y λ.	s λ.	$\frac{t}{t_0}$.	$\frac{t^3}{t_0}$.	$K\,s$.	$k\,s^2$.
3+	4.2	1	1	0	31.434
4	3.2	1.297	2.1818	3.7934	26.956
5	2.2	2.188	10.475	7.1673	12.503
5.5	1.7	2.8563	23.302	5.3031	6.4903
6	1.2	3.673	49.552	3.4931	2.7323

11.—Functions ϕ and g.—The choice of a function, ϕ, to be used in the expression, $z = U\,\phi$, has been indicated: ϕ is the same as ϕ_I, defined by Equation (71) and shown in Fig. 158. The derivatives of ϕ needed for the computations are stated in Table 34. Numerical values are given in Table 35.

The function, g, is chosen as follows:

$$g = \frac{315}{128}\,s^3\,\phi \qquad (98)$$

One finds:

$$\int_0^s x^n\,g\,d\,x = \frac{5 \times 7 \times 9\ s^{n+4}}{16\,(n + 1)\,(n + 3)\,(n + 5)} \qquad (99)$$

The integrals stated in the last column in Table 34 are computed readily by use of Equation (99).

12.—Conditions at the Top.—Let,

U_0 = value of U at the top;

U'_0 = value of U' at the top.

By applying the principle stated in Equations (78) to the expressions in Equations (82) and (83), one obtains two conditions which the function, U, must satisfy at the top. By use of the expressions in the right column in Table 34, one finds, with $\mu = 0.15$, $s = 7.2$, $\frac{\mu\, s^2}{r^2} = 0.08$:

$$U'' = 1.37\, U_0\, s^{-2} + U_0'\, s^{-1} = 0.026427\, U_0 + 0.13889\, U_0' \ldots (100)$$

and,

$$U''' = 4.315\, U_0\, s^{-3} + 8.47\, U_0'\, s^{-2} + 1.5\, U''\, s^{-1}$$
$$= 0.017066\, U_0 + 0.19232\, U_0' \ldots\ldots\ldots\ldots (101)$$

TABLE 34.—Function, ϕ, with Derivatives and Integrals.*

$f(x)$.	$\int_0^s f(x)\, g(x)\, dx$.
$\phi = 1 - 2 s^{-2} x^2 + s^{-4} x^4$	s^4
$\phi_1 = 4 (s^{-3} x^2 - s^{-5} x^4)$	$0.5\, s^3$
$\phi_2 = 4 (-3 s^{-4} x^2 + 5 s^{-6} x^4)$	$-s^2$
$\phi_3 = 8 (6 s^{-5} x^2 - 15 s^{-7} x^4)$	$1.5\, s$
$\phi_4 = 8 (-30 s^{-6} x^2 + 105 s^{-8} x^4)$	7.5
$\phi'' = 4 (-s^{-2} + 3 s^{-4} x^2)$	$-3 s^2$
$\phi_1'' = 8 (s^{-3} - 6 s^{-5} x^2)$	$1.5\, s$
$\phi_2'' = 8 (-3 s^{-4} + 30 s^{-6} x^2)$	13.5
$\phi^{IV} = 24 s^{-4}$	31.5

$$* \phi_m = \frac{\delta m\, \phi}{\delta\, s^m};\ \phi_m'' = \frac{\delta m + 2}{\delta\, x^2\, \delta\, s^m}.$$

13.—The Differential Equation for U.—By applying the principle stated in Equation (77) to the expression in Equation (85), one obtains the ordinary differential equation of fourth order, the equation for U, which replaces the equation of flexure. This equation assumes the form,

$$A\, U + A_1\, U' + A_2\, U'' + A_3\, U''' + A_4\, U^{IV} + C = 0 \ldots (102)$$

TABLE 35.—Values of ϕ and Its Derivatives.

$\frac{x}{s}$.	ϕ.	$\phi_1\, s$.	$\phi_2\, s^2$.	$\phi''\, s^2$.	$\phi_3\, s^3$.	$\phi_1''\, s^3$.	$(\phi_1'' + \phi_3)\, s^3$	$(\phi^{IV} + 2\phi_2'' + \phi_4)\, s^4$
0	1.0000	0	0	−4.00	0	8.00	8.000	−24.000
0.1	0.9801	0.0396	−0.118	−3.88	0.468	7.52	7.988	−21.516
0.2	0.9216	0.1536	−0.448	−3.52	1.728	6.08	7.808	−13.056
0.3	0.8281	0.3276	−0.918	−2.92	3.348	3.68	7.028	4.404
0.4	0.7056	0.5376	−1.408	−2.08	4.608	0.32	4.928	35.904
0.5	0.5625	0.7500	−1.750	−1.00	4.500	− 4.00	0.500	88.500
0.6	0.4096	0.9216	−1.728	0.32	1.728	− 9.28	− 7.552	171.264
0.7	0.2601	0.9996	−1.078	1.88	− 5.292	−15.52	− 20.812	295.284
0.8	0.1296	0.9216	0.512	3.68	−18.432	−22.72	− 41.152	473.664
0.9	0.0361	0.6156	3.402	5.72	−39.852	−30.88	− 70.732	721.524
1.0	0	0	8.000	8.00	−72.000	−40.00	−112.00	1056.00

The coefficients, A, A_1, etc., are computed by use of the integrals in Table 34. The formulas,

$$\int_0^s \phi\, d\, x = \frac{8}{15}\, s;\ \int_0^s g\, d\, x = \frac{21}{16} = s_4 \ldots\ldots\ldots\ldots (103)$$

are also used. By dividing each coefficient, as obtained directly from Equation (77), by s^2, one finds the following values:

$$
\left.
\begin{aligned}
A &= \frac{8.4\, s^3}{r^2\,(s+l)\,t^2} + \left(66 - 3.15\,\frac{s^2}{r^2} - \left(3 + 0.075\,\frac{s^2}{r^2}\right) K\, s \right. \\
&\qquad \left. - \left(1.45 - 0.15\,\frac{s^2}{r^2}\right) k\, s^2 \right) s^{-2} \\
A_1 &= \left(-\,12 - 0.15\,\frac{s^2}{r^2} - \left(6 - 0.15\,\frac{s^2}{r^2}\right) K\, s - k\, s^2\right) s^{-1} \\
A_2 &= -\,12 + 0.15\,\frac{s^2}{r^2} - 1.5\, K\, s + k\, s^2 \\
A_3 &= (-\,2 + K\, s)\, s \\
A_4 &= s^2 \\
C &= -\,\frac{21\, s^2}{16\, N}\,(w + P_y\, r'' + w_c\, t\, r') - \frac{63\, s^3\, \varepsilon_T\, T}{4\, r\,(s+l)\,t^2}
\end{aligned}
\right\} \ldots (104)
$$

The present problem is to determine the influence of the water pressures in the case of a full reservoir. Accordingly, in the expression for C the terms containing w_c, T, and P_y will be ignored.

The water pressure per unit of area of the cylinder with radius 99 ft. (upon which dx is measured), becomes,

$$w = 624\,\frac{\text{lb.}}{\text{ft.}^2} \times \frac{100\ \text{ft.}}{99\ \text{ft.}} \times \frac{y}{\lambda} = 630.30\,\frac{y}{\lambda}\,\frac{\text{lb.}}{\text{ft.}^2} \ldots\ldots\ldots\ldots (105)$$

Thus, one finds, in the case of a full reservoir,

$$C = 0.280783\,\frac{y}{\lambda}\left(\frac{s}{\lambda}\right)^2\left(\frac{t}{t_0}\right)^{-3}\ \text{in.}\ \lambda^{-2} \ldots\ldots\ldots\ldots\ldots (106)$$

Table 36 contains numerical values according to these formulas.

TABLE 36.—COEFFICIENTS IN DIFFERENTIAL EQUATION FOR U (EQUATION (102)).

y λ.	A λ^{-2}.	A_1 λ^{-1}.	A_2.	A_3 λ.	A_4 λ^2.	C in. λ^{-2}.
0	99.974	− 1.6776	−11.931	−14.4	51.84	0
1	73.609	− 1.9450	−11.941	−12.4	38.44	−10.7933
2	51.799	− 2.3156	−11.959	−10.4	27.04	−15.185
3−	34.834	− 2.8636	−11.973	− 8.4	17.64	−14.859
3+	32.299	−10.3479	19.461	− 8.4	17.64	−14.859
4	10.2344	−28.623	1.782	21.739	10.24	− 5.271
5	6.9668	−30.664	−10.241	11.368	4.84	− 0.64868
5.5	14.546	−29.582	−13.460	5.6153	2.89	− 0.19153
6	35.905	−29.737	−14.505	1.7917	1.44	− 0.048959

14.—Numerical Integration.—Let h be the height of a narrow horizontal strip, like any of those shown in Fig. 159. The values, U, U', $\ldots U^{IV}$, refer to the depth, y. Let the same symbols with the index, $y - h$, denote the corresponding values at the depth, $y - h$. The latter values are assumed to be known. The problem is to determine the values at the depth, y. Assume that the interval from $y - h$ to y is small enough to make it possible to

express U within this interval, with the desired degree of accuracy, as a polynomial of fifth degree in y. The constant fifth derivative of this polynomial will be denoted by u^V.

Let the quantities, $u, u', \ldots u^{IV}$, be defined as follows:

$$\left.\begin{aligned} u &= U_{y-h} + h\,U'_{y-h} + \frac{h^2}{2}\,U''_{y-h} + \frac{h^3}{6}\,U'''_{y-h} + \frac{h^4}{24}\,U^{IV}_{y-h} \\ u' &= U'_{y-h} + h\,U''_{y-h} + \frac{h^2}{2}\,U'''_{y-h} + \frac{h^3}{6}\,U^{IV}_{y-h} \\ u'' &= U''_{y-h} + h\,U'''_{y-h} + \frac{h^2}{2}\,U^{IV}_{y-h} \\ u''' &= U'''_{y-h} + h\,U^{IV}_{y-h} \\ u^{IV} &= U^{IV}_{y-h} \end{aligned}\right\} \ldots (107)$$

Then, one finds, by Taylor's formula,

$$\left.\begin{aligned} &U = u + \frac{h^5}{120}\,u^V;\; U' = u' + \frac{h^4}{24}\,u^V;\; U'' = u'' + \frac{h^3}{6}\,u^V; \\ &U''' = u''' + \frac{h^2}{2}\,u^V;\; U^{IV} = u^{IV} + h\,u^V \end{aligned}\right\} \ldots (108)$$

Denote:

$$A_5 = -\,h\,A_4 - \frac{h^2}{2}\,A_3 - \frac{h^3}{6}\,A_2 - \frac{h^4}{24}\,A_1 - \frac{h^5}{120}\,A \ldots\ldots (109)$$

By substituting the values of $U, U', \ldots, U^{IV}$. (Equation (108)), in Equation (102) one finds,

$$A_5\,u^V = A\,u + A_1\,u' + A_2\,u'' + A_3\,u''' + A_4\,u^{IV} + C \ldots\ldots (110)$$

After dividing by A_5, Equation (110) assumes the form:

$$u^V = a\,u + a_1\,u' + a_2\,u'' + a_3\,u''' + a_4\,u^{IV} + c \ldots\ldots\ldots (111)$$

Table 37 contains numerical values of the coefficients in Equation (111). Since U^{IV} changes suddenly at the depth, $y = 3$, Equation (111) will be applied at the depth, $y = 3\,-$, but not at the depth, $y = 3\,+$. The values stated in Table 37 for $y = 0$ and $y = 3\,+$ are obtained by dividing the corresponding coefficients in Table 36 by $-\,A_4$. The equation thus obtained,

$$a\,U + a_1\,U' + a_2\,U'' + a_3\,U''' - U^{IV} + c = 0 \ldots\ldots\ldots (112)$$

gives a simple computation of U^{IV} at these particular elevations when U, U', U'', and U''' are known.

TABLE 37.—COEFFICIENTS IN EQUATIONS FOR u^{IV} AND u^V.*

y λ.	a λ^{-5}.	a_1 λ^{-4}.	a_2 λ^{-3}.	a_3 λ^{-2}.	a_4 λ^{-1}.	c in. λ^{-5}.
0	− 1.9285	0.032361	0.23015	0.27778	−1	0
1	− 2.8913	0.063186	0.38792	0.40283	−1.2488	0.35063
2	− 2.5666	0.11474	0.59256	0.51531	−1.3398	0.75240
3−	− 2.9989	0.24653	1.03078	0.72317	−1.5187	1.2792
3+	− 1.8310	0.58662	−1.1032	0.47619	−1	0.84235
4	− 0.50418	1.4101	−0.087787	−1.07093	−0.50445	0.25968
5	− 0.91698	4.0360	1.3479	−1.4963	−0.63705	0.085380
5.5	− 8.1116	16.496	7.5060	−3.1814	−1.6116	0.106807
6	−62.587	51.835	25.284	−3.1232	−2.5101	0.085342

*Equation (112) for $y = 0$ and $3\,+$, otherwise Equation (111).

The solution is stated in the form,

$$U = c_1\,U_I + c_2\,U_{II} + U_{III} \quad \ldots\ldots\ldots\ldots\ldots\ldots (113)$$

in which, c_1 and c_2 are constants, and U_I, U_{II}, and U_{III} are particular solutions of Equation (102), each satisfying the conditions at the top, as stated in Equations (100) and (101). U_I is the particular function, U, defined by the further requirements, $U_0 = 1$, $U'_0 = 0$, $C = c = 0$. U_{II} is the particular function, U, defined by the requirements, $U_0 = 0$, $U_0' = 1$, $C = c = 0$. U_{III} is the particular integral defined by the conditions, $U_0 = U'_0 = 0$, and by the values of C and c which are stated in Equation (106) and the tables. The three particular solutions do not satisfy the conditions at the bottom. Only the combined solution in Equation (113) satisfies all the conditions. It may be observed that c_1 and c_2 are the resultant values of U_0 and U'_0, respectively.

Each of the three particular solutions is obtained by the following process: For $y = 0$ express U'' and U''' by Equations (100) and (101), and compute U^{IV} by Equation (112), using the coefficients stated in Table 37.

Compute the quantities, $u, u', \ldots$, for $y = 1$, $h = 1$, according to Equations (107). Determine u^V for the interval, $y = 0$ to $y = 1$, by Equation (111). Compute $U, U', \ldots U^{IV}$, for $y = 1$ by Equations (108). The process is systematized by arranging the terms as shown in Table 38.

TABLE 38.—SAMPLE OF COMPUTATIONS OF THE FUNCTION, U.
(Determination of Values for $y = 1$ from Values for $y = 0$ in the Particular Solution Defined by $U_0 = 1\,000$, $U'_0 = w = 0$.)

y	u a $a\,u$ $\frac{1}{120}\,u^V$ U w	u' a_1 $a_1\,u'$ $\frac{1}{24}\,u^V$ U'	u'' a_2 $a_2\,u''$ $\frac{1}{6}\,u^V$ U'' $\frac{1}{2}\,U''$	u''' a_3 $a_3\,u'''$ $\frac{1}{2}\,u^V$ U''' $\frac{1}{2}\,U'''$ $\frac{1}{6}\,U'''$	u^{IV} a_4 $a_4\,u^{IV}$ u^V U^{IV} $\frac{1}{2}\,U^{IV}$ $\frac{1}{6}\,U^{IV}$ $\frac{1}{24}\,U^{IV}$
0	.... 1 000 0	 0	.... 26.427 13.214	.... 17.066 8.533 2.844	.... — 1 917.7 — 958.85 — 319.62 — 79.904
1	936.15 — 2.3913 —2 238.6 — 8.1875 927.96 0	— 284.66 0.063186 — 17.987 — 40.94 — 325.60	— 915.36 0.38792 — 355.09 — 163.75 —1 079.11	— 1 900.6 0.40283 — 755.62 — 491.3 — 2 391.9	— 1 917.7 — 1.2488 2 394.8 — 982.5 — 2 900.2 ....

Proceed according to the same pattern from $y = 1$ to $y = 2$, and then from $y = 2$ to $y = 3$—.

In passing from $y = 3-$ to $y = 3+$, the quantities, U, U', U'', and U''', remain unchanged, but U^{IV} changes suddenly. Compute the value of U^{IV} for $y = 3 +$ by Equation (112).

Proceed according to the pattern in Table 38 from $y = 3+$ to $y = 4$.

The next two strips in Fig. 159 have the height $h = \frac{1}{2}$. Table 39 shows the corresponding pattern of computations. Proceed accordingly from $y = 5$ to $y = 5.5$, and then from $y = 5.5$ to $y = 6$.

The three solutions obtained in this manner are stated numerically in Table 40, in the order, U_I, U_{II}, U_{III}.

TABLE 39.—SAMPLE OF COMPUTATIONS OF THE FUNCTION, U.

(Determination of Values for $y = 5.5$ from Values for $y = 5$ in the Particular Solution Defined by $U_0 = V_0 = 0$. Reservoir Full.)

y	u a $a\,u$ c $\frac{1}{3\,840}\,u^{V}$ U	u' a_1 $a_1\,u'$ $\frac{1}{384}\,u^{V}$ U' $\frac{1}{2}\,U'$	u'' a_2 $a_2\,u''$ $\frac{1}{48}\,u^{V}$ U'' $\frac{1}{2}\,U''$ $\frac{1}{8}\,U''$	u''' a_3 $a_3\,u'''$ $\frac{1}{8}\,u^{V}$ U''' $\frac{1}{2}\,U'''$ $\frac{1}{8}\,U'''$ $\frac{1}{48}\,U'''$	u^{IV} a_4 $a_4\,u^{IV}$ u^{V} $\frac{1}{2}\,u^{V}$ U^{IV} $\frac{1}{2}\,U^{IV}$ $\frac{1}{8}\,U^{IV}$ $\frac{1}{48}\,U^{IV}$ $\frac{1}{384}\,U^{IV}$
	....	....	....	....	....
5	10.9753	12.115 6.0575	14.855 7.1775 1.7944	22.947 11.474 2.8684 0.47806	37.569 18.785 4.696 0.7827 0.09784
     5.5	19.403 — 8.1116 —157.39 0.106807 0.067 19.470	22.944 16.496 378.48 0.675 23.619	30.525 7.5060 229.12 5.398 35.923	41.782 — 3.1314 —130.68 32.886 74.118	37.569 — 1.6116 — 60.546 259.09 129.545 167.11 ....

On account of the many numbers involved, the computations were carried out with five significant figures in each number which begins with figures between 11 and 99, and with six significant figures in each number which begins with figures between 100 and 109.

The polynomials expressing U for the different strips join each other at the edges of the strips without discontinuities in U, U', U'', and U'''. U^{IV} is dis-

continuous only at the depth, $y = 3$. The diagrams for U^{IV} have the appearance of string polygons, except for the discontinuity at $y = 3$.

The polynomials satisfy the differential equation for U at the edges of the strips. The degree of accuracy is the greater, the narrower the strips, and the smoother the curves which represent the functions. In view of the latter condition, the combined expression for U in Equation (113) can be expected to have a higher degree of accuracy than the individual solutions, U_I, U_{II}, and U_{III}.

TABLE 40.—THREE SOLUTIONS OF THE DIFFERENTIAL EQUATION FOR U.

y λ.	U in.	U' in. λ^{-1}.	U'' in. λ^{-2}.	U''' in. λ^{-3}.	U^{IV} in. λ^{-4}.
	VALUES FOR $U_0 = 1$ Inch; $U'_0 = 0$; $w = 0$.				
0	1.0000	0	0.026427	0.017066	— 1.9177
1	0.92796	— 0.32560	— 1.07911	— 2.3919	— 2.9002
2	— 0.46452	— 3.1232	— 5.0776	— 5.7617	— 3.8394
3—	— 7.2087	— 11.531	— 11.996	— 7.3121	0.7385
3+	— 7.2087	— 11.531	— 11.996	— 7.3121	16.187
4	— 5.603	— 26.088	— 17.626	— 10.3571	— 22.227
5	— 64.359	— 58.610	— 63.140	—104.689	—166.39
5.5	—104.453	—109.550	—158.82	—323.14	—707.44
6	—189.57	—263.24	—562.03		
	VALUES FOR $U_0 = 0$; $U'_0 = 1$ Inch λ^{-1}; $w = 0$.				
0	0	1.0000	0.13889	0.19232	0.11775
1	1.0861	1.1531	— 0.01613	— 0.90858	— 2.31955
2	1.9427	0.09415	— 2.8923	— 5.6514	— 7.1662
3—	— 0.73833	— 7.2607	— 13.897	— 18.127	— 17.785
3+	— 0.73833	— 7.2607	— 13.897	— 18.127	3.7917
4	— 17.950	— 30.283	— 32.905	— 22.665	— 12.867
5	— 71.236	— 87.848	— 106.731	—169.71	— 281.23
5.5	—133.28	—173.44	— 267.91	—557.33	—1269.2
6	—272.33	—442.68	—1018.56		
	VALUES FOR $U_0 = U'_0 = 0$; FULL RESERVOIR.				
0	0	0	0	0	0
1	0.0029219	0.014610	0.058438	0.17532	0.35063
2	0.095486	0.24367	0.50719	0.82031	0.93935
3—	0.77693	1.3592	1.9635	2.2585	1.9369
3+	0.77693	1.3592	1.9635	2.2585	—0.87354
4	3.4847	4.4403	4.3209	2.9920	2.3405
5	10.9753	12.115	14.355	22.947	37.569
5.5	19.470	23.619	35.923	74.118	167.11
6	38.249	59.314	133.765		

15.—Conditions at the Bottom.—The true abutment is at the depth, $y = 6$, whereas the imagined abutment, B, is at the depth, $y = 6.6$. It is in keeping with the idea of the imagined abutment to state the conditions at the bottom in the following manner, with the indices representing the values of y:

$$U_6 = 0,\ U_{6\cdot 6}' = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (114)$$

Moreover, it is in keeping with this idea to express $U_{6\cdot 6}'$ in terms of a constant value of U'', imagined to exist between $y = 6$ and $y = 6.6$; that is,

$$U_{6\cdot 6}' = U_6' + 0.6\ U_6'' \ldots\ldots\ldots\ldots\ldots\ldots\ldots (115)$$

The three solutions in Table 40 give the following values of this quantity: $U'_6 = -600.46, -1053.82$, and 139.57, respectively. By referring to Equation (113), with $c_1 = U_0$ and $c_2 = U_0'$, one finds the following two equations, representing the conditions at the bottom:

$$-189.57\,U_0 - 272.33\,U_0' + 38.249 = 0 \qquad (116)$$

$$-600.46\,U_0 - 1053.82\,U_0' + 139.57 = 0 \qquad (117)$$

Equations (116) and (117) give:

$$U_0 = c_1 = 0.063407;\; U_0' = c_2 = 0.096313 \qquad (118)$$

16.—The Resultant Function, U.—The combined solution may now be stated in the form,

$$U = 0.063407\,U_I + 0.096313\,U_{II} + U_{III} \qquad (119)$$

In computing the values of U and its derivatives according to Equation (119), the results applying to the lower part of the dam appear as fairly small differences between much larger numbers. The condition of "small differences" does not exist here in an objectionable form, yet it was considered desirable, partly for the purpose of checking the computations, to compute an additional particular integral, U_{IV}, using the same method by which the other particular solutions were obtained. This particular integral is defined by the conditions stated in the title of Table 41. The results are given in this table. They should be the same as those defined by Equation (119), but on account of the natural limitation in the accuracy, resulting from the use of only five significant figures in most of the numbers, small errors accumulate, and discrepancies may be noticed in the results referring to the lower part of the dam.

TABLE 41.—SOLUTION FOR $U_0 = 0.063407$ INCH; $U_0' = 0.096313$ INCH λ^{-1}. FULL RESERVOIR.

y λ.	$10^3\,U$ in.	$10^3\,U'$ in. λ^{-1}.	$10^3\,U''$ in. λ^{-2}.	$10^3\,U'''$ in. λ^{-3}.	$10^3\,U^{IV}$ in. λ^{-4}.
0	63.407	96.313	15.053	19.605	−110.253
1	166.37	105.026	− 11.537	− 63.853	− 56.663
2	253.14	54.716	− 93.329	− 89.340	5.697
3− 3+	248.75	− 71.289	−135.64	48.887	270.76 517.99
4	132.46	−130.72	33.97	152.04	−311.69
5	33.092	− 62.609	70.435	− 38.82	− 70.04
5.5	9.6353	− 33.353	45.069	− 57.046	− 2.865
6	−2.6265	− 18.233	14.395		

The results defined by Table 40 and Equation (119) are more nearly correct than those stated in Table 41. The particular integral, U_{IV}, however, may be used in the same manner as was previously U_{III}. Thus, one may write,

$$U = c_1\,U_I + c_2\,U_{II} + U_{IV} \qquad (120)$$

The constants, c_1 and c_2, represent the additions to be made to the values of U_0 and U'_0 in Table 41. By taking into account the conditions at the bottom in the same manner as before (see Equations (114) to (118), inclusive), one

finds the new values of c_1 and c_2, which, substituted in Equation (120), define the resultant solution,

$$10^3\ U = -\ 0.0042643\ U_I - 0.0066761\ U_{II} + 10^3\ U_{IV} \ldots\ldots\ldots (121)$$

Table 42 contains the resultant values computed according to Equation (121). The results agree closely with those obtained from Equation (119). Fig. 160 shows the function, U, obtained in this manner.

TABLE 42.—RESULTANT VALUES OF U AND ITS DERIVATIVES.

y. λ	$10^3\ U$ in.	$10^3\ U'$ in. λ^{-1}.	$10^3\ U''$ in. λ^{-2}.	$10^3\ U'''$ in. λ^{-3}.	$10^3\ U^{IV}$ in. λ^{-4}.
0	63.403	96.306	15.052	19.604	− 110.25
1	166.36	105.020	− 11.532	− 63.837	− 56.635
2	253.13	54.729	− 93.288	− 89.278	5.761
3− / 3+	248.79	− 71.191	−135.50	49.039	270.88 / 517.90
4	132.69	−130.41	34.27	152.24	− 311.51
5	33.842	− 61.773	71.417	− 37.24	− 67.452
5.5	10.081	− 31.728	47.535	− 51.947	8.625
6	0	− 14.155	23.592		

A comment may be made here concerning the influence of the conditions at the bottom. If these conditions are changed so that instead of the values in the last line in Table 42 the noticeably different values in Table 41 are produced, the corresponding changes at the top, as indicated by the differences between the values in the first lines in the two tables, will be negligible. The conclusion is indicated that the conditions at the bottom, in general, have no great influence upon the upper part of the dam. It is a simple matter to compute new values of the constants, c_1 and c_2, in Equation (120) corresponding to conditions other than those of Equations (114). Computations of this sort show, for example, that by moving the fixed abutment from the position, $y = 6.6$, to the position, $y = 6$, the top of the dam will be affected only slightly. The influence in the lower part of the dam, on the other hand, will be considerable.

17.—Supplementary Loads.—The numerical solution which has been obtained, calls for additional computations.

The dam will deflect according to the law, $z = U\ \phi$, under the influence of certain loads. The loads which must be added to these in order to produce the true loads are called the supplementary loads.

Equations (82) and (83) define the values of the couples, M_y, and the forces, R_y, which must be applied at the top in order to produce the deflections, $z = U\ \phi$. The true values of the couples and forces at the top are zero. Thus, the supplementary couples and forces may be expressed as,

$$M' = -\ M_y;\ R' = -\ R_y \ldots\ldots\ldots\ldots\ldots\ldots (122)$$

By substituting the values given in the first line in Table 42, one finds:

$$10^3\ \frac{M'}{N} = 15.149\ \phi - 26.752\ \phi_1\ s + 1.2231\ \phi_2\ s^2 + 0.18346\ \phi''\ s^2 \ldots (123)$$

$$10^3\ \frac{R'}{N} = 19.751\ \phi - 6.2851\ \phi_1\ s + 5.5733\ \phi_2\ s^2$$
$$+\ 3.4368\ \phi''\ s^2 - 0.16987\ \phi_3\ s^3 - 0.31426\ \phi_1''\ s^3 \ldots\ldots\ldots (124)$$

Let w'' = supplementary distributed load, per unit of area;

P'' = corresponding supplementary arch thrust (that is, the amount to be added to P);

$$w' = w'' - \frac{P''}{r}.$$

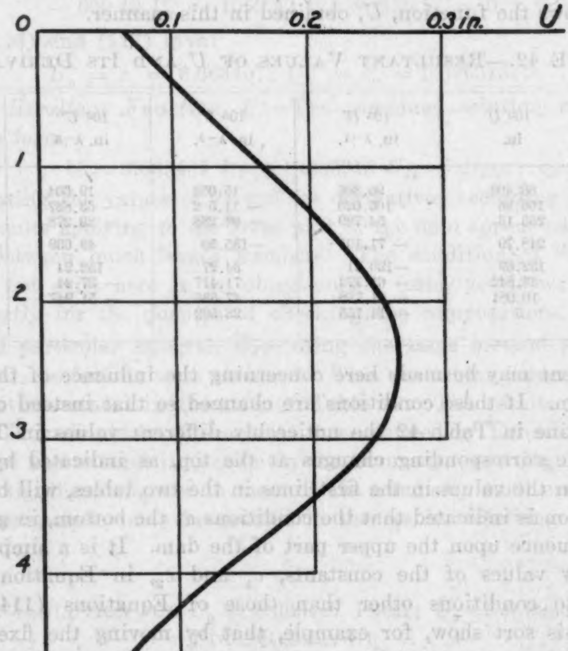


FIG. 160.—THE FUNCTION, U.

Equation (85) gives, with the numerical values substituted from the first line in Table 42, for $y = 0$:

$$10^3 \frac{w'}{N} = 110.23\,\phi + 10.9321\,\phi_1\, s - 1.7440\,\phi_2\, s^2 - 0.59319\,\phi''\, s^2$$
$$+ 1.03208\,(\phi_1'' + \phi_3)\, s^3 - 0.023593\,(\phi^{IV} + 2\,\phi_2'' + \phi_4)\, s^4 - 92.004 \quad ..(125)$$

Table 43 and Fig. 161 show the values computed from Equations (123), (124), and (125), by use of Table 35.

The equation of flexure does not give information concerning the variation of P''. There is reason to assume that P'' at the top is negative in the middle portion and positive near the abutments. In this case the total variation of w'' will be less than that of w'.

TABLE 43.—SUPPLEMENTARY LOADS AT THE TOP OF THE DAM.

	Couples.	Forces.	Distributed Load.
$\frac{x}{s}$	$10^3 \frac{M'}{N}$ in. λ^{-2}	$10^3 \frac{R'}{N}$ in. λ^{-3}	$10^3 \frac{w'}{N}$ in. λ^{-4}
0.	14.415	3.490	29.422
0.1	12.933	2.674	27.724
0.2	8.658	0.439	22.499
0.3	2.123	− 2.580	13.341
0.4	− 5.797	− 5.322	− 0.421
0.5	−13.867	− 6.301	− 19.728
0.6	−20.505	− 3.610	− 45.790
0.7	−23.774	5.084	− 80.087
0.8	−21.390	22.539	−124.37
0.9	−10.712	51.936	−180.65
1.0	11.253	96.881	−251.21

The quantity, w', representing the combined influence of of w'' and P'', is in a sense a supplementary distributed load, and is so designated in Table 43.

18.—Up-Stream Deflections.—The supplementary couples, M', tend to decrease the deflections at the top in the central portion and increase the deflections closer to the abutments. These effects, however, can be shown to be only small, and they may be ignored in the present discussion. The supplementary forces, R', show a tendency to produce up-stream (negative) deflections of a part of the top, but these effects, too, are relatively unimportant, and will be ignored.

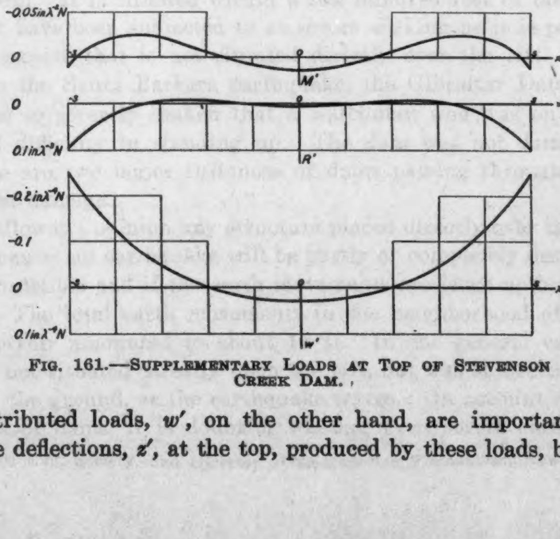


FIG. 161.—SUPPLEMENTARY LOADS AT TOP OF STEVENSON CREEK DAM.

The distributed loads, w', on the other hand, are important. One may estimate the deflections, z', at the top, produced by these loads, by considering

an arch with the same dimensions as those of the top of the dam. Since the influence lines for the thrusts in this arch have nearly the shape of the diagram representing ϕ, and since $\int_0^s w' \phi \, d x = 0$, the arch thrusts produced by the load, w', will be unimportant. The arch, therefore, will deflect approximately as a straight fixed beam, according to the law.

$$\frac{d^4 z'}{d x^4} = \frac{w'}{N} \qquad \qquad (126)$$

This equation may be interpreted as a special, greatly simplified form of Equation (56).

The approximate values of z' given in Fig. 162 were obtained by replacing the four successive integrations, called for by Equation (126), by four summations, with $d x$ represented by 0.1 s. The diagram shows also the values of $U \phi$ and $U \phi + z'$.

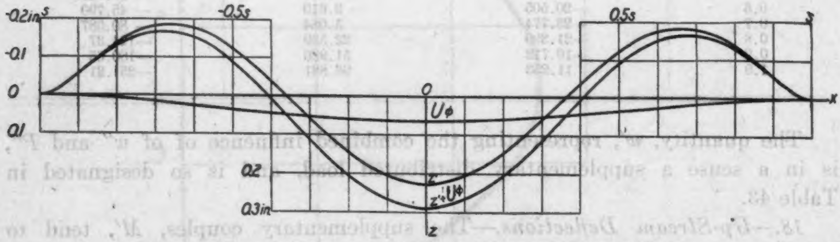


FIG. 162.—DEFLECTIONS AT TOP OF STEVENSON CREEK DAM.

Since the top of the dam is not an independent arch, the up-stream deflections, thus obtained, are exaggerated. With this nature of the results kept in mind, they may be compared with the results of the tests. The conclusion is indicated that the general structural behavior of the Stevenson Creek Dam can be accounted for in terms of the theory of flexure of an elastic shell.

PART IX.—EARTHQUAKES, ICE PRESSURE, AND DETERIORATION OF CONCRETE

By Alfred D. Flinn,* M. Am. Soc. C. E.

1.—Effects of Earthquakes

Experimental determination of the effects of earthquakes upon arch dams has not appeared feasible. Since, however, this question is important, the Committee gives herein some records of experience, with comments. It is fortunate that the American Society of Civil Engineers has a Committee on Effects of Earthquakes on Engineering Structures, which has been at work since 1923. In a communication dated May 13, 1927, J. D. Galloway, M. Am. Soc. C. E., Chairman of that Committee, gives the following records and opinion.

In the reports on the effects of earthquakes on engineering structures which the Special Committee has collected, there are only two instances of destruction of dams, and both of these were earth dams. In the 1906 earthquake in California, the earthquake rift passed directly through a small earth dam of the Spring Valley Water Company. This was an old dam at the Crystal Springs Reservoir site that had been submerged by the building of the larger masonry dam. There was water on both sides of it and the earthquake rift ruptured the dam, but no exterior damage resulted, on account of its situation. In the Santa Barbara earthquake of 1925 a small dam was destroyed and some damage resulted from the water washing down the canyon. There was considerable criticism of the construction methods of this dam.

The positive evidence as to the behavior of masonry dams is that they are not injured in any earthquake of the intensity of the 1906 California shock. The Crystal Springs Dam is built of concrete masonry of gravity section, but of an arch form. It is situated within a few hundred feet of the earthquake rift and must have been subjected to as severe shaking as it is possible for a structure to receive that is not situated directly over the rift. It was not damaged. In the Santa Barbara earthquake, the Gibraltar Dam, an arched structure, was so severely shaken that a watchman who was on the dam at the time had difficulty in standing up. The dam was not damaged by the shake. These are two major instances of dams passing through earthquake shocks without damage.

In Mr. Galloway's opinion any structure placed directly over the rift in the ground that causes an earthquake will be partly or completely destroyed. The forces are irresistible and if the earth movements are large nothing can withstand them. The total earth movements in the neighborhood of the Crystal Springs Reservoir amounted to about 15 ft. In the general case, however, the dam was not situated directly upon the rift, but was subjected only to the vibrations of the ground, or the earthquake waves. On account of the nature and positions of dams, it is doubtful whether even severe earthquakes will

* Secy., United Eng. Society, and Director of Engineering Foundation, New York, N. Y.

damage them to any great extent. In some earthquakes there is a tendency for the earth sides of canyons to move toward each other, but under the usual condition a masonry dam would be founded upon bed-rock with bed-rock abutments. No movements of bed-rock in canyons has ever been observed other than the slight movements when subjected to earthquake waves. It is the general opinion that material 20 ft. or more below the surface of the ground does not move very much. It is also his opinion from a study of the subject that a properly built masonry arch, subjected to earthquakes of the intensity of that of California in 1906, or greater, will not be materially injured.

The Earthquake Committee is collecting all the information available on the effect of earthquakes on dams, and will present these data in its report, to be published by the American Society of Civil Engineers.

Secretary Noetzli, of the Committee on Arch Dam Investigation, contributes the following note: The Corfino Arch Dam in Tuscany, Italy, was subjected to a violent earthquake which was of such intensity that most of the buildings in surrounding towns were destroyed. The dam itself did not develop any cracks, nor did it suffer any other damage. The structure is about 130 ft. high. It has a vertical water face curved on a constant radius of 77 ft. The thickness at the crown is 5 ft. and at the base, 23 ft. The dam was built in 72 days.*

2.—Ice Pressure

In situations where a strong, thick sheet of ice can form on a reservoir and where there may be large changes of temperature below the freezing point, the ice may exert great pressure upon a dam. Although the combination of conditions producing maximum pressure may occur with great infrequency, provision should be made in some way for the worst probability, unless the dam is so situated that its failure could not cause a disaster. For thin dams especially careful consideration is necessary.

If the water level in the reservoir at times when strong ice can form will always be a number of feet below full reservoir level, the possibility of damage is greatly reduced.

Ice has done serious injury to dams in a number of instances. The maximum thrust is to be expected where a thick, sound sheet of strong ice has some abutment against which to re-act when its temperature rises rapidly from a point far below freezing, for example, an island in the reservoir, or an inward bend in its shore, not far away, with steep slopes at ice level. Ice has about eight times as great capability for thermal expansion as steel. Its coefficient of expansion is given as 0.0000640 per degree centigrade, or 0.0000356 per degree Fahrenheit. Pressure of an ice field may be increased by wind friction. Ice has a modulus of elasticity reported to range from 180 000 to 360 000 lb. per sq. in. It may expand 0.00025 to 0.00066 ft. per lin. ft. of reach. The force to be considered may be its ultimate crushing strength, which varies widely and is given by authorities as from 12 to 25 tons per sq. ft. Ice pressure may be ignored where the mean January temperature is above 40° Fahr.

* An account of the remarkable effect on the dam was given in *Schweizerische Bauzeitung*, December 27, 1924, and in "Die Staumauern," by Dr. N. Kelen, 1926, p. 184.

Allowance for it has sometimes been made indirectly, but for a number of carefully designed dams definite values ranging from 12 to 25 tons per lin. ft. of dam were used in computations of stability.

Wherever serious ice pressure may occur, either the possibility of a thrust against the proposed dam must be eliminated or else suitable provision should be made in the design of the dam.*

3.—Deterioration of Concrete

Deterioration is insidious and often does not begin to manifest itself for months or years after completion of concrete structures. Its causes and probabilities escape the kind of construction inspection which has been prevalent. Precautions are especially necessary in hydraulic structures exposed to the weather in places having cold winters, where frost action may hasten the damage originating from other causes. Careful selection of cement, aggregates, and water, faithfulness and skill in mixing and placing the concrete in water-tight forms, and intelligent care during the period of hydration, all are essential. Very dense concrete made of good materials is remarkably resistant; but density alone is not sufficient, although dense concrete of any given materials will resist deterioration longer than permeable concrete of the same materials. Some studies in America and other countries indicate that for durability of concrete to be exposed to water under pressure, or to sea water, or to alkali in soils, it is most important to select a suitable cement. Cements which may be acceptable for structures not so exposed may quickly deteriorate under the conditions named. In other words, given honest and intelligent craftsmanship, deterioration of concrete originates in chemical reactions resulting from use of unsuitable cement or other ingredients. In addition to density, it appears to be necessary also to utilize all practicable means to prevent or minimize the entrance of water into the "pores" of the concrete.

There may be conditions so severe that concrete made of the best materials economically available at the site and with the measure of intelligence in supervision which can be commanded, should not be used for thin dams. Another type of dam may be necessary.

* For technical information on ice see "Ice Engineering," by Prof. Howard T. Barnes, McGill Univ., Renouf Pub. Co., Montreal, Que., Canada.

En
So
vic
Th
eff

en
su
Ta
to
re
Mc

of
O.
ut
of
at
st

Di
ur
te
on
in

Cr

av
th

we
M
L.
Fo
A
Sp
in
ta

To

ACKNOWLEDGMENTS, CONTRIBUTORS, AND PERSONNEL

1.—ACKNOWLEDGMENTS

The Arch Dam Investigation was supported, financially and otherwise, by Engineering Foundation, the National Bureau of Standards, the American Society of Civil Engineers, U. S. Bureau of Reclamation, and all the individuals and organizations contributing to the Stevenson Creek Test Dam. The information presented in this report was collected through the combined efforts of an unusually large number of persons and organizations.

Members of the American Society of Civil Engineers and many other engineers in this and foreign countries supplied valuable information and suggestions. William Cain, M. Am. Soc. C. E., and G. F. Swain and A. N. Talbot, Past-Presidents, Am. Soc. C. E., acted at various times as counsellors to the Committee, and important communications and suggestions were received from Thaddeus Merriman, F. R. McMillan, and Duff A. Abrams, Members, Am. Soc. C. E., about concrete.

Generous support was given by George K. Burgess, Director of the Bureau of Standards, and members of the staff, notably Messrs. L. B. Tuckerman, O. S. Peters, and H. L. Whittemore. The Bureau of Standards also contributed the services of W. A. Slater, M. Am. Soc. C. E., during the whole period of construction and testing of the dam, as well as those of Mr. O. S. Peters at the site of the test in connection with the installation of the electrical strain-gauges.

The U. S. Bureau of Reclamation, Elwood Mead, M. Am. Soc. C. E., Director, and Raymond F. Walter, M. Am. Soc. C. E., Chief Engineer, undertook with the collaboration of the University of Colorado the extensive tests of models of arch dams. The Bureau and its engineers also made tests on some of its dams in service and under construction, besides contributing in other ways.

The U. S. Department of Agriculture granted the use of the Stevenson Creek Dam site, which is on a Government reservation.

The co-operation of the Southern California Edison Company and the availability of its personnel, equipment, and water supply, greatly facilitated the construction and test of Stevenson Creek Dam.

The members of the Stevenson Creek testing crew, the majority of whom were on duty throughout most of the construction and all the testing, were Messrs. G. H. Barry, R. W. Carlson, A. W. DeYoe, K. M. Fenwick, Inge Lyse, L. J. Marchand, J. J. Stout, and B. E. Wilson. All this crew, except Messrs. Fenwick and Wilson, participated in the preparation of Part IV of this report. All worked intelligently and loyally to make the test and the report successful. Special recognition is due Messrs. Carlson and Lyse for a high degree of initiative in the interpretation of the results, and to Mr. Lyse for his assistance in formulating the report.

The University of California, Stanford University, California Institute of Technology, University of Colorado, Princeton University, and Massachusetts

Institute of Technology in a spirit of co-operation have made available to the Committee their laboratory facilities. Members of the staffs of these institutions on numerous occasions have given freely of their time.

Information on arch and multiple-arch dams in this country and abroad received from numerous sources are acknowledged with thanks.

The laboratory tests at the University of California have been made possible through substantial grants from the research funds of the University, liberal contributions by the Portland Cement Association, and the active support of C. Derleth, Jr., M. Am. Soc. C. E., Dean of the College of Civil Engineering and Director of the Materials Testing Laboratory. G. E. Troxell, Assoc. M. Am. Soc. C. E., Assistant Professor of Civil Engineering, organized and supervised the testing of the concrete made from materials used in the Stevenson Creek Dam, and analyzed many of the test data. The special apparatus and equipment were designed and constructed by Val Arntzen, Head Mechanician of the Materials Testing Laboratory. Messrs. J. L. Banville, H. E. Davis, K. B. Wolfskill, R. P. Bronson, G. A. Sedgwick, and C. J. Woodbridge collected data and compiled results.

The Gates Rubber Company, of Denver, Colo., is co-operating with the Sub-Committee on Models in the manufacture of the rubber mercury bag, and in the investigation of hard rubber as a possible material for the construction of models.

Foreign Correspondents.—The interest of engineers in foreign countries in the Arch Dam Investigation was manifested by many communications, some containing valuable information on methods of design and results of testing of arch dams, among them: B. A. Smith, M. Am. Soc. C. E., R. A. Sutherland, Assoc. M. Am. Soc. C. E., Australia; George Beurle, Austria; P. Joye, Canada; P. Coufourier, France; N. Kelen, A. Ludin, F. Maier, E. Probst, Germany; S. Nishida, Japan; R. J. H. Hudson, India; F. Baratta, P. Bonetti, C. Bonomi, A. Danusso, G. Fantoli, G. Ganassini, C. Guidi, G. Ippolito, E. Kalman, L. Kambo, L. Luiggi, Hon. M. Am. Soc. C. E., U. Puppini, A. Toscani, Italy; F. Vogt, Norway; F. F. Gubin, U. S. S. R., Russia; B. Hellstrom, Sweden; H. E. Grüner, M. Am. Soc. C. E., K. E. Hilgard, M. Am. Soc. C. E., A. Rohn, A. Stucky, Switzerland.

2.—Contributors and Co-Operators

Allis-Chalmers Company
Aluminum Company of America
American Society of Civil Engineers
Bankers' Group:
- Blyth, Witter & Company
- Bond & Goodwin & Tucker
- Coffin & Burr, Inc.
- First Securities Company
- Harris, Forbes & Company, Inc., Boston, Mass.
- Harris, Forbes & Company, Inc., New York, N. Y.
- Harris Trust & Savings Bank
- Mercantile Trust Company
- National City Company
- Rollins, E. H. & Sons
- Security Trust & Savings Bank
- Staats, Wm. R. & Company

Bent Brothers
Blaw-Knox Company
Burns & McDonnell Engineering Company
Byllesby Engineering & Management Corporation

Cain, William
California Institute of Technology
California-Oregon Power Company
California Portland Cement Company
California, University of
Colorado, University of
Cooper, Hugh L. & Company
Crane, A. S.
Creager, W. P.
Davis, Arthur P.
Electric Bond and Share Company:
 Carolina Power & Light Company
 Idaho Power Company
 Minnesota Power & Light Company
 Pacific Power & Light Company
 Pennsylvania Power & Light Company
 Utah Power & Light Company
Electro - Metallurgical Company (Union Carbide Company)
Engineering Foundation
Flinn, Alfred D.
Foundation Company
Gannett, Seelye & Fleming
General Electric Company
Giant Powder Company
Great Western Power Company
Halsey, Stuart & Company, Inc.
Ingersoll-Rand Company
Insley Manufacturing Company
International Paper Company
Kellogg, M. W., Company
Lietz, The A., Company
Los Angeles, County of
Massachusetts Institute of Technology
Middle West Utilities Company
Monolith Portland Cement Company
National Bureau of Standards
Noetzli, Fred A.
Oregon, State of
Pacific Coast Steel Company
Pacific Gas & Electric Company
Parsons, H. deB.
Pelton Water Wheel Company
Portland Cement Association
Portland Electrical Power Company (Portland Railway, Light & Power Company)
Power Division, American Society of Civil Engineers
Princeton University
Rensselaer Valve Company
Riverside Portland Cement Company
Robinson, Dwight P. & Company, Inc.
Roebling's Sons Company, John A.
Sanderson & Porter
San Joaquin Light & Power Corporation
Southern California Edison Company
Southern Pacific Railway
Stanford University
Stone & Webster, Inc.
Sugar Pine Lumber Company
Swain, George F.
Talbot, Arthur N.
U. G. I. Contracting Company
U. S. Bureau of Reclamation
U. S. Coast and Geodetic Survey
U. S. Department of Agriculture
U. S. Indian Service
Utica Gas & Electric Company
Walbridge, H. D., & Company
Washington Water Power Company
Water Works Supply Company
Western Pipe & Steel Company
Westinghouse Electric & Manufacturing Company
Wheeler, William
White, J. G., Engineering Corporation

3.—Committee on Arch Dam Investigation

Members

Professor Charles D. Marx, *Chairman,* Stanford University, California.

Professor C. Derleth, Jr., University of California, Berkeley, Calif., *Chairman,* until December, 1925.

Alternate: Professor R. E. Davis, University of California, Berkeley, Calif.

FRED A. NOETZLI, *Secretary,* Consulting Engineer, 928 Central Building, Los Angeles, Calif.

JOHN L. SAVAGE, Designing Engineer, U. S. Bureau of Reclamation, Denver, Colo.

Alternate: IVAN E. HOUK, Engineer, U. S. Bureau of Reclamation, Denver, Colo.

F. E. WEYMOUTH, formerly Chief Engineer, U. S. Bureau of Reclamation, now Chief Engineer, J. G. White Engineering Corporation, on Irrigation Works, Government of Mexico, City of Mexico, Mexico.

Alternate: JULIAN HINDS, formerly Research Engineer, U. S. Bureau of Reclamation, now with J. G. White Engineering Corporation, Pabéllon, Aguascalientes, Mexico.

PAUL BAILEY, Chief Engineer, Orange County Flood Control District, formerly State Engineer of California, Sacramento, Calif.

M. M. O'SHAUGNESSY, City Engineer, San Francisco, Calif.

Alternate: R. P. MCINTOSH, Designing Engineer, San Francisco, Calif. (Died January 26, 1928.)

H. HOBART PORTER, Sanderson & Porter, New York, N. Y.

Alternate: WYNNE MEREDITH, of Sanderson & Porter, San Francisco, Calif.

SILAS H. WOODARD, Consulting Engineer, New York, N. Y.

H. HAWGOOD, Consulting Engineer, Los Angeles, Calif.

D. C. HENNY, Consulting Engineer, Portland, Ore.

H. W. DENNIS, Chief Civil Engineer, Southern California Edison Company, Los Angeles, Calif.

ALFRED D. FLINN, Director, Engineering Foundation (formerly Deputy Chief Engineer, Catskill Aqueduct), New York, N. Y.

W. F. MCCLURE, formerly State Engineer of California. (Died June 22, 1926.)

Sub-Committee on Instruments and Program: F. E. WEYMOUTH, *Chairman,* R. E. Davis, C. B. Wing, Fred A. Noetzli.

Sub-Committee on Test Dam: H. HAWGOOD, *Chairman,* 722 H. W. Hellman Building, Los Angeles, Calif., G. S. Binckley, H. W. Dennis, L. C. Hill, Fred A. Noetzli, J. W. Reagan.

Special Sub-Committee on Instruments: G. S. BINCKLEY, 7831 Hillside Avenue, Los Angeles, Calif.

Trustee of Fund for Test Dam Construction and Experiments: W. A. BRACKENRIDGE, Senior Vice-President, Southern California Edison Company, Los Angeles, Calif.

In Charge of Construction of Test Dam: H. W. DENNIS, Los Angeles, Calif.

In Charge of Tests at Dam and Interpretation of Results: W. A. SLATER, Chief of Section of Masonry Construction, U. S. Bureau of Standards, Washington, D. C.

Sub-Committee on Models: J. L. SAVAGE, *Chairman,* 1441 Welton Street, Denver, Colo.; George E. Beggs, Raymond E. Davis, F. R. Dungan, C. L. Eckel, H. J. Gilkey, D. C. Henny, Julian Hinds, Ivan E. Houk, Fred A. Noetzli, and W. A. Slater.

Sub-Committee on Dam No. 6, Southern California Edison Company: H. HAWGOOD, *Chairman;* H. W. Dennis, George S. Binckley.

Sub-Committee on Emigrant Creek Dam: D. C. Henny, *Chairman;* F. C. Dillard, H. M. Chadwick, O. Arnspiger.

Sub-Committee on Clear Creek Dam: D. C. Henny, *Chairman;* J. L. Lytle, Chauncy Wernecke.

Sub-Committee on Gerber Dam: Julian Hinds, *Chairman;* J. L. Savage, *Alternate;* H. D. Newell, E. L. Stevens, D. C. Henny.

Sub-Committee on Lake Eleanor Multiple Arch Dam: M. M. O'Shaughnessy, *Chairman.*

Sub-Committee on Gibson Dam: J. L. Savage, *Chairman.*

Sub-Committee on Bull Run Dam: D. C. Henny, *Chairman;* R. E. Davis, B. S. Morrow, Fred A. Noetzli, W. A. Slater, B. E. Torpen.

Sub-Committee on Coolidge Multiple-Dome Dam: Fred A. Noetzli, *Chairman,* C. R. Olberg, H. C. Neuffer, E. L. Rose.

Sub-Committee on [illegible] Creek Dam: D. C. Henny, Chairman; [illegible], H. M. Chadwick, O. Arnspiger.

Sub-Committee on Clear Creek Dam: D. C. Henny, Chairman; J. L. Lytle, Chauncey Wernecke.

Sub-Committee on Garber Dam: Julian Hinds, Chairman; J. L. Savage, Alternate; H. D. Newell, E. L. Stevens, D. C. Henny.

Sub-Committee on Lake Eleanor Multiple Arch Dam: M. M. O'Shaughnessy, Chairman.

Sub-Committee on Gibson Dam: J. L. Savage, Chairman.

Sub-Committee on Bull Run Dam: D. C. Henny, Chairman; L. H. Davis, R. S. Morrow, Fred A. Noetzli, W. A. Slater, B. E. Torpen.

Sub-Committee on Coolidge Multiple-Dome Dam: Fred A. Noetzli, Chairman; Carl Oberg, H. C. Nuttall, E. L. Stevens.

BIBLIOGRAPHY ON ARCH AND MULTIPLE-ARCH DAMS*

1.—Theoretical and General Discussions

AMERICAN ARCHED DAM WITH TANGENT ENDS, CANYON CREEK DAM, UTAH. (*The Engineer*, v. 131, January 28, 1921, pp. 88-90).

AMSTEG STAUMAUER. Trigonometrische Beobachtung der elastischen Deformationen der Staumauer am Pfaffensprung des Kraftwerkes Amsteg der S. B. B. Mitteilung der Section für Geodæsie der Eidg. Landestopographie, Bern. (*Schweizerische Bauzeitung*, v. 81, January 20, 1923, p. 21.)

ATCHERLEY, L. W. Some Disregarded Points in the Stability of Masonry Dams. (*Minutes of Proceedings*, Inst. C. E., v. 162, 1905, p. 456.)

BARATTA, F. Dighe ad archi multiplipmetodo approssimato per determinare le proporzioni piu economiche fra archi e pilastri. (Reprint from *Giornale* del Genio Civile, v. LVII, 1919.)

BARONI, M., GRANZOTTO U., KAMBO, L., MARZOLI U. La Diga del Gleno. Milan, Capriolo and Massimino, 1924

BARRAGES EN ARC. Contribution à l'étude des. (*Nouvelles Annales de la Construction*, Paris Livraisons de novembre et decembre 1924, janvier, février, mars, avril, et mai, 1925.)

BATICLE, E. Les barrages à voûtes multiples et la securité. (*Le Genie Civil*, v. 85, July, 12, 1924, pp. 36-37.)

BELLET, H. Barrages en maçonnerie et murs de réservoirs. Publications de la Houille Blanche, Grenoble, Gratier et Rey, 1907.

BERWALD, FR. R. OCH HELLSTROM, BO. Om temperaturvariationer och temperaturspænningar; betongkonstruktioner. A. B. Svenska Teknologforeningen Förlag, Stockholm, 1921.

BLIGH, W. G. Dams and Weirs. Chic., Am. Technical Soc., 1915.

BOEGH, C. Beitrag zur Berechnung der Spannungsverteilung in Stuetzpfeilern von Stauwehren aufgeloester Bauweise. (*Beton und Eisen*, v. 26, September 20, 1927, p. 331.)

BONETTI, P. Contributo allo studio delle dighe a gravita. Dighe a volte multiple ad asse verticale. (*Giornale* del Genio Civile, v. 61, 1923, p. 593.)

BONNET, M. F. Cours de barrages. Librairie d'enseignement technique, Paris, 1920.

BONOMI, C. Dighe e serbatoi construiti e in construzione a tutto il 1923. (Estratto dagli *Annali delle utilizzazioni delle Acque*, v. I, No. 1, 1924, p. 35.)

BRULE RIVER DAM. Tests of Water Pressures under. (*Engineering News-Record*, v. 96, February 18, 1926, p. 274.)

CAIN, WILLIAM. Stresses in Masonry Dams. (*Transactions*, Am. Soc. C. E., v. 64 (1909), p. 208.)

——The Circular Arch under Normal Loads. (*Transactions*, Am. Soc. C. E., v. 85 (1922), p. 233.)

——Discussion on "Stresses in Thick Arches of Dams." (*Transactions*, Am. Soc. C. E., v. 90 (1927), p. 522.)

CARTON, F. Calcul des barrages en arc; influence des variations de température. (*Le Genie Civil*, v. 85, October 25, 1924, p. 381.)

——Calcul des barrages en arc; voûtes circulaires soumises a des pressions hydrostatiques. (*Le Genie Civil*, v. 85, August 30, 1924, pp. 192-195.)

CAUFOURIER, P. Barrage à voûtes multiples, sur le Tirso, à San Chiara d'Ula (Sardaigne). (*Le Genie Civil*, v. 81, July 22, 1922, pp. 77–82.)

——Le calcul des barrages arqués. (*Le Genie Civil*, v. 81, August 19, 1922, pp. 182–183.)

——Les vannes automatiques à secteurs du barrage de San Chiara d'Ula, sur le Tirso (Sardaigne). (*Le Genie Civil*, v. 81, October 7, 1922, pp. 326–327.)

——Le barrage d'Essais de Stevenson Creek. (*Le Génie Civil*, v. 88 1926, p. 9.)

CHRISTIANS, G. W. Asphalt Grouting under Hales Bar Dam. (*Engineering News-Record*, v. 96, May 20, 1926, p. 798.)

CONCRETE ARCH DAM BUILT FROM SUPERPOSED TRESTLE. (*Engineering News-Record*, v. 84, Feburary 26, 1920, pp. 421–422.)

COTTEN, S. M. Foundation Pressures in Multiple-Arch Dams. (*Engineering News-Record*, v. 89, July 13, 1922, p. 78.)

CREAGER, W. P. Engineering for Masonry Dams. N. Y., Wiley, 1917.

——Hydraulic Structures, Masonry Dams, in "Reinforced Concrete and Masonry Structures," by G. A. Hool and W. S. Kinne. N. Y., McGraw-Hill, 1924.

CROSS, H. Temperature Deformations in Concrete Arches. (*Engineering News-Record*, v. 96, February 4, 1926, p. 190.)

DAVIS, A. P. Irrigation Works Constructed by the United States Government. N. Y., Wiley, 1917.

DAVIS, A. P. and WILSON. Irrigation Engineering. N. Y., Wiley, 1919.

* Compiled and arranged with the assistance of Engineering Societies Library.

DILLMAN, G. L. A Proposed New Type of Masonry Dam. (*Transactions*, Am. Soc. C. E., v. 49 (1902), p. 94.)

DUMAS, A. Etude théoretique et pratique sur barrages-réservoirs. (Barrage du Rio Grande.) (*Le Génie Civil*, v. 27, 1895, p. 237.)

EFFENBERGER, W. Ueber eine neue Berechnung der Spannungsverteilung in zylindrischen Behaetterwaenden (*Beton und Eisen*, Jahrgang 16, 1917.)

EGER. Talsperren von Mauerwerk und von Eisen. (*Zentralblatt der Bauverwaltung*, v. 18, 1898, pp. 525, 529.)

FAILURE OF DAM IN WALES DUE TO WASHOUT UNDER FOUNDATION. (*Engineering News-Record*, v. 96, January 7, 1926., p. 12.)

FANTOLI, G. Intorno ai problemi delle Dighe per serbatoi e del loro tipo nelle applicazioni italiane. (*L'Energia Elettrica*, v. I, No. 1, Milan, October, 1924, p. 3.)

FARIES, C. W. Some Valuable Precautions in the Design and Construction of Dams. (*Western Construction News*, v 2, No. 16, August 25, 1927, p. 38.)

FEDERHOFER, K. Graphisches Verfahren für die Ermittlung der Spannungsverteilung in zylindrischen Behaelterwaenden. (*Beton und Eisen*, v. 8, December 16, 1909, p. 387.)

FILLUNGER, P. Die Spannungsverteilung in keilfoermigen Koerpern, auf welche eine Einzelkraft einwirkt. (*Zeitschrift für Mathematik und Physik*, v. 59, 1910, p. 44.)

——Drei wichtige ebene Spannungszustaende des keilfoermigen Koerpers. (*Zeitschrift für Mathematik und Physik*, v. 60, 1912, p. 273.)

——Ueber die Anwendung des Trapezgesetzes zur statischen Berechnung von Talsperren. (*Oesterreichische Wochenschrift für den öffentlichen Baudienst*, No. 45, 1913.)

——Neuere Grundlagen für die statische Berechnung von Talsperren. (*Zeitschrift* des österreichischen Ingenieur und Architekten Vereins, v. 66, June 5, 1914, p. 441.)

FLINN, ALFRED D., WESTON, ROBERT S. and BOGERT, CLINTON L. Waterworks Handbook. Third Edition. N. Y., McGraw-Hill Book Co., 1927.

FORCHHEIMER, P. Die Berechnung ebener und gekruemmter Behaelterboeden. Berlin, Ernst, 1909.

FOWLER, F. H. A Graphic Method for Determining the Stresses in Circular Arches under Normal Loads by the Cain Formulas. (*Proceedings*, Am. Soc. C. E., October, 1927, Papers and Discussions, p. 1893.)

GANASSINI, G. and DANUSSO, A. Relazione peritale sopra le cause che hanno determinato la rovina della diga del pian di Gleno in Val di Scalve, crollata la mattina del 1 Dicembre 1923. (*Annali dei Lavori Pubblici*, v. 62, No. 5, Rome, 1924.)

GODFREY, E. Thin Arch Dams. (*Engineering and Contracting*, v. 65, November, 1926, p. 518.)

GOULD, C. A. Building the Lake Humphreys Concrete Arch Dam. (*Engineering News-Record*, v. 94, February 19, 1925, pp. 315–319.)

GRUNER, H. C. La construction des barrages-voûtes en Suisse; barrages de Broc, sur la Jogne, et d'Amsteg, sur la Reuss. (*Le Genie Civil*, v. 87, August 29, 1925, pp. 181–186.)

GUIDI, C. Statica delle dighe per laghi artificiali. Torino, Avalle, 1921.

——Dighe di sbarramento per laghi artificiali. (Estratto dagli *Atti*, Società Italiana per il Progresso delle Scienze, Riunione Napoli, Pavia, 1924.)

——Tensioni interne in una volta da ponte, prodotte della sua parziale ò totale sommersione in un liquido stagnante. (Estratto dagli *Atti* della Reale Accademia delle Scienze di Torino, v. 60, 1925.)

——Studi sperimentali zu costruzioni in cemento armato. (Estratto dal rendiconto dell' XI Riunione dell' Associazione Italiana per gli studi sui materiali da costruzione, Turin, Bona, 1926.)

——Esperienze termiche su di una diga a volta. (*Atti* della Reale Accademia delle Scienze di Torino, v. 62, 1927.)

HARRIS, E. G. A Study on Reinforced Concrete Dams. (*Technical Series Bulletin*, v. 8, No. 3, May, 1925, School of Mines and Metallurgy, Univ. of Missouri.)

HARRISON, C. L., and WOODARD, S. H. Lake Cheesman Dam and Reservoir. (*Transactions*, Am. Soc. C. E., v. LIII (1904), p. 89.)

HELLSTROM, BO. Valvdamm av armerad betong vid Gideäbacka. *Teknisk Tidskrift*, Väg-och Vatten, v. 49, 1919, p. 25.)

——Serievalvdamm vid Melby. (*Teknisk Tidskrift*, Väg-och Vatten, v. 53, 1923, p. 13.)

——Recent Swedish Arch Dam Design. (Section B, Sweden, The First World Power Conference, July 4, London, 1924, v. 2, p. 667.)

HILLBERG, A. G. Hydraulic Structures, Dams. In Concrete Engineers' Handbook by Hool and Johnson. New York, McGraw-Hill, 1918.

IPPOLITO, G. Sul Calcolo delle dighe ad arco. Milano, Hoepli, 1923.

JAKOBSEN, B. F. Volume Relation of Constant Angle Arch Dams and Gravity Dams. (*Engineering and Contracting*, v. 54, December 8, 1920, p. 554.)

——To Investigate Arch Dams. (*Engineering News-Record*, v. 91, November 8, 1923, p. 755.)

——Stresses in Multiple-Arch Dams. (*Transactions* Am. Soc. C. E., v. 87 (1924), p. 276.)

——Arching not Effective in Slightly Arched Gravity Dams. (*Engineering News-Record*, v. 92, June 12, 1924, pp. 1020–1023; Discussion, v. 93, July 31, 1924, p. 193.)

——Stresses in Thick Arches of Dams. (*Transactions*, Am. Soc. C. E., v. 90 (1926), p. 475.)

JOHNSTONE, TAYLOR F. Arched Concrete Dam for Tasmanian Barrage. (*Concrete*, v. 24, June, 1924, p. 247–248.)

JORGENSEN, L. R. The Constant-Angle Arch Dam. (*Transactions*, Am. Soc. C. E., v. 78 (1915), p. 685.)

——Construction Features of a Multiple-Arch Dam. (*Journal of Electricity*, v. 41, December 1, 1918, p. 506.)

——Improving Arch Action in Arch Dams. (*Transactions*, Am. Soc. C. E., v. 83 (1919–20), p. 316.)

JOYE, P., and CHRISTEN A. Recherches sur les variations et la répartition de la temperature dans le barrage de Montsalvens. (Extrait du *Bulletin Technique de la Suisse Romande*, Lausanne, 1922.)

JUILLARD, H. Influence de l'encastrement latéral dans les grands barrages. (*Schweizerische Bauzeitung*, v. 78, December 3, 1921, p. 271.)

——Calcul des barrages arqués. (*Schweizerische Bauzeitung*, v. 81, January 13, 1923, p. 11.)

KAMBO, L. Laghi artificiali e dighe ad archi multipli con particolare riflesso al servatoio in construzione sul fiume Tirso in Sardegna. (*Annali* del Consiglio Superiore delle Acque, v. 3, No. 1, 1921.)

——La diga di S. Chiara d'Ula sur Tirso in Sardegna. (*Annali* del Consiglio Superiore delle Acque, v. 4, No. 1, 1922.)

——Confronto tra le dighe a gravità e le dighe ad archi multipli. (*Energia Elettrica*, v. 2, June, 1925, p. 517.)

——Il circolo di equilibrio applicato al calcolo dei muri di ritenuta. (*Annali* della Società degli Ingegneri e degli Architetti Italiani.)

KELEN, N. Grundriss-anordnung der Gewoelbereihendaemme. (*Deutsche Wasserwirtschaft*, No. 10, 1924.)

——Die Spannungsverhaeltnisse in Staumauern. (*Beton und Eisen* v. 24, 1925, p. 287.)

——Die Staumauern. Berlin, Springer, 1926.

KIENAST-CURTI, W. Ueber ein Verfahren zur Dimensionierung von Gewicht-Staumauern. (*Schweizerische Bauzeitung*, v. 83, May 17, 1924, p. 229.)

KREUTER, F. Beitrag zur Berechnung und Ausfuehrung der Staumauren. Muenchen, Oldenbourg, 1917.

——Beitrag zur Berechnung und Ausfuehrung der Staumauern. (*Die Wasserkraft*, 1925, Nos. 14, 15, und 16.)

LEVY, M. Sur l'équilibre élastique d'un barrage en maçonnerie à section triangulaire. (*Comptes rendus des séances* de l'Academie des Sciences de Paris, v. 127, 1898, p. 10; also, *Annales des Ponts et Chaussées*, 1897.)

LEWE, V. Die statische Berechnung der Flüssigkeitsbehälter. (Handbuch für Eisenbetonbau. Edition 3, v. 5. Berlin, Ernst, 1923.)

——Zylindrische Wasserbehälter mit Zwischenwänden und ringfoermige Abschlusswände (Bogentalsperren). (*Beton und Eisen*, v. 22, 1923, p. 7.)

LINK, E. Die Bestimmung der Querschnitte von Staumauern und Wehren aus dreieckigen Grundformen. Berlin, Springer, 1910.

LIPPINCOTT, J. B. Large Multiple-Arch Dam Being Built in Italy. (*Engineering News-Record*, v. 95, October 29, 1925, pp. 710–711.)

LONGEST MULTIPLE-ARCH DAM IN THE WORLD IS COMPLETED BY SOUTHERN CALIFORNIA EDISON COMPANY. (*Journal of Electricity*, v. 57, September 1, 1926, p. 182.)

LOSSIER, H. Le Calcul des Réservoirs Circulaires. *Le Génie Civil*, v. 57, 1910, p. 247.)

LUDIN, A. Der Einsturz der Gleno-talsperre. (*Deutsche Wasserwirtschaft*, v. 19, No. 2, February 20, 1924.)

——Neuere Fortschritte in Konstruktion und Bauausführung der Staumauern. (*Deutsche Wasserwirtschaft*, v. 19, No. 9, September 20, 1924.)

——Zum Einsturz der Gleno-talsperre. (*Deutsche Wasserwirtschaft*, v. 20, No. 8, August 20, 1925.)

——Modellversuche für das Shannonwehr bei Parteen Villa. (*Der Bauingenieur*, v. 8, 1927. p. 730.)

MACPHAIL, J. M. Tensile Reinforcement in Concrete Dams. (*Engineering and Contracting*, v. 62, July, 1924, p. 88.)

MAIER, F. Die Staumauer des Kraftwerkes Voehrenbach. (*Beton und Eisen*, v. 23, 1924, pp. 13, 25.)

——Staumauern in Eisenbeton. Beton-Kalender, Berlin, 1924.

MARQUARDT, E. Staudaemme und Talsperren. (Handbuch für Eisenbetonbau, v. 4, Wasserbau. Edition 3. Berlin, Ernst, 1926.)

MEISSNER, E. Beanspruchung und Formänderung zylindrischer Gefässe mit linear veränderlicher Wandstärke. (*Vierteljahrsschrift* der naturforshenden Gesellschaft in Zürich, Jahrgang 62, 1917, pp. 153–168.)

MERKLE, G. Wasserdurchässigkeit von Beton in Abhängigkeit von seinem Aufbau und vom Druckgefaelle. Berlin, Springer, 1927.

MEYER-PETER, E. Die technische Entwicklung der hydro-elektrischen Anlagen in der Schweiz in der Darstellung durch die E. T. H. an der Internationalen Ausstellung in Basel 1926. III. Die Entwicklung der baulichen Bestandteile der hydro-elektrischen Werke. (*Schweizerische Bauzeitung*, v. 89, February 26, 1927, p. 107.)

MILLARD, B. T. Setting Form Points on Constant-Angle Arch Dams. (*Engineering News-Record,* v. 98, April 14, 1927, pp. 618–619.)

MITCHELL, J. H. On Some Elementary Distributions of Stress in Three Dimensions. (*Proceedings,* London Mathematical Soc., v. 32, 1900.)

MOERSCH, E. Berechnung kreisförmiger Gewölbe gegen Wasserdruck. (*Schweizerische Bauzeitung,* v. 51, May 2, 1908.)

MOHR, O. Der Spannungszustand einer Staumauer. (*Zeitschrift* des Oesterreichischen Ingenieur und Architekten Verein, v. 60, 1908, pp. 641, 661.)

——Abhandlungen aus dem Gebiete der technischen Mechanik. Second Edition. Berlin, Ernst, 1914.

MORRISON, C. E., and BRODIE, O. L. Masonry Dam Design. Second Edition. New York, Wiley, 1916.

MULTIPLE-ARCH DAM. (*Scientific American,* v. 136, April 27, 1927, pp. 271–272.)

NOETZLI, FRED A. The Reinforced Concrete Arch Dam. (*Journal of Electricity,* v. 45, July 15, 1920, p. 79.)

——Gravity and Arch Action in Curved Dams. (*Transactions,* Am. Soc. C. E., v. LXXXIV (1921), p. 1.)

——Arch Action in Curved Dams. (*Engineering and Contracting,* v. 55, April 13, 1921, p. 372.)

——The Relation Between Deflections and Stresses in Arch Dams. (*Transactions,* Am. Soc. C. E., v. LXXXV (1922), p. 284.)

——Possible Full-Size Dam Tests at Boulder Canyon. (*Engineering News-Record,* v. 88, April 13, 1922, p. 625.)

——Foundation Pressures in Multiple-Arch Dams. (*Engineering News-Record,* v. 88, June 15, 1922, p. 1009.)

——Durchbiegungen und Spannungen in Gewölbe-Staumauern. (*Schweizerische Bauzeitung,* v. 80, August 5, 1922, p. 57.)

——Arch Dam Temperature Changes and Deflection Measurements. (*Engineering News-Record,* v. 89, November 30, 1922, p. 930.)

——Deflection Measurements on Arch Dam in Switzerland. (*Engineering and Contracting,* v. 60, September, 1923, p. 530.)

——Improved Type of Multiple-Arch Dam. (*Transactions,* Am. Soc. C. E., v. LXXXVII (1924), p. 342.)

——The Experimental Arch Dam on Stevenson Creek, California. (*Engineering* (Lond.), v. 122, November 5, 1926, p. 562.)

——Tests on Stevenson Creek Dam. (*Engineering News-Record,* v. 99, July 14, 1927, p. 66.)

OTTLEY, Sir JOHN W., and BRIGHTMORE, A. On Stresses in Masonry Dams. (*Minutes of Proceedings,* Inst. C. E., v. 172, 1907–08, p. 89.)

PANETTI, M. Studio statico dei serbatoi cilindrici in ferro et in cemento armato. (*Giornale* del Genio Civile, v. 44, 1906, p. 117.)

PARSONS, H. de B. Multiple-Arch Dam at Sherman Island on the Hudson River. (*Engineering and Contracting,* v. 91, No. 12, September 20, 1923.)

PASTERNAK, P. Formeln zur raschen Berechnung der Biegebeanspruchung in kreisrunden Behältern. (*Schweizerische Bauzeitung,* v. 86, September 12, 1925, p. 129.)

——Die praktische Berechnung biegefester Kugelschalen kreisrunder Fundamentplatten auf elastischer Bettung und kreiszylindrischen Wandungen in gegenseitiger monolither Verbindung. (*Zeitschrift für angewandte Mathematik und Mechanik,* v. 6, 1926, p. 1.)

PAUL, C. H., and MAYHEW, A. B. Temperature Changes in Mass Concrete. (*Transactions,* Am. Soc. C. E., v. LXXIX (1915), p. 1225.)

PEARCE, C. E. The Analysis of Circular Arch Dams by Cain's Method, Including Shear. (*Western Construction News,* v. 2, No. 17, September 9, 1927, p. 35.)

PERKINS, W. A. Analysis of Arch Dams. (*Western Construction News,* v. 2, No. 7, April 10, 1927, p. 43.)

PLATZMANN, F. Ueber den Querschnitt der Staumauern. Leipzig, Engelmann, 1908, p. 24.

POESCHL, TH. Die Berechnung der Spannungen in zylindrischen Behälterwänden mit veränderlichem Querschnitt. (*Sitzungsberichte* der Kaiserl. Akademie der Wissenschaften in Wien, June, 1912.)

——und **TERZAGHI, K.** Berechnung von Behältern nach neueren analytischen und graphischen Methoden. Berlin, Springer, 1926.

POSTUVANSCHITZ, F. Rechnerische Bestimmung und Auswertung der Elastizitätsellipse in ihrer Anwendung auf die Bogenträger. Leipzig, Engelmann, 1910.

PRIEST, R. M. Repairing Laguna Dam. (*Engineering and Contracting,* v. 63, January, 1925, p. 119.)

PUPPINI, U. Variazioni di temperatura entro le dighe a volta. (*Il Monitore Technico,* v. 28, No. 32, 1922.)

REISSNER, H. Ueber die Spannungsverteilung in zylindrischen Behaelterwaenden. (*Beton und Eisen,* v. 7, 1908, p. 150.)

RESAL, M. Formes et dimensions des grands barrages en maçonnerie. (*Annales des Ponts et Chaussées,* I, March–April, 1919, p. 165.)

RITTER, H. Die Berechnung von bogenförmigen Staumauern. Karlsruhe, Lang, 1913.

ROHN, A. Beitrag zur Berechnung massiver Staumauern. (*Schweizerische Bauzeitung*, v. 79, March 11, 1922, p. 126.)

——Zur Berechnung der Gewoelbe-Staumauern. (*Schweizerische Bauzeitung*, v. 87, January 2, 1926.)

RUNGE, C. Ueber die Formänderung eines zylindrischen Wasserbehälters durch den Wasserdruck. (*Zeitschrift für Mathematik und Physik*, v. 51, 1904, p. 254.)

SAMSIOE, A. F. Measures Taken in Sweden Against Ice Troubles at Water-Power Plants. (Section B, Sweden, The First World Power Conference, London, v. 2, 1924, p. 806.)

SHAW, P. A. Arch Dam at Marlboro, New Hampshire. (*Journal*, Boston Soc. Civ. Engrs., v. 14, June 27, 1927, pp. 377–379.)

SLATER, W. A. Interesting Features of the Construction and Testing of the Stevenson Creek Arch Dam. (*Modern Irrigation*, v. 3, No. 6, June, 1927, p. 22.)

SMITH, B. A. Arched Dams. (*Transactions*, Am. Soc. C. E., v. LXXXIII (1919-20), p. 2027.)

——Experimental Deformation of a Cylindrical Arched Dam. (*Transactions*, Am. Soc. C. E., v. 91 (1927), p. 705.)

STUCKY, A. Etude sur les barrages arqués. (*Bulletin technique de la Suisse Romande*, Lausanne, v. 48, 1922, pp. 1, 25, 49, 85, 97.)

——Der Talsperrenbruch im Val Gleno. (*Schweizerische Bauzeitung*, v. 83, February 9, 1924, p. 74.)

STUDER, H. Das Kraftwerk Amsteg der S. B. B. (*Schweizerische Bauzeitung*, v. 86, 1925, November 14 and 21, 1925, pp. 229, 241, 256, 285, 299, 313, 325.)

SUORVA LAKES DAMS, LAPLAND, The. (*Water-Works*, v. 61, February, 1924, p. 361.)

TEMPERATURE CHANGES ON MASONRY, The Effect of. (*Transactions*, Am. Soc. C. E., v. LXXXIII (1919-20), p. 590.)

THUERNAU. Die Bewegung der Temperatur in der Sperrmauer der Waldecker Talsperre. (*Deutsche Wasserwirtschaft*, v. 19, 1924.)

TIMOSHENKO, S. Calcul des arcs élastiques. Paris, 1922.

——On the Distribution of Stresses in a Circular Ring Compressed by Two Forces Acting Along a Diameter. (*Philosophical Magazine*, ser. 6, v. 44, November, 1922, p. 1014.)

TOSCANI, A. Impressioni di un costruttore dalla visita ad opere e lavori specialmente per l'utilizzazione delle acque negli Stati Uniti d'America e nel Messico. (*L'Energia Elettrica*, v. 2, October, 1925, p. 1008.)

VISCHER, H. and WAGONER, L. On the Strains in Curved Masonry Dams. (*Transactions*, Technical Soc. of the Pacific Coast. v. 6, December, 1889, p. 75.)

VOGT, F. Ueber die Berechnung der Fundamentdeformation. (Det Norske Videnskaps Akademi, Oslo, Mathematiske Natürvidenskapelige Klasse, 1925, No. 2.)

WADE, L. A. B. Concrete and Masonry Dam Construction in New South Wales. (*Minutes of Proceedings*, Inst. C. E. v. 178, 1908-09, p. 1.)

——Curved Masonry Dams in New South Wales. (*Engineering News*, v. 63, May 19, 1910, p. 588.)

WEGMANN, E. The Design and Construction of Dams. Eighth Edition. N. Y. Wiley, 1927.

WILSON, S. and GORE, W. Stresses in Dams. (*Minutes of Proceedings*, Inst. C. E., v. 172, 1907-08, p. 107.)

WOLF, K. Zur Integration der Gleichung $\Delta \Delta F = 0$ durch Polynome im Fall des Staumauerproblems. (*Sitzungsberichte* der Kaiserl. Akademie der Wissenschaften in Wien, 1914, v. 123, No. 2.)

WOODARD, S. H. Dams, Aqueducts, Canals, Shafts, Tunnels. In American Civil Engineers' Handbook. Edition 4. N. Y., Wiley, 1920.

ZIEGLER, P. Ueber ausgeführte Gewoelbe-Talsperren in New Suedwales. Berechnungen, Rissbildungen, Druchbiegung-messungen. (*Beton und Eisen*, v. 11, 1912, pp. 14, 44.)

——Der Talsperrenbau, v. I, II, III. Third Edition. Berlin, Ernst, 1925.

2.—Descriptions of Dams

AGNEW LAKE DAM.
Multiple-Arch Dams on Rush Creek, California. L. R. Jorgensen. (*Transactions*, Am. Soc. C. E., v. LXXXI (1917), p. 850.)

AMSTEG DAM.
Deformations of Arch Dam Measured by Triangulation. (*Engineering News-Record*, v. 91, July 12, 1923, p. 63.)

Das Kraftwerk Amsteg der Schweizerischen Bundesbahnen. H. Studer. (*Schweizerische Bauzeitung*, v. 86, November 7, 1925, pp. 229, 241, 256, 285, 299, 313, 325.)

ANYOX DAM.
British Columbia Dams. E. Davis and E. G. Marriott. (*Water-Works*, v. 65, February, 1926, p. 60.)

BAKER RIVER DAM.
Annual Salmon Run Taken Over 265-Ft. Baker River Dam. (*Engineering News-Record*, v. 97, October 21, 1926, p. 652.)

BLUEWATER CANYON DAM.
Bluewater-Toltec Project Presents Striking Example of Arch Type Dam. V. L. Sullivan. (*Modern Irrigation*, v. 3, No. 6, June, 1927, p. 32.)

BOWMAN SOUTH DAM.
Nevada Irrigation District, California. (*Western Construction News*, v. 1, October 10, 1926, p. 88.)

BUCKS CREEK DAM.
Bucks Creek Dam. C. W. Faries. (*Western Construction News*, v. 1, No. 4, February 25, 1926, p. 28.)

BULLARDS BAR DAM.
Bullards Bar Dam. C. W. Faries. (*Western Construction News*, v. 2, No. 8, April 25, 1927, p. 39.)

BRULE RIVER DAM.
Tests of Water Pressures under Brule River Dam. (*Engineering News-Record*, v. 96, February 18, 1926, p. 274.)

CAMPBELL'S DAM.
Deflections and Stresses in Arch Dams. F. A. Noetzli; Discussion by G. S. Williams. (*Transactions*, Am. Soc. C. E., v. LXXXV (1922), p. 345.)

COOLIDGE DAM.
Multiple-Dome Design Adopted for Coolidge Dam. (*Engineering News-Record*, v. 96, May 27, 1926, p. 865.)
The Coolidge Multiple-Dome Dam. C. R. Olberg. (*Western Construction News*, v. 1, December 25, 1926, p. 33.)
The Coolidge Multiple-Dome Dam. (*Western Construction News*, v. 1, July 10, 1926, p. 34.)
Multiple-Dome Staumauer-Typ. (*Schweizerische Bauzeitung*, v. 88, October 16, 1926, p. 227.)
Die projectierte Coolidge-Staumauer. N. Kelen. (*Der Bauingenieur*, v. 8, May 28, 1927, p. 385.)

CUSHMAN DAM.
Cushman Project Provides Large Storage at Low Cost. B. E. Torpen. (*Engineering News-Record*, v. 98, April 14, 1927, p. 608.)

DALTON DAM.
Dalton Multiple Arch Dam. (*Western Construction News*, v. 2, No. 20, October, 1927, p. 44.)

EAST CANYON CREEK DAM.
The East Canyon Creek Dam. A. F. Parker. (*Transactions*, Am. Soc. C. E., v. LXXXIII (1921), p. 574.)

FULLY DAM.
Die Wasserkraftanlage Fully. H. Chenaud and L. Dubois. (*Schweizerische Bauzeitung*, v. 80, November 25, 1922, p. 247.)

FLORENCE LAKE DAM.
Florence Lake Multiple-Arch Dam. (*Western Construction News*, v. 2, No. 14, July 25, 1927, p. 36.)

GEM LAKE DAM.
Multiple-Arch Dams on Rush Creek, California. L. R. Jorgensen. (*Transactions*, Am. Soc. C. E., v. LXXXI (1917), p. 850.)
Multiple-Arch Concrete Dams Exposed to Frost. S. W. Stewart. (*Engineering News-Record*, v. 95, September 3, 1925, p. 396.)
Multiple-Arch Dam Disintegrates Under Low Temperatures. (*Engineering News-Record*, v. 95, July 2, 1925, p. 22.)
Multiple-Arch Dam at Gem Lake on Rush Creek, California. F. O. Dolson and W. L. Huber. (*Transactions*, Am. Soc. C. E., v. 89 (1926), p. 713.)
Was Gem Lake Disintegration Due to Underburned Cement? L. R. Jorgensen. (*Engineering News-Record*, v. 95, October 8, 1925, p. 646.)
Sheffield and Gem Lake Failures. M. M. O'Shaughnessy. (*Engineering News-Record*, v. 95, July 30, 1925, p. 194.)

GERBER DAM.
Concrete Arch Dam with Central Spillway Built on Klamath Project. (*Engineering News-Record*, v. 96, February 25, 1926, p. 331.)
Gerber Dam, Klamath Project, Oregon. Details of Design and Construction. A. L. Darr. (*Western Construction News*, v. 1, No. 5, March 10, 1926, p. 29.)

GIBSON DAM.
The Gibson Dam. (*Western Construction News*, v. 1, No. 16, August 25, 1926, p. 43.)
Concrete to be Poured Soon for Gibson Dam, Sun River Project's Arch-Type Structure. B. W. Steele. (*Modern Irrigation*, v. 3, No. 6, June, 1927, p. 38.)

GLENO DAM.
Official Report on Collapse of Gleno Dam. (*Engineering News-Record*, v. 93, August 7, 1924, p. 213.)
Pros and Cons of the Gleno Dam Failure. (*Water Works*, v. 63, June, 1925, p. 1275.)
Der Talsperrenbruch im Val Gleno. A. Stucky. (*Schweizerische Bauzeitung*, v. 83, February 9, 1924, p. 74.)
Failure of the Gleno Dam in Italy. G. Rodio. (*Water Works*, v. 61, February, 1924, p. 375.)
Zum Talsperrenbruch im Val Gleno. (*Schweizerische Bauzeitung*, v. 83, June 21, 1924, p. 295.)
Zwei Expertenberichte über die Ursachen des Einsturzes der Gleno-Staumauer in Oberitalien. (*Schweizerische Bauzeitung*, v. 85, May 30, 1925, p. 279.)

HALLIGAN DAM.
The Halligan Dam: A Reinforced Masonry Structure. G. N. Houston. (*Transactions*, Am. Soc. C. E., v. LXXV (1912), p. 112.)

HARTEBEESTPOORT DAM.
Hartebeestpoort Dam. (*Water-Works*, v. 61, February, 1924, p. 340.)
Construction Features of Hartebeestpoort Dam of South African Irrigation Department. (*Engineering and Contracting*, v. 57, February 8, 1922, p. 125.)

HUACAL DAM.
Huacal Dam, Sonora, Mexico. H. Hawgood. (*Transactions*, Am. Soc. C. E., v. LXXVIII (1915), p. 564.)

HUBBART, UPPER DAM.
Main Features of the Hubbart Dam. (*Water-Works*, v. 63, June, 1925, p. 1258.)

KERCKHOFF DAM.
Hydro-Electric Plant Built in 15 Months. R. C. Starr. (*Electrical World*, v. 77, February 26, 1921, p. 471.)

LANIER LAKE DAM.
Two Arch Dams Fail Through Undermining of Abutments. (*Engineering News-Record*, v. 97, October 14, 1926, p. 616.)

LOST CREEK DAM.
Lost Creek Dam, California. C. W. Faries. (*Western Construction News*, v. 2, No. 11, June 10, 1927, p. 42.)

MANITOU DAM.
Arch Dam Repaired by Fills Above and Below Structure. John E. Field. (*Engineering News-Record*, v. 95, December 10, 1925, p. 958.)

MELONES DAM.
Melones Dam Good Example of Present Day Less Than Gravity Type of Structure. F. P. Basler. (*Modern Irrigation*, v. 3, No. 6, June, 1927, p. 27.)
The Melones Dam on the Stanislaus River, California. E. C. Eaton. (*Western Construction News*, v. 2, No. 10, May 25, 1927, p. 34.)

MONTEJAQUE DAM.
High Arch Type Dam Built in Southern Spain. M. E. Wegenstein. (*Engineering News-Record*, v. 93, July 24, 1924, p. 128.)

MORMON FLAT DAM.
Arch Dam of Thin Section Now in Service on Salt River. (*Engineering News-Record*, v. 96, May 13, 1926, p. 778.)

MOYIE DAM.
Two Arch Dams Fail Through Undermining of Abutments. (*Engineering News-Record*, v. 97, October 14, 1926, p. 616.)

OAK GROVE DAM.
Oak Grove Power Plant of the Portland Electric Power Company. M. E. Reed. (*Western Construction News*, v. 1, No. 7, April 10, 1926, p. 20.)

PACOIMA DAM.
Construction Started on Highest Dam in World: The Pacoima Dam for the Los Angeles County Flood Control District. (*Western Construction News*, v. 1, No. 6, March 25, 1926, p. 39.)
Stresses in Thick Arches of Dams. B. F. Jakobsen. (*Transactions*, Am. Soc. C. E., v. 90 (1927), p. 475.)
The Los Angeles County Flood Control District. J. W. Reagan. (*Western Construction News*, v. 1, No. 16, August 25, 1926, p. 32.)

PALMDALE DAM.
Multiple-Arch Dam, California. E. C. Eaton. (*Western Construction News*, v. 1, No. 22, November 25, 1926, p. 22.)

PATHFINDER DAM.
Flood and Drought Control on the North Platte. (*Scientific American*, v. 126, May, 1922, p. 315.)
Third and Fourth Annual Reports of the Reclamation Service of the United States Geological Survey. (*Engineering News*, v. 60, October 29, 1908, p. 461; *Engineering Record*, v. 58, November 7, 1908, p. 508.)
Stresses in High Masonry Dams of Short Span. G. T. Wisner and E. T. Wheeler. (*Engineering News*, v. 54, August 10, 1905, p. 141.)

PLEASANT LAKE DAM.
Highest Multiple-Arch Dam in United States. C. E. Griggs. (*Western Construction News*, v. 1, No. 6, March 25, 1926, p. 23.)
Lake Pleasant Dam, near Phœnix, Will Be the Highest Multiple Arch Dam in the World. (*Modern Irrigation*, v. 3, No. 6, June, 1927, p. 30.)

SALMON CREEK DAM.
Improving Arch Action in Arch Dams. L. R. Jorgensen. (*Transactions*, Am. Soc. C. E., v. LXXXIII (1919–20), p. 316.)
Deflections and Stresses in Arch Dams. F. A. Noetzli. (*Transactions*, Am. Soc. C. E., v. LXXXV (1922), p. 296.)

SAWPIT DAM.
Sawpit Dam for Los Angeles County Flood Control District. (*Western Construction News*, v. 1, No. 24, December 25, 1926, p. 45.)

SHERMAN ISLAND DAM.
Sherman Island Dam and Power House. H. de B. Parsons. (*Transactions*, Am. Soc. C. E., v. 88 (1925), p. 1257.)

SHOSHONE DAM.
Closing 42-In. Outlet Pipes in Shoshone Dam Under 200 Feet of Water, with Wooden Balls. J. S. Longwell. (*Engineering and Contracting,* September 13, 1922, p. 257.)

SIX MILE CREEK (ITHACA) DAM.
Deflections and Stresses in Arch Dams. F. A. Noetzli. Discussion by G. S. Williams. (*Transactions,* Am. Soc. C. E., v. LXXXV (1922), p. 342.)

SPAULDING DAM.
(*Transactions,* Am. Soc. C. E., v. LXXVIII (1915), p. 710.)

STEVENSON CREEK DAM.
Stevenson Creek Experimental Arch Dam Withstands Flood Three Feet Deep Over Crest Without Injury. (*Western Construction News,* v. 1, No. 23, December 10, 1926, p. 40.)

Proposed Experimental Arch Dam on Stevenson Creek. (*Engineering News-Record,* v. 94, May 14, 1925, p. 816.)

Le barrage d'Essais de Stevenson Creek. P. Caufourier. (*Le Génie Civil,* v. 88, January 2, 1926, p. 9.)

SUTHERLAND DAM.
Sutherland Dam for San Diego Water Supply. (*Western Construction News,* v. 2, No. 1, January 10, 1927, p. 46.)

SWEETWATER DAM.
Sweetwater Dam Enlarged for the Third Time. H. N. Savage. (*Engineering News-Record,* v. 82, May 15, 1919, p. 948.)

TIRSO DAM.
Record Height Multiple-Arch Dam Under Way in Italy. I. Gutmann. (*Engineering News-Record,* v. 90, May 10, 1923, p. 820.)

Die Ausnuetzung der Wasserkraefte Sardiniens und die Tirsotalsperre. N. Kelen. (*Deutsche Wasserwirtschaft,* v. 19, No. 3, 1924.)

VELHAS, RIO DAS, DAM.
Das Wasserkraftwerk Rio de Pedras, Brazil. E. Kemnitz and Cia., Ltd. (*Beton und Eisen,* v. 26, February 5, 1927, p. 41.)

VOEHRENBACH DAM.
Das Kraftwerk Voehrenbach. Die erste Eisenbetonstaumauer in Deutschland. F. Maier und Kammueller. (*Der Bauingenieur,* v. 4, 1923, p. 110.)

WARM SPRINGS DAM.
Design, Details and Field Methods on Thin Arch Dam. (*Engineering News-Record,* v. 84, March 4, 1920, p. 474.)

APPENDIX

GENERAL INDEX OF STEVENSON CREEK TEST DATA AND COMPUTATIONS

(Filed in Office of Engineering Foundation, New York, N. Y.)

PROCEEDINGS

of the

American Society

of

Civil Engineers

AMERICAN SOCIETY OF CIVIL ENGINEERS FOUNDED 1852

THREE PARTS

PART 3

MAY, 1928

REPORT ON

ARCH DAM INVESTIGATION

VOL. 1.